Electric Power Systems Research 162 (2018) 134–141

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Highly accurate computation of Carson formulas based on exponential approximation

Ivan Krolo*, Slavko Vujević, Tonći Modrić

University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, Rudjera Boskovica 32, HR-21000 Split, Croatia

A R T I C L E I N F O

Keywords:
Exponential approximation
Carson formulas
Self impedance
Mutual impedance
Overhead power lines

A B S T R A C T

In this paper, analytical expressions for a highly accurate computation of Carson formulas are developed. Highly accurate analytical expressions for per-unit-length (pul) self and mutual impedance corrections are now reduced to two linear combinations of 50 functions. Coefficients of these linear combinations are computed using exponential approximation of integrals' kernel function. Proposed algorithm has simplified computation in comparison with other highly accurate numerical algorithms. Results computed by proposed algorithm and by two approximation methods are compared with 7-digit accurate results computed by piecewise quadratic approximation for large frequency range.

1. Introduction

Computation of per-unit-length (pul) self and mutual impedance of power line conductors is a long-standing problem. Formulas for computation of pul self and mutual impedances of infinitely long parallel conductors were proposed by Carson [1] and they contain complex-valued integrals with infinite upper limits [1,2], which can be transformed into real-valued integrals [3,4]. These integrals can be computed using various approximation methods, e.g. Carson infinite series [1], single-term approximation of infinite Carson series [1], Gary-Dubanton formulas [2,5–7], Alvarado–Betancourt formulas [8], etc. Overall overview of the proposed methods along with a few improvements are given in Ref. [9], and also in Ref. [10], where displacement currents were considered. Recent studies have also discussed the development of more accurate expressions valid at high frequencies [11].

This paper presents a unified and efficient computation procedure based on the successful numerical approximation of integrals' kernel function which appears in Carson formulas. Carson formulas can be considered as low frequency approximation of Sunde formula since they neglect the displacement currents [10,12]. They consider homogeneous soil and only then the kernel function is identical in all cases. Most importantly, it is also independent of soil characteristics and frequency, in comparison to Sunde formula [12]. Kernel function of Sunde formula, as well as similar formulas developed for multi-layered earth [13,14], contain frequency and soil characteristics and thus they are not appropriate formulas for unified approximation algorithm. On the other hand, numerical algorithms proposed in Refs. [3,4] can be successfully implemented in case of multi-layered earth.

Using the proposed exponential algorithm, kernel function is approximated by a linear combination of small number of real exponential functions. The approximated kernel function is then multiplied by the rest of the integrands and then analytically integrated. The efficiency of the proposed numerical algorithm is shown in the numerical examples where results obtained by proposed algorithm at a large frequency range are compared with 7-digit accurate results obtained by numerical algorithm explained in Ref. [4]. Two approximate methods: single-term approximation method proposed by Carson [1] and Gary–Dubanton formulas [2,5–7] are also presented and compared with the same 7-digit accurate results at a large frequency range.

2. The Carson formulas

The formulas for computation of pul self and mutual impedances of conductors are presented by Carson [1]. In these formulas displacement currents are neglected. In Fig. 1, two cylindrical infinitely long parallel conductors placed in the air along with their images obtained by the image method are presented.

The Carson formulas can be divided into two parts [3,4]:

$$\overline{Z}_{ii}^{1} = \overline{Z}_{ii}^{1pcg} + \overline{Z}_{ii}^{1cor} \quad (1)$$

$$\overline{Z}_{ik}^{1} = \overline{Z}_{ik}^{1pcg} + \overline{Z}_{ik}^{1cor} \quad (2)$$

where $\overline{Z}_{ii}^{1pcg}$ and $\overline{Z}_{ik}^{1pcg}$ are pul self and mutual impedances of perfectly conducting ground, whereas $\overline{Z}_{ii}^{1cor}$ and $\overline{Z}_{ik}^{1cor}$ are pul self and mutual impedance corrections. Pul self and mutual impedances of perfectly conducting ground are described by following expressions:

* Corresponding author.
E-mail address: ikrolo@fesb.hr (I. Krolo).

https://doi.org/10.1016/j.epsr.2018.05.008
Received 31 August 2017; Received in revised form 5 April 2018; Accepted 8 May 2018

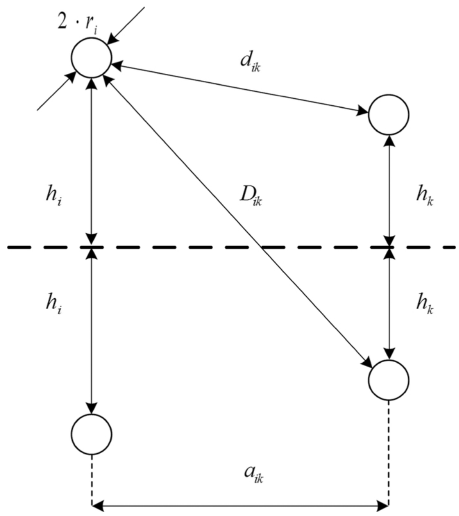

Fig. 1. Two cylindrical conductors in the air, their images and all relevant distances between them.

$$\overline{Z}_{ii}^{1pcg} = \overline{Z}_i^1 + j\cdot\frac{\omega\cdot\mu_0}{2\cdot\pi}\cdot\ln\frac{2\cdot h_i}{r_i} \tag{3}$$

$$\overline{Z}_{ik}^{1pcg} = j\cdot\frac{\omega\cdot\mu_0}{2\cdot\pi}\cdot\ln\frac{D_{ik}}{d_{ik}} \tag{4}$$

where $\overline{Z}_i^1$ is the pul internal impedance of conductor, h_i is the ith conductor height, D_{ik} is the distance between ith conductor and the kth conductor image, d_{ik} is the distance between ith and kth conductor, r_i is the ith conductor radius, ω is the angular frequency and μ_0 is magnetic permeability of vacuum. Distances D_{ik} and d_{ik} are defined by:

$$D_{ik} = \sqrt{a_{ik}^2 + (h_i + h_k)^2} \tag{5}$$

$$d_{ik} = \sqrt{a_{ik}^2 + (h_i - h_k)^2} \tag{6}$$

where h_k is the kth conductor height and a_{ik} is the horizontal distance between ith and kth conductors. Highly accurate computation of pul internal impedance of single-layer and two-layer cylindrical conductors using scaled modified Bessel functions are described in Refs. [15,16].

Formulas for pul self and mutual impedance corrections can be written as [1,3,4]:

$$\overline{Z}_{ii}^{1cor} = \frac{\omega\cdot\mu_0}{\pi}\cdot\int_0^\infty(\sqrt{\lambda^2+j} - \lambda)\cdot e^{-p_{ii}\cdot\lambda}\cdot d\lambda \tag{7}$$

$$\overline{Z}_{ik}^{1cor} = \frac{\omega\cdot\mu_0}{\pi}\cdot\int_0^\infty(\sqrt{\lambda^2+j} - \lambda)\cdot e^{-p_{ik}\cdot\lambda}\cdot\cos(q\cdot\lambda)\cdot d\lambda \tag{8}$$

where:

$$p_{ii} = 2\cdot h_i\cdot\gamma \tag{9}$$

$$p_{ik} = (h_i + h_k)\cdot\gamma \tag{10}$$

$$q = a_{ik}\cdot\gamma \tag{11}$$

Complex wave propagation constant $\overline{\gamma}$ and its magnitude γ are described by:

$$\overline{\gamma} = \frac{\gamma}{\sqrt{2}}\cdot(1+j) \ ; \ \gamma = \sqrt{\frac{\omega\cdot\mu_0}{\rho}} \tag{12}$$

where ρ is soil resistivity. Eqs. (7) and (8) contain the same kernel function of the integrals which leads to universal computation algorithm because these expressions exclude soil characteristics and frequency.

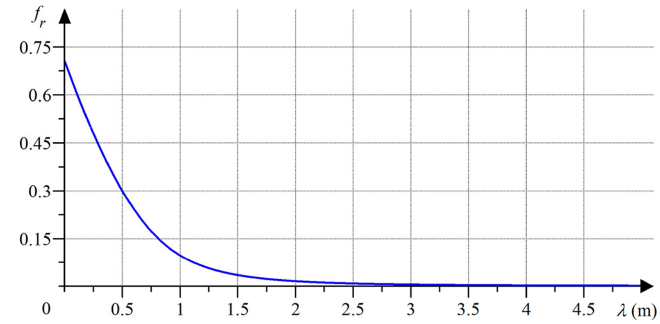

Fig. 2. Real part of the kernel function.

3. Exponential approximation of the kernel function

The kernel function of integrals (7) and (8) can be divided into real and imaginary parts [3,4]:

$$\overline{f} = \sqrt{\lambda^2 + j} - \lambda = f_r + j\cdot f_i \tag{13}$$

It is necessary to point out that only solution of square root of complex function in the first quadrant is considered [17]. Functions f_r and f_i are shown in Figs. 2 and 3, and they are described by:

$$f_r = \frac{1}{(\sqrt{2}\cdot g + 2\cdot\lambda)\cdot g^2} \ ; \ f_i = \frac{1}{\sqrt{2}\cdot g} \tag{14}$$

where:

$$g = \sqrt{\lambda^2 + \sqrt{\lambda^4 + 1}} \tag{15}$$

For achieving high accuracy, based on many numerical experiments, semi-infinite integration interval of function f_r is divided into two subintervals, where division point is $\lambda = 20$. Function f_i and both subintervals of function f_r are approximated by a linear combination of exponential base functions [18–22]. These approximations are described by the following equations:

$$\tilde{f}_r^f = \sum_{n=1}^{20}\alpha_n\cdot e^{-\eta_n\cdot\lambda};\ 0 \le \lambda \le 20 \tag{16}$$

$$\tilde{f}_r^s = \sum_{n=1}^{10}\beta_n\cdot e^{-\xi_n\cdot\lambda};\ \lambda \ge 20 \tag{17}$$

$$\tilde{f}_i = \sum_{n=1}^{20}c_n\cdot e^{-\tau_n\cdot\lambda};\ \lambda \ge 0 \tag{18}$$

where α_n, β_n and c_n are unknown coefficients, whereas η_n, ξ_n and τ_n are chosen parameters. Highly accurate approximation in all cases is achieved only if the kernel function is the same in all cases. Values of parameters η_n, ξ_n and τ_n are chosen on the basis of numerous numerical tests. These parameters are described by:

$$\eta_n = 0.05\cdot1.4^{n-1};\ n = 1, 2, \ldots, 20 \tag{19}$$

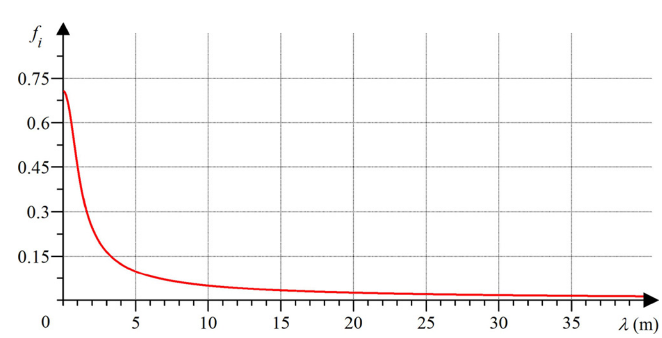

Fig. 3. Imaginary part of the kernel function.