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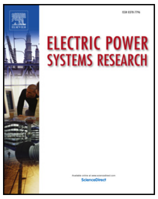

A vertical conductor circuit model including up- and down-ward traveling waves

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ABSTRACT

A model of a vertical conductor for a circuit analysis is proposed based on calculation results by the Finite-Difference Time-Domain (FDTD) method. The model expresses a frequency-dependent effect of a vertical conductor. In addition, differences in its propagation characteristics of the upward and downward traveling waves are taken into consideration. An experiment is carried out using a reduced-size wind turbine tower model. The simulation results by Electromagnetic Transients Program (EMTP) accurately reproduce the experimental results.

1. Introduction

Most of the wind turbine power plants in Japan are built on top of hills or mountains to obtain good wind conditions. However, the towers located at these places are often struck by lightning. For optimal lightning protection designs of new wind power plants and lightning-risk evaluations of the existing wind power plants, transient characteristics of the tower, i.e., a vertical conductor should be clarified [1]. An electromagnetic field analysis is suitable for this investigation since the lightning surge characteristics of towers are obtained with difficulty by experiments due to their required time and cost. The method becomes practical by improvements of computational abilities. They are, however, not sufficient for the lightning surge estimation of power systems such as wind farms. Although circuit analysis method is appropriate for the analysis, the vertical conductor has to be represented by a numerical model.

Various circuit models of the vertical conductor or the tower have been proposed based on the Neumann's formula [2–4], experimental results [5,6], and numerical electromagnetic analysis results [7–9]. Some models [6,8] take the frequency-dependent effect of the vertical conductor into account by applying the Semlyen's line model installed in Electromagnetic Transients Program (EMTP).

This paper presents a modeling method of a vertical conductor using results by the Finite-Difference Time-Domain (FDTD) method. The FDTD method has been employed in Refs. [7–9] to consider the frequency-dependent characteristics of the towers or vertical conductors in circuit analysis. In additions to this, the proposed model enables to consider direction-dependent characteristics of traveling waves along a single vertical conductor. The circuit model is installed in EMTP by using a MODELS language. An experiment using a reduced-size wind tower model is carried out to confirm the validity of the proposed model.

2. FDTD simulation

Virtual Surge Test Lab (VSTL), which is developed by Central Research Institute of Electric Power Industry (CRIEPI), is used for the FDTD calculations [10]. The FDTD calculations are carried out in the following two cases: (A) a current wave expressed by (1) is injected into the bottom of a vertical conductor, and (B) the current is injected into the top of the vertical conductor. In Cases (A) and (B), the vertical conductor is set afloat above a perfectly conducting ground.

$$i_{in}(t) = I_{in}\{1 - \exp(-t/\tau_i)\} \tag{1}$$

where I_{in} and τ_i are set to 1 A and 1 ns, respectively.

The rising time of the injected current is fast enough to observe the transient characteristic of the vertical conductor in a high frequency region.

The simulation spaces are illustrated in Fig. 1. The current is injected at the center of the conductor by a current source through a thin wire. The FDTD analysis space is $8.125 \times 8.125 \times 6.175\ \mathrm{m}^3$ $(= x \times y \times z)$ and is divided into cubic cells whose side length is 32.5 mm. The second order Liao's absorbing boundary condition is applied to the boundary. A calculation time step Δt is set to 62.5 ps based on the Courant condition.

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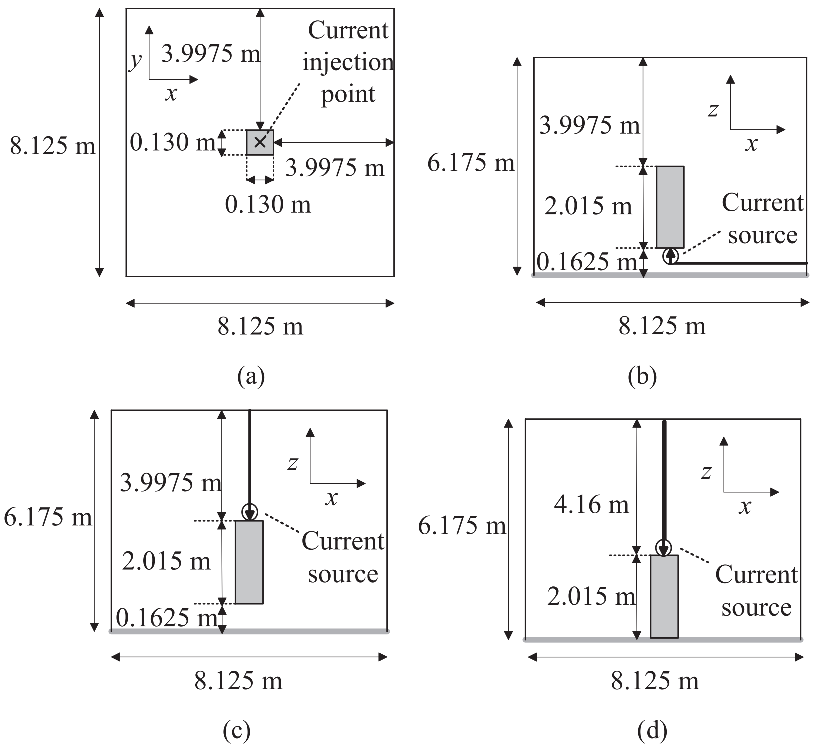

Fig. 1. Configuration of the FDTD analysis. (a) illustrates the x–y plane of the analysis space of all cases, (b) illustrates the x–z plane of Case (A): the current is injected into the bottom of the floated vertical conductor, (c) is for Case (B): the current is injected into the top of the floated vertical conductor, and (d) is Case (C): the current is injected into the top of the grounded vertical conductor.

The vertical conductor is a 1/40 reduced-size model of a 2.5 MW class wind turbine tower. It is expressed by a square perfect conductor, whose length and side length are 2.015 m (62 cells) and 130 mm (4 cells), respectively. The circumference of the tower model is equal to that of the pipe, which is used for an experiment in Section IV-A. The conductor is arranged above 162.5 mm (5 cells) from the earth surface.

Each conductor voltage is defined as an integration of the electric field from the absorbing boundary to the conductor. The injected, sending-end, and receiving-end currents are respectively derived as an integration of the magnetic field around each conductor.

An additional case is appended for a reliability test of the circuit model: (C) the current is injected into the top of a vertical conductor, which is directly grounded to the perfectly conducting ground.

3. Circuit model of a vertical conductor

3.1. Derivation of model parameters

The current and voltage waveforms calculated by means of the FDTD method should be transformed from the time domain into the frequency domain in order to obtain the model parameters expressing the vertical conductor. In a numerical Fourier or Laplace transform, the truncation and aliasing errors are unavoidable. In this paper, the current and the voltage waveforms are transformed into the frequency domain using analytical Laplace transform to avoid the numerical errors. For the analytical calculation, the voltage waveforms are approximated by exponential functions expressed in (2) and are transformed into s-domain [11].

$$v_{apr}(t) = u(t - t_d)\left[\begin{array}{l} V_0\{1 - \exp(-(t - t_d)/\tau_i)\} + \\ \displaystyle\sum_{k=1}^{N} V_k \left\{\begin{array}{l} \exp(-(t - t_d)/\tau_i) \\ -\exp(-(t - t_d)/\tau_k) \end{array}\right\} \end{array}\right]$$

$$V_{apr}(s) = \mathscr{L}\{v_{apr}(t)\} = \frac{\exp(-st_d)}{1 + s\tau_i}\left(\frac{V_0}{s} + \sum_{k=1}^{N} V_k \frac{\tau_k - \tau_i}{1 + s\tau_k}\right) \tag{2}$$

where $\mathscr{L}$ and s denote Laplace transform and its operator.

The receiving- and sending-end voltage waveforms are approximated until the arrival of the reflection waves. The delay time t_d expresses the propagation time of the traveling wave for the receiving-end voltages and is defined by the length and a light velocity. The delay t_d is zero for the sending-end voltages. The parameters V_0, V_k, and τ_k are determined by a nonlinear least squares method using modified Newton method or sequential quadratic programing.

The approximated voltage shown in (2) contains the time constant τ_i of the current in order to make the impedances independent of the injected current waveform. The calculated waveforms by the FDTD method and their approximation results are shown in Fig. 2.

The sending-end current waveform has to be approximated by the function shown in (1) because some of the injected current leaks via a stray capacitor, i.e., all of the current cannot be injected into the vertical conductor. The sending-end current is also approximated until the arrival of the reflection wave. Even if the receiving-end is open-circuited, the current at the node leaks via a stray capacitor. The injected, sending-end and receiving-end currents are shown in Fig. 3.

The characteristic impedance is given in the frequency domain as a ratio between the approximated sending-end voltage and the current.

$$Z_0(s) = V_s(s)/I_s(s) \tag{3}$$

The propagation characteristic for the traveling wave propagating toward each direction is defined as a ratio between the sending-end voltage $V_s(s)$ and the traveling wave at the receiving-end $V_r(s)/(1 + \theta_r)$.

$$\exp(-\Gamma l) = \frac{V_r(s)}{(1 + \theta_r)}\frac{1}{V_s(s)},$$
$$\theta_r = (Z_r - Z_0)/(Z_r + Z_0) \tag{4}$$

where Γ is the propagation constant, l is the conductor length, θ_r is the reflection coefficient, and Z_r is the impedance connected to the vertical conductor at the receiving end.

Though the vertical conductor illustrated in Fig. 1 is open circuited in the FDTD calculation, a stray capacitor C_s has to be considered as Z_r in (4) to express the leakage current at each node. The stray capacitor C_s is calculated from the slope of the integrated current waveform vs. the voltage at the receiving-end. The stray capacitances of 2.5 pF and 4.0 pF are derived from Cases (A) and (B), respectively.

A phase rotation n in (5) has to be taken account in the calculation