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Semidefinite programming solution of economic dispatch problem with non-smooth, non-convex cost functions

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ABSTRACT

The paper presents a solution to economic dispatch (ED) problems with non-convex, non-smooth fuel cost functions, which characterize practical generating units. A method involving a unified semidefinite programming (SDP) formulation of different ED problems through cost function decomposition was presented. The solution of the resulting rank-relaxed SDP problem was refined to achieve the rank constraint using the method of convex iteration and branch-and-bound technique. The SDP method was investigated on some test problems in the literature. The results showed that the SDP method compared favorably with other methods, and can efficiently solve non-convex and non-smooth ED problems.

1. Introduction

The general economic dispatch (ED) problem involves optimum utilization of the capacities of committed generating units with the goal of minimizing the cost of satisfying total load demand and system constraints. Apart from various fixed costs, the cost of generation is tightly linked with fuel cost. Consequently, the problem is posed as an optimization of an objective function of fuel costs. A practical ED problem is characterized by non-convexity and non-differentiability in the objective and constraint functions. This is usually simplified as a convex quadratic fuel cost function which fails to capture various practical effects exhibited in the actual operation of power generating units. Solutions obtained in this situation often underestimates the optimum operating condition. In order to account for practical operating conditions, the objective function should incorporate effects such as multiple fuel options, valve point loading and combined cycled co-generation in the operation of power plants. Additionally, the constraints should account for prohibited operating zones (POZs), ramp-rate limits and non-convex equality forms. These effects complicate the ED problem and result in multiple local optimal solutions.

1.1. Related work

Several methods have been proposed to solve the ED problem including the lambda iteration [1], the base point and participation factor method [2], and gradient methods [3]. These methods have the drawback of slow convergence to a solution. The dynamic programming method proposed in [4] has the attraction of being indifferent to the shape of the cost function but suffers the "curse of dimensionality" [5]. Various heuristic and evolutionary approaches have also been considered to solve the ED problem, among which are the genetic algorithm (GA) [6], particle swarm optimization (PSO) [7,8], evolutionary programming (EP) [9] and group search optimizer [10]. These methods enjoy ease of implementation and indifference to the shape of the cost function. However, their stochastic nature does not allow a guaranteed global optimal solution. Multiple runs are required to capture a range of results which are reported statistically.

Another approach, which has gained increased popularity in solving various power system optimization problems, is semidefinite programming (SDP); a convex optimization method [11–19]. Most optimization problems in power systems have polynomial objective functions and constraints [20] and can be formulated as a semidefinite program with addition of non-convex constraints, most of the time, rank constraints. Since they are generally non-convex, finding a direct solution can be somewhat difficult. The semidefinite programming (SDP) relaxation method simplifies and allows such problems to be solved by convex optimization techniques [16], usually through the provision of an easily computable lower bound of the minimum value.

The relaxation techniques used by SDP has been very successful because the relaxed problem, under suitable rank conditions, is guaranteed to have a

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global optimum solution in polynomial time, which is not achieved by other methods [14]. Furthermore, while heuristic approaches depend on tuning several parameters to obtain desirable outcome, SDP does not have such a requirement [21]. Finally, the same optimal solution is achieved irrespective of the number of runs of the algorithm and this obviates the need to report results in statistical average terms.

1.2. Our contribution

Previous studies that use SDP to solve stand-alone ED problems (or their multi-objective formulations) have been limited to problems with convex and smooth fuel cost functions (e.g. [11,16,17]) and non-convex constraint set (e.g. [18]). Operational features of practical power plants that make the fuel cost objective function non-convex and non-differentiable were not considered. In this paper, non-convexities and non-differentiability in the objective cost function are addressed through a unified formulation that uses the decomposition method, and solved iteratively as convex sequences.

The paper is organized as follows. Section 2 discusses various practical features of generating units and how these affect the fuel cost objectives of the ED problem for such units. The decomposition of the problem induced by the non-differentiability and non-convexity in the cost objectives is considered, along with a unified representation of all such features, in Section 3. Section 4 contains an introduction of some theoretical frameworks in semidefinite programming such as semi-definite relaxation, handling of rank constraint by convex iteration and branch-and-bound technique for rank-constrained SDP. Section 5 shows the decomposition of the unified ED problem and the SDP relaxation of the resulting problem. In Section 6, the test systems considered for the evaluation of the proposed method and the SDP solvers used are presented. Section 7 presents the various results, and the comparative analysis of the result with those of other methods presented in the literature. Finally, we conclude in Section 8.

2. Non-convex economic dispatch models

The economic dispatch problem can be formulated as follows:

$$\text{minimize } C(P) = \sum_{i=1}^{p} C_i(P_i) \tag{1a}$$

$$\text{subject to: } \sum_{i=1}^{p} P_i = P_D + P_L(P), \qquad P = [P_1, \ldots, P_p]^T \tag{1b}$$

$$P_i^{\min} \le P_i \le P_i^{\max}, \qquad i = 1, \ldots, p \tag{1c}$$

where P, the decision variable, is the vector of power outputs, $C(\cdot)$ is the fuel cost objective, $P_i^{\min}$ and $P_i^{\max}$ are the respective minimum and maximum power generation limit of the i-th unit. The transmission loss, $P_L(P)$, can be calculated using the Kron's loss formula:

$$P_L(P) = \sum_{i=1}^{p}\sum_{j=1}^{p} P_i\ B_{ij}\ P_j + \sum_{i=1}^{p} B_{i01}\,P_i + B_{00}, \tag{2}$$

where B_{ij}, B_{i01} and B_{00} are B-coefficients.

In the conventional handling of the ED problem, the cost function is represented by quadratic polynomial functions [22]. The total fuel cost, $C(P)$, is expressed as

$$C(P) = \sum_{i=1}^{p} a_i P_i^2 + b_i P_i + c_i, \tag{3}$$

where P_i is the real power output of the i-th generating unit, and a_i, b_i, and c_i are the corresponding fuel cost coefficients.

In order to complete the practical model considered in this paper, three features of thermal power generation units that introduce non-convexity and non-differentiability into the objective cost function are now described.

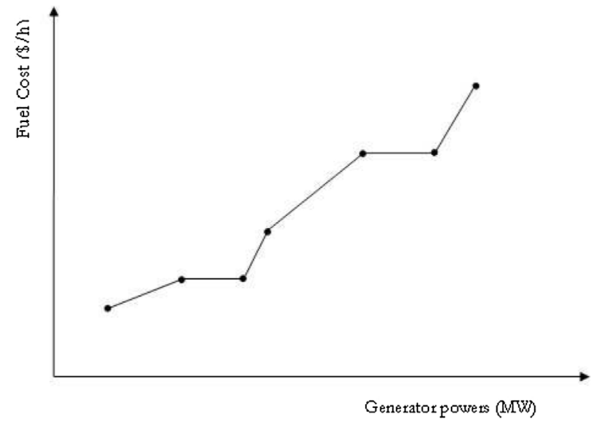

Fig. 1. Fuel cost characteristic for CCCP unit.

2.1. Co-generation plant fuel cost

A combined cycle co-generation plant (CCCP) consists of one or more gas and steam turbines interconnected to generate electric power. Different configurations of the gas and the steam turbines therefore lead to different cost functions over regions of the unit's range of operation. The fuel cost characteristic for the CCCP unit with q number of configurations (see Fig. 1) is non-smooth and non-differentiable [2] and is given as [23]:

$$C_i(P_i) = \begin{cases} b_{i1}P_i + c_{i1}, & P_i^{\min} \le P_i \le \chi_{i1} \quad \text{(a)} \\ b_{i2}P_i + c_{i2}, & \chi_{i1} \le P_i \le \chi_{i2} \quad \text{(b)} \\ \vdots & \\ b_{iq}P_i + c_{iq}, & \chi_{iq-1} \le P_i \le P_i^{\max} \quad \text{(c)} \end{cases} \tag{4}$$

where χ_{ij} is the power output of unit i when it switches from configuration j to $j+1$.

2.2. Multiple fuel option

Generation units may operate on multiple fuel sources and using a particular fuel for a range of power output may be economically advantageous. In this case the cost function is defined to reflect the possible mix of fuel choices by representing it as several piecewise convex quadratic functions. This makes the determination of the most economical fuel type complicated as the ED problem is now non-continuous [24]. The fuel cost function for q fuel types can be expressed as [25]:

$$C_i(P_i) = \begin{cases} a_{i1}P_i^2 + b_{i1}P_i + c_{i1}, & P_i^{\min} \le P_i \le \chi_{i1} \quad \text{(a)} \\ a_{i2}P_i^2 + b_{i2}P_i + c_{i2}, & \chi_{i1} \le P_i \le \chi_{i2} \quad \text{(b)} \\ \vdots & \\ a_{iq}P_i^2 + b_{iq}P_i + c_{iq}, & \chi_{iq-1} \le P_i \le P_i^{\max} \quad \text{(c)} \end{cases} \tag{5}$$

where P_i denotes the output of unit i which can be in one of the q regions defined by the fuel types (see Fig. 2). P_i^{min} and P_i^{max} define the respective lower and upper limits of power, P_i, generated by each of the units in operation.

2.3. Valve point loading effect

Large steam turbine generators have a number of steam admission valves that are opened in sequence to achieve increased output of the unit. Upon initial opening of a valve, the throttling losses increase rapidly and the unit's incremental cost rate rises suddenly [26]. The fuel cost function for a unit i with valve point loading is modelled by adding a recurring rectified sine component to the basic quadratic cost function (as shown in Fig. 3) [27]: