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Group consensus in multi-agent systems with switching topologies and time delays ⋆

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Abstract: In this paper, we investigate the group consensus problem for discrete-time multi-agent systems with switching topologies and bounded time delays. The analysis in this paper is based on nonnegative matrix theory and graph theory. Under the assumption of common inter-group influence, the group consensus problem is proved to be solvable, if the union of the communication topology across any time interval with some given length contains group spanning trees. It is also shown that the nonzero in-degree groups finally converge to convex combination of the consensus states of the zero in-degree groups. The effectiveness of the theoretical results is finally demonstrated by the simulation example.

Keywords: Distributed control; Discrete-time systems; Matrix algebra; Network topologies; Linear analysis.

1. INTRODUCTION

As one type of critical problems for distributed coordination, consensus problem has been studied from various perspectives, see Qin et al. (2014), Ren and Beard (2005), Qin et al. (2011), Hu and Lin (2010), Qin et al. (2011) and Qin and Yu (2014). Its objective is reaching an agreement regarding a certain quantity of interest. Recently, a more general kind of consensus, group/cluster consensus, has attracted increasing attention from researchers, see Yu and Wang (2009), Yu and Wang (2010), Qin and Yu (2013), Xie and Liu (2014), Ji et al. (2014), Han et al. (2012), Han et al. (2013) and Shang (2013). In a complex network consisting of multiple subgroups, group consensus means that the agents in each subgroup reach consensus asymptotically, but the consistent states of different subgroups may not be the same. Group consensus is more appealing for nature and human society, since the agreements are often different with the changes of of environments, situations, cooperative tasks or even time.

Under the in-degree balanced assumption, Yu and Wang (2009) presented some sufficient conditions in terms of linear matrix inequalities (LMIs) for guaranteeing the group consensus with fixed topology, the result was later extended to the multi-agent network with switching topologies in Yu and Wang (2010). Qin and Yu (2013) further showed that under directed acyclic topology, the group consensus can be achieved regardless of the magnitudes of the coupling strengths among the agents. Besides, by utilizing a distributed protocol with Cartesian coordinate coupling matrix, Xie and Liu (2014) established some necessary and sufficient group consensus criteria for continuous-time multi-agent systems. Moreover, Han et al. (2012) extended the concept of spanning tree to that of group spanning trees, and discussed the group consensus problem of continuous-time multi-agent systems. The group consensus problems under group spanning trees were also considered in Han et al. (2013), Shang (2013). Note that most of the aforementioned works did not take into consideration communication delays. However, it is more realistic especially when time delays are accounted for. In addition, the communication topology may be time-varying in the real-world networks of agents.

Inspired mainly by the above analysis, this paper focuses on the group consensus problem with switching topologies and time delays. According to the topology structure, the groups under group spanning trees are classified into two categories: zero in-degree groups and nonzero in-degree groups. As a result, it is proved that under the assumption of common inter-group influence, the multi-agent systems with bounded time delays can realize group consensus, if the union of the communication topology across any time interval with some given length contains group spanning trees. Furthermore, the nonzero in-degree groups are shown to finally converge to convex combination of the consensus states of the zero in-degree groups.

The rest of the paper is organized as follows. Some preliminaries and the problem formulation are provided in Section 2. In Sections 3, we establish our main results. The effectiveness of the theoretical findings is demonstrated in Section 4.

2. PRELIMINARIES AND FORMULATION

Directed graphs will be used to model the interaction topology among agents. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph composed of a vertex set $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and an edge set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, where vertex v_i represents agent i. An edge in $\mathcal{E}$ is denoted by an ordered pair (v_j, v_i), where

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$(v_j, v_i) \in \mathcal{E}$ if and only if that agent i can access the state information of agent j. If $(v_i, v_i) \in \mathcal{E}$, it is called a self-loop on vertex v_i. The index set of neighbors of vertex v_i is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}, j \neq i\}$. A directed path is a sequence of edges in a directed graph of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots$, where $v_{i_j} \in \mathcal{V}$. If there exists a path from vertex v_j to vertex v_i, we say that v_i is reachable from v_j. A directed tree is a directed graph, where there exists a vertex, called the root, such that any other vertex of the digraph can be reached by and only one path starting at the root. A spanning tree of a digraph is a directed tree formed by graph edges that connect all the vertexes of the graph. Let $\mathcal{G}(t)$ corresponds to the communication topology at time t, then the union of a set of directed graphs $\Gamma = \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_k\}$ with a common vertex set $\mathcal{V}$ is a directed graph with the edge set given by the union of the edge sets of $\mathcal{G}_i$, $i = 1, \ldots, k$.

A nonnegative matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ can be associated with a directed graph $\mathcal{G}(A)$ in such a way that A is specified as the weighted adjacency matrix. In detail, for $v_i, v_j \in \mathcal{V}$, $v_j \in \mathcal{N}_i \Leftrightarrow a_{ij} > 0$. Moreover, $a_{ii} > 0$ if there is a self-loop on vertex v_i.

Let $\mathbb{Z}_+$ be the set of non-negative integers, I denotes an identity matrix with a compatible dimension, and $\mathbf{1}_n$ denotes an all 1 vector with dimension n. A vector norm of a vector x is denoted by $\|x\|$. Given a matrix $A \in \mathbb{R}^{n\times n}$, $\|A\|$ represents the matrix norm of A induced by the vector norm $\|\cdot\|$. A square nonnegative matrix is called a (row) stochastic matrix if all its rows sums are 1. Let $\prod_{k=v}^{s} A(k)$ denotes the successive matrix product from s to v, that is, $\prod_{k=v}^{s} A(k) = A(s)A(s-1)\cdots A(v)$.

To study the group consensus problem, we first give a grouping of multi-agent systems. Without loss of generality, we assume that the multi-agent systems consisting of n agents are divided into K disjoint groups. Specifically, a partition of the multi-agent systems, denoted by $G = \{G_1, \ldots, G_K\}$, is a sequence of subsets of $\mathcal{V}$ such that $\bigcup_{p=1}^{K} G_p = \mathcal{V}$ and $G_p \bigcap G_q = \emptyset$ for $p \neq q$.

Definition 1. (Han et al. (2013)). For a given partition G, a graph $\mathcal{G}$ is said to have group spanning trees with respect to G if for each group G_p, $p = 1, \ldots, K$, there is a vertex $v_p \in \mathcal{V}$ such that there exist paths in $\mathcal{G}$ from v_p to all vertexes in G_p. The vertex v_p is called the root of the group G_p.

It is worth pointing out that the root vertex of G_p does not necessarily belong to G_p. Therefore, the definition of the group spanning tree can be regarded as a generalization of that of spanning tree.

Let $x_i(t) \in \mathbb{R}$ represent the state of agent i at time t, then the system is said to solve a group consensus problem in Yu and Wang (2009) if for any initial states, the states of agents satisfy:

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0,$$

for all $i, j \in G_p$ and $p = 1, \ldots, K$.

To solve the group consensus problem, the definition of Hajnal diameter (see Hajnal (1976)) is extended to the group case in Han et al. (2013). In detail, given a matrix A with row vectors $A_1, A_2, \ldots, A_n$ and a partition G, the group Hajnal diameter of A is

$$\Delta_G(A) = \max_{p=1,\ldots,K} \max_{i,j\in G_p} \|A_i - A_j\|,$$

for some vector norm $\|\cdot\|$. It is clear that $\Delta_G(A)$ measures the differences between the rows of A. Similarly, given a vector $x = (x_1, x_2, \ldots, x_n)^T$ and a partition G, the group Hajnal diameter of x is

$$\Delta_G(x) = \max_{p=1,\ldots,K} \max_{i,j\in G_p} \|x_i - x_j\|.$$

Also we need to recall the following definition.

Definition 2. (Han et al. (2013)) Given a partition $G = \{G_1, \ldots, G_K\}$, a stochastic matrix A is said to have common inter-group influence if for each pair of p and q, $p, q = 1, 2, \ldots K$, $\sum_{j\in G_q} a_{ij}$ only depends on the group indices p and q but is independent of i with $i \in G_p$.

For a multi-agent system consisting of K groups, we divide the vertexes under group spanning trees into zero in-degree groups and nonzero in-degree groups. Without loss of generality, suppose the number of zero in-degree groups is m, denoted by G_1, $G_1, \ldots, G_m$: $G_1 = \{v_1, v_2, \ldots, v_{l_1}\}$, $G_2 = \{v_{l_1+1}, v_{l_1+2}, \ldots, v_{l_2}\} \ldots$, $G_m = \{v_{l_{m-1}+1}, v_{l_{m-1}+2}, \ldots, v_{l_m}\}$, respectively. And let the vertexes of nonzero in-degree group be $G_{m+1} = \mathcal{V}(\mathcal{G})\backslash(\cup_{i=1}^{m} G_i) = \{v_{l_m+1}, v_{l_m+2}, \ldots, v_n\}$. For convenience, let $n_1 = l_1$, $n_2 = l_2 - l_1$, $\ldots$, $n_{m+1} = n - l_m$.

If all the groups in the topology $\mathcal{G}(A)$ have zero in-degree, then there is no information exchange between each other, so they can be considered separately. Therefore, we assume that there are vertexes which belong to nonzero in-degree groups, that is, $l_m < n$.

3. MAIN RESULTS

In this paper, we study the following discrete-time model of the n agents:

$$x_i(t+1) = \sum_{j=1}^{n} a_{ij}(t)x_j(t - \tau_{ij}(t)), \quad i = 1, \ldots, n, \quad (1)$$

where $A(t) = (a_{ij}(t))$ is an $n\times n$ stochastic matrix, $a_{ii}(t) > 0$, $\tau_{ij}(t) \in \mathbb{Z}_+$, $\tau_{ij}(t) \leq \hat{\tau}$ is the transmitted information delays from agent j to i at time t, and $\tau_{ii}(t) \equiv 0$ for $i = 1, 2, \ldots, n$. For any i, j, t, there exists a constant $e > 0$ such that either $a_{ij}(t) = 0$ or $a_{ij}(t) \geq e$ holds.

For the global consensus problem in which all the agents converge to a common consistent state, Xiao and Wang (2007) derived the following result:

Lemma 3. (Xiao and Wang (2007)) If there exists $T \geq 0$ such that for all $t \geq 0$, the union of graph $\mathcal{G}(t)$ across time interval $[t, t+T]$ contains a spanning tree, then system (1) solves the consensus problem.

Because of the limited number of vertexes in $\mathcal{G}(t)$, all the possible communication topologies constitute a finite set, denoted by $\Gamma = \{\mathcal{G}_1, \mathcal{G}_2, \ldots, \mathcal{G}_l\}$. Given each pair of groups: G_p and G_q, $p, q = 1, \ldots, K$, we make the following assumptions about each graph in Γ and the matrix $A(t)$:

($\mathfrak{C}1$) either there is no edge connecting G_p and G_q;
($\mathfrak{C}2$) or for each vertex in G_p and each graph $\mathcal{G}_i$, $i = 1, \ldots, l$, there is at least an edge from G_q to it.