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Accuracy Evaluation of Least-Squares Method for a Class of Systems with Multi-Errors in Input and Output ⋆

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Abstract: This paper aims to evaluate the identification error of Least Square (LS) method for a class of systems with bias error, scale error and random noise in the measurement of both control input and output. According to the relation between the LS estimation error and the sensor deviations, this paper proposes an accuracy evaluation method to provide the bound of the estimation error for LS. Besides, the robustness of the evaluation result to the prior information is analysed. The theoretical analysis is verified by the simulation on the linearized longitudinal aircraft aerodynamic model.

Keywords: least-squares method, multi-errors, accuracy evaluation, robustness

1. INTRODUCTION

Least-Squares (LS) method has been the dominant algorithm for parameter estimation due to its simplicity in concept and convenience in implementation. Besides, the LS estimation is the unbiased and minimum variance estimation when the system is only corrupted by the Guass white measurement noise in output, see Chen (1986). However, this assumption usually can not be satisfied because most practical dynamic systems are contaminated by input and output noise errors, which leads to the dynamic errors-in-variables (DEV) problem, see Deistler (1989) and Schoukens (1991). Unfortunately, the conventional Least-Squares (LS) method yields biased estimates when applied to identify linear DEV systems even if the measurement deviation is Guass white noise.

This problem has received considerable attention. One way of estimating the parameters of a linear DEV model would be the extended least-square (ELS) method, see Huang (1990), which can treat the situation of colored noise. In fact, the DEV problem can be equivalently transformed to the parameter estimation of an equation error model, see Ljung (1999), corrupted by colored noise. Another approach would be the bias-eliminated least-squares (BELS) method proposed in Zheng (1989) and Zheng (2002). It is based on pre-correcting the bias according to the noise variances. These approaches can efficiently achieve the unbiased estimation of parameters under some conditions, but they only consider the random noise. Since the dynamics of the sensor model is complicated and contains various kinds of errors, such as bias error, scale error and random noise, which bring bias for estimation of LS and other methods. And this bias may not be pre-corrected because it not only needs the magnitudes but also the signs of the deviations to compensate the bias. Thus it is necessary to analyze the effect of these multiple measurement errors on the estimation accuracy of the parameters.

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Moreover, as the true values of the parameters are unavailable, it is important to evaluate the estimation accuracy, that is to offer the bound of the estimation error via prior information, which is of fundamental importance from both theoretical and practical points of view. However, few articles concern this problem.

This paper investigates parameter identification via LS method for the system with bias error, scale error and random noise in the input and output. It reveals the relationship between the LS estimation error and the bias error, the scale error and the random noise variance. According to the relationship, we propose a novel method to evaluate the estimation accuracy, which can offer the confidence interval of estimate error for LS algorithm. Besides, we analyze the robustness of the evaluation precision to the prior information, which is of practical meaning.

The paper is organized as follows. In Section 2, the background of the dynamic model is introduced. In Section 3, the relationship between the LS estimation error and the measurement errors is given. In Section 4, The evaluation approach of the identification accuracy for LS method is given. In Section 5, the simulation results of the evaluation approach on the linearized longitudinal aircraft aerodynamic model are given. The concluding remarks are shown in Section 6.

2. PROBLEM FORMULATION

Consider the multi-input multi-output nonlinear descrete system described by the following model:

$$\begin{cases} x_{k+1} = A(\theta)x_k + B(\theta)u_k, \\ y_k = x_k, k = 1, 2, ... \end{cases} \tag{1}$$

where $x_k \in R^m$ is the state vector, $\theta \in R^n$ denotes the unknown parameter vector to be estimated, the matrix $A(\theta) \in R^{m\times m}$ and $B(\theta) \in R^{m\times p}$ are the linear matrixes of θ, $u_k \in R^p$ is the control input, and $y_k \in R^m$ is the output.

The measurements of both the output and the control input are obtained through the sensors which may introduce multiple

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kinds of measurement errors. In this paper we consider the following three main measurement errors:

1.$n_k \in R^m$ and $n_{u,k} \in R^p$ denote the output and input noise, which are assumed to be zero-mean white noises with standard deviation being $\sigma(n_{k,i}) = \sigma_i, i = 1,2,...,m$ and $\sigma(n_{u,k,j}) = \sigma_{u,j}, j = 1,2,...,p$ respectively. $\{n_k\}_{k=1}$ and $\{n_{u,k}\}_{k=1}$ are independent of each other. Note that the noise in one instrument is usually independent of that in another, then this independence assumption is likely to be met.

2.$T \in R^{m\times m}$ and $T_u \in R^{p\times p}$ denote the scale error for the measurement of the output and control input respectively.

3.$B \in R^m$ and $B_u \in R^p$ denote the bias error for the measurement of the output and control input respectively.

Thus the measurement output is expressed as:

$$y_{m,k} = (I_m + T)y_k + B + n_k, k = 1,2\ldots,N. \tag{2}$$

The measurement of the control input is expressed as:

$$u_{m,k} = (I_p + T_u)u_k + B_u + n_{u,k}, k = 1,2\ldots,N, \tag{3}$$

where N is the number of the measurement data.

Assume that the control input u_k is implemented by both the measurement output feedback and the input excitation as follows:

$$u_k = Ky_{m,k} + \delta(kh) \tag{4}$$

where $K \in R^{p\times m}$ is the feedback coefficient matrix, $\delta(kh)$ is the input excitation.

Remark 1. The output feedback will bring the output measurement errors to the model, resulting in the correlation between the state and the measurement noise, which generates bias in LS estimation.

We want to use LS method to estimate the parameter vector θ from the available data $\{y_{m,k}, u_{m,k}, k = 1,2,\ldots N\}$. Thus the following questions are mainly concerned with:

Q1. What is the performance of LS method for the closed system (1)-(4) when the bias error, the scale error and the random noise exist in the measurements of control input and output?

Q2. If the estimation of LS method is biased for this system, how to evaluate the bias, that is to say, how to give the confidence interval of the bias without the true values of the parameters?

In the following sections, Q1-Q2 will be discussed.

3. LEAST-SQUARES METHOD DESIGN AND ESTIMATION ACCURACY ANALYSIS

Since the right side of (1) is a linear function of θ, it can be described as

$$A(\theta)x_k + B(\theta)u_k = H_1(x_k,u_k)\theta + H_2(x_k,u_k), \tag{5}$$

where $H_1(x_k,u_k) \in R^{m\times n}$ and $H_2(x_k,u_k) \in R^m$ are linear matrix functions of x_k and u_k.

Denote $Y_k \triangleq x_{k+1} - H_2(x_k,u_k), H_k \triangleq H_1(x_k,u_k)$, we can rewrite (1) as

$$Y_k = H_k\theta. \tag{6}$$

Direct closed-loop identification is aimed at estimating the plant parameter vector θ from the sampled measurements $\{y_{m,k}, u_{m,k}, k = 1,2,\ldots N\}$ via LS method, thus we replace $\{x_k, u_k\}$ with $\{y_{m,k}, u_{m,k}\}$ and obtain the following linear regression model:

$$Y_{m,k} = H_{m,k}\theta + V_k, \tag{7}$$

where $Y_{m,k} = y_{m,k+1} - H_2(y_{m,k}, u_{m,k})$ and $H_{m,k} = H_1(y_{m,k}, u_{m,k})$ is the regression matrix.

$$V_k = (H_k - H_{m,k})\theta + Y_{m,k} - Y_k, \tag{8}$$

is the remainder term generated from the measurement errors.

In the following analysis, we assume that the system (7) satisfies:

A1. The matrix $\frac{1}{N}\sum_{k=1}^{N}(H_{m,k})^T H_{m,k}$ is nonsingular.

A2. The bias error, the scale error and the random noise satisfy:

$$\begin{aligned}
&B_i \in [\underline{B}_i, \overline{B}_i], i = 1,2,...,m,\\
&B_{u,i} \in [\underline{B}_{u,i}, \overline{B}_{u,i}], i = 1,2,...,p,\\
&T_i \in [\underline{T}_i, \overline{T}_i], i = 1,2,...,m^2,\\
&T_{u,i} \in [\underline{T}_{u,i}, \overline{T}_{u,i}], i = 1,2,...,p^2,\\
&\sigma_i \in [\underline{\sigma}_i, \overline{\sigma}_i], i = 1,2,...,m,\\
&\sigma_{u,i} \in [\underline{\sigma}_{u,i}, \overline{\sigma}_{u,i}], i = 1,2,...,p,
\end{aligned} \tag{9}$$

where $\underline{B}, \overline{B}, \underline{B}_u, \overline{B}_u, \underline{T}, \overline{T}, \underline{T}_u, \overline{T}_u, \underline{\sigma}, \overline{\sigma}, \underline{\sigma}_u, \overline{\sigma}_u$ are known.

A3. The true values of the parameters satisfy:

$$\theta_i \in [\underline{\theta}_i, \overline{\theta}_i], i = 1,2,...,n. \tag{10}$$

where $\underline{\theta}, \overline{\theta}$ are known.

A1 means that the sequence of input vectors satisfies the persistent excitation conditions (PE conditions), which is widely used for LS identification problem, see Moore (1978). A2 indicates that the bias error, the scale error and the standard deviation of the random noise are bounded. Those bounds can be obtained by the performance index of the sensors. A3 means that the true values of the parameters are bounded, and the ranges can be obtained by the physical meaning and the prior information of these parameters. Thus A2 and A3 are reasonable for most practical plants.

According to the regression model (7), the off-line LS estimate of the parameter vector θ is given by

$$\hat{\theta}_N = \left[\sum_{k=1}^{N}(H_{m,k})^T H_{m,k}\right]^{-1}\left[\sum_{k=1}^{N}(H_{m,k})^T Y_{m,k}\right]. \tag{11}$$

Hence, the estimation error is

$$\begin{aligned}
\hat{\theta}_N - \theta &= \left[\sum_{k=1}^{N}(H_{m,k})^T H_{m,k}\right]^{-1}\left[\sum_{k=1}^{N}(H_{m,k})^T (Y_{m,k} - H_{m,k}\theta)\right]\\
&= \left[\sum_{k=1}^{N}(H_{m,k})^T H_{m,k}\right]^{-1}\left[\sum_{k=1}^{N}(H_{m,k})^T (V_k)\right].
\end{aligned} \tag{12}$$

If the system (1)-(3) is an ARX model with only Guass white noise in output measurement:

$$y_{m,k+1} = A(\theta)y_{m,k} + B(\theta)u_{m,k} + n_{k+1}, \tag{13}$$

then $V_k = n_{k+1}$, which is zero mean and uncorrelated with $H_{m,k}$. Thus

$$E\left[\hat{\theta}_N - \theta\right] = 0.$$

In this way, LS estimation $\hat{\theta}_N$ is the unbiased minimum variance estimation Ljung (1999). However, in the presence of the input and output measurement errors which contain the bias error,