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Assessment of different formulations for the ground return parameters in modeling overhead lines

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ABSTRACT

There is a growing concern regarding a more accurate assessment of the transient behavior of overhead lines and underground cables. Traditionally, displacement currents are neglected in the evaluation of the ground return correction terms, thus only the transmission line impedance being affected. However, if these currents are to be considered, the ground cannot be assumed as an equipotential and there are some ambiguities depending on the definition used to determine the conductors' voltages. Furthermore, given the numerical challenges associated with the infinite integrals needed to obtain the ground return parameters, it is rather common to use closed-form approximations instead. In this paper, we assess the influence that the different formulations may have in modelling two distinct overhead lines. Both frequency and time-domain results are presented and discussed. The results indicated that there are some mismatches between induced voltages calculated using the voltage formulation with quasi-TEM approximation and those using the potential formulation with closed-form approximations.

1. Introduction

Originally, in overhead lines and underground cables modelling for transient studies, the determination of the ground return parameters was made assuming the soil as a good conductor [1–3]. However, there are several scenarios where one may need to include ground displacement currents such as very high frequency phenomena or the inclusion of frequency dependent ground parameters. A feature not commonly known is that the inclusion of ground displacements currents implies that the ground no longer can be assumed as an equipotential. This implies that there are different possible definitions for the overhead conductor voltages: the correct definition (here referred as voltage formulation), the definition where the magnetic vector potential is disregarded (here referred as potential difference formulation) and the definition where the zero potential occurs at a remote ground (here referred as potential formulation). Independently of the adopted formulation, complex infinite integrals are involved in evaluating the transmission line parameters.

To reduce the computational burden, a common practice is using closed-form approximations instead of the traditional infinite integrals expressions [4–7]. There are some possibilities to derive closed-form approximation and among them, the usage of the so-called image methods provide simpler expressions based on logarithmic functions. As shown in Refs. [8,9], even when ground displacement currents are considered, it is possible to derive simple expressions to include the ground return admittance. Unfortunately, as presented in Ref. [11], the usage of closed-form approximations considering either voltage or potential difference formulations may lead to numerical instabilities in the modeling of short lines as small passivity violations in the nodal admittance matrix may arise in the high frequency range, i.e., above a few MHz. There are two possible mitigation techniques to overcome this limitation, either use the Numerical Laplace Transform, as the complex frequency causes eigenvalues to shift to the left-hand side of the complex plane or consider the potential formulation to derive the per unit length parameters. Further investigations for the inclusion of ground displacement currents were carried out in Ref. [12] but considering ground-wire to be grounded, i.e. Kron reduction was applied to derive ground return impedance and admittance matrices. Thus, the soil plays a less important role than it should, as the return circuit will also include the ground-wires. The obtained results are summarized in Appendix A. To further access the different formulations and investigate the accuracy and adequacy of the closed-form approximation, scenarios

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where the ground-wires are explicitly represented, and the influence of the soil parameters is addressed in this paper.

The paper is organized as follows: Section 2 presents the expressions for the ground return impedance and admittance matrices. Section 3 details the closed-form approximations used in the line parameters determination. Section 4 shows the two test cases considered, to provide a more general overview, involving circuits with "vertical" and "horizontal" configuration. Frequency domain responses of the modal propagation constant, modal characteristic admittance and modal propagation function together with the actual phase domain characteristic admittance and propagation function matrices are used to assess the mismatches between the formulations. Time responses considering voltage injection at a phase-conductor and at a ground-wire are used to evaluate the impact and the adequacy of the approximated formulation. To avoid instability issues in the Method of Characteristic due to inaccurate interpolation of modal travel times [13], the time responses are obtained using the Numerical Laplace Transform [14,15].

2. Impedance & admittance expressions

For the simple case of a conductor above a lossy soil, a complete and rigorous characterization of the electromagnetic field associated can be obtained using the so-called full-wave formulation [16–18] which demands a solution of an mixed potential integral equation (MPIE). A rigorous extension of MPIE to a multi-phase configuration has not been developed, nevertheless some approximations have been considered in the literature such as the usage of an asymptotic approximation [19] or segmenting the configuration. In the latter the integral equation is then converted to an algebraic equation and then solved using numerical methods such as FDTD (finite-difference time-domain), MoM (Method of Moments) or FEM (finite element method) [20–22]. Alternatively, one may avoid dealing with all these issues altogether by resorting to a quasi-TEM (transverse electromagnetic) approximation A quasi-TEM approximation occurs when the propagation constant appearing in the infinite integrals in a MPIE assumes a pre-defined value [8,9]. In this work we adopt 10 MHz as the upper frequency limit.

Consider an overhead line with infinitely long conductors i and j, both at a constant height, h_i and h_j respectively, and with radius r as depicted in Fig. 1. Both air and soil are characterized by a permittivity ε_i, conductivity σ_i where i = 1 for air and i = 2 for the soil, permeability $\mu_1 = \mu_2 = \mu_0$, and propagation constant $\gamma_i = \sqrt{j\omega\mu_0(\sigma_i + j\omega\varepsilon_i)}$.

2.1. Voltage formulation

The most correct formulation is based on calculating the voltage between the overhead conductor and the ground surface point below, using a vertical path and considering both the electric scalar potential and magnetic vector potential [9]. Then, for conductor i, the voltage U_i is given by,

$$U_i = V_i + j\omega\int_0^{h_n} A_y(r_i, \zeta)d\zeta \tag{1}$$

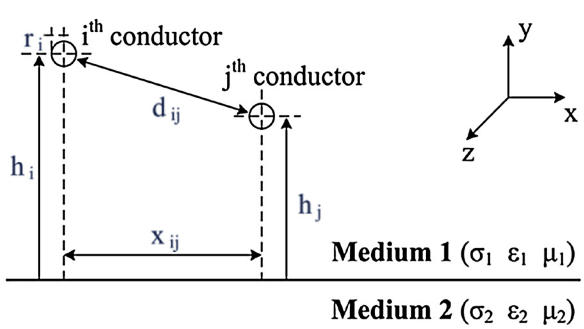

Fig. 1. Configuration of the conductors for an overhead line.

where r_i is the radius of conductor i, A_y is the vertical component of the magnetic vector potential $\boldsymbol{A}(x, y)$, and V_i is the difference of the electric scalar potential $\varphi_i(x, y)$ between conductor n and ground point given by,

$$V_i = \varphi_i(r_i, h_i) - \varphi_i(0, 0) \tag{2}$$

the electric scalar potential being defined with respect to a remote ground.

For the inclusion of the conductor losses we must assume that its propagation constant γ_c is such that $\gamma_c \gg \gamma$, where γ is the overall propagation constant of a given overhead line. For power transmission circuit this condition is true for a wide frequency range, thus one can easily include the skin effect in the conductoŕs internal impedance.

Assuming a thin wire, i.e., only axially directed currents with a uniform azimuthal distribution are assumed [23] and quasi-TEM approximations, and being $\mu_1 = \mu_2 = \mu_0$, the per-unit-length impedance and admittance matrices are then,

$$\boldsymbol{Z} = \boldsymbol{Z}_i + \tfrac{j\omega\mu_0}{2\pi}[\boldsymbol{P} + \boldsymbol{S}_1 - (\boldsymbol{S}_2 + \boldsymbol{S}_3)]$$
$$\boldsymbol{Y} = 2\pi(j\omega\varepsilon_0)[\boldsymbol{P} - \boldsymbol{S}_3]^{-1} \tag{3}$$

where $\mathbf{Z}_i$ stands for the internal impedance using Bessel functions, and the elements in $\mathbf{P}$ are given by,

$$\boldsymbol{P}_{ii} = \ln\tfrac{2h_i}{r_i} \quad \boldsymbol{P}_{ij} = \ln\tfrac{D_{ij}}{d_{ij}} \tag{4}$$

with $D_{ij} = \sqrt{\ell_{ij}^2 + x_{ij}^2}$, $\ell_{ij} = h_i + h_j$ and d_{ij} and x_{ij} are as shown in Fig. 1. The elements of $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$ are given by,

$$S_{1\,ij} = \int_{-\infty}^{\infty}\frac{\exp(-\ell_{ij}\lambda)}{\lambda + \bar{u}}\exp(j\lambda x_{ij})d\lambda \tag{5}$$

$$S_{2\,ij} = \int_{-\infty}^{\infty}\frac{\exp(-\ell_{ij}\lambda)}{n^2\lambda + \bar{u}}\exp(j\lambda x_{ij})d\lambda \tag{6}$$

$$S_{3ij} = \int_{-\infty}^{\infty}\frac{\bar{u}}{\lambda}\left(\frac{\exp\left(-\ell_{ij}\frac{\lambda}{2}\right) - \exp(-\ell_{ij}\lambda)}{n^2\lambda + \bar{u}}\right)\exp(jx_{ij}\lambda)d\lambda \tag{7}$$

where $\bar{u} = \sqrt{\lambda^2 + \gamma_2^2 - \gamma_1^2}$ and n is the refractive index of the ground. The expression of $\mathbf{S_1}$ are a simple extension of the ground return model of Pollaczek [1] and Carson [2] and $\mathbf{S_2}$ was proposed by Bridges and Shafai [23], $\mathbf{S_3}$ appears to include the non-null electric potential at the ground surface.

2.2. Potential formulation

Simpler expressions are obtained if one considers only $\varphi(r_i, h_i)$ to define the parameters which then leads to the following,

$$\boldsymbol{Z} = \boldsymbol{Z}_i + \tfrac{j\omega\mu_0}{2\pi}[\boldsymbol{P} + \boldsymbol{S}_1]$$
$$\boldsymbol{Y} = 2\pi(j\omega\varepsilon_0)[\boldsymbol{P} + \boldsymbol{S}_2]^{-1} \tag{8}$$

This formulation corresponds to considering a remote ground for potential reference. It is worth mentioning that if the ground displacement currents are neglected $\mathbf{S}_2$, $\mathbf{S}_3$ and $\mathbf{S}_4$ tend to zero and all formulations lead to the well-known results, which correspond to the ground being assumed as a good conductor.

Given that, even in this case, complex infinite integrals are involved, the most common practice is to use closed-form approximations to represent the ground return impedance and more recently the ground return admittance of overhead lines [8,9].

3. Closed-form approximations

Regardless of the formulation used to define the line parameters, the main issue lies in the need to deal with infinite integrals. One may