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Group Consensus for Multi-agent Systems Under the Effect of Coupling Strength Among Groups ⋆

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Abstract: This paper investigates a group consensus problem for multi-agent systems with fixed topologies. We focus on discussing the effect of the coupling strength among the agents in different groups, and obtain some conditions in form of linear matrix inequalities (LMIs) guaranteeing the group average consensus by using Lyapunov function methods. In order to study the effect of the coupling strength, we introduce two different parameters γ_1, γ_2 in the network topologies. Finally, illustrative examples are provided to demonstrate the effectiveness of the theoretical results with the two parameters.

Keywords: Linear systems, Co-operative control, Graphic theory, Lyapunov functions, Matrix algebra.

1. INTRODUCTION

The research booms for consensus problems of multi-agent systems caused by DeGroot's literature cited in DeGroot et al. (1974). Following it there were numerous researchers from mathematics, physics, biology, sociology, control science, computer science. Consensus problems have been studied extensively in the recent literature. Among various studies of linear-consensus algorithms, a noticeable phenomenon is that algebraic graph theory and linear matrix inequalities(LMIs) play an important role in dealing with consensus problems.

There has been tremendous amount of interest in complete consensus problems of multi-agent systems. Vicsek et al. (1995) obtained the result that all agents eventually move in the same direction by using the nearest neighbours' information. Jadbabaie et al. explained the phenomenon mentioned in Vicsek et al. (1995) by theoretical techniques like undirected graph theory, algebraic theory and the special properties of stochastic matrices in Jadbabaieet al. (2003). The multi-agent distributed complete consensus problems have been intensively investigated, since the theoretical framework of consensus problems for first-order multi-agent networks was posed and solved by Olfati-Saber and Murray in Saber et al. (2004). They presented the conditions on consensus in terms of graphes for three cases. Furthermore, Ren and Beard presented looser conditions to guaranteeing the complete consensus based on the algebraic graph theory in Ren et al. (2005) than Jadbabaieet al. (2003) Saber et al. (2004).

Due to reality applications, some researchers analysed the stability and the convergence performance of networks under the protocol with communication delays (Sun et al. (2009), Xiao et al. (2008),Xiao et al. (2006)). Sun et al. (2009) complete consensus problems for multi-agent systems in directed networks with nonuniform time-varying delays including three cases were investigated. Many researchers also were interested in discrete-time multi-agent systems(Xiao et al. (2008),Xiao et al. (2006)). Xiao et al. (2008) proposed the necessary and sufficient conditions to guaranteeing the complete consensus for fixed and switching topologies by using the algebraic theory. Furthermore, they studied the consensus problems with time-varying delays. For more complex multi-agent systems, Xie and Wang designed a second-order algorithm which cause all the agents to converge to the same trajectory in Xie et al. (2007). They analyzed the consensus problem based on undirected graph and obtained the sufficient conditions eventually. Another typical second-order model was designed by Ren et al. (2007). After that there were numerous results on consensus problem, and for details please refer references(Gao et al. (2010),Gao et al. (2009).Sun et al. (2009),Lin et al. (2009), Yuan et al. (2010),Pan et al. (2014)).

All the mentioned publications above are about the complete consensus. However, complex networks in real worlds may be composed of interaction smaller subnetworks. The phenomenon of group (cluster) consensus is observed when an ensemble of oscillators splits into several subgroups, called group consensus throughout this paper. Group consensus has been studied extensively in the recent literature. Chen et al. (2011) solved the cluster consensus problem for first-order discrete-time multi-agent system based on the stochastic matrix theory. Qin et al. (2013) investigated the cluster consensus control for generic linear multi-agent system under directed interaction topology via distributed

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feedback controller. Furthermore, Yu et al. (2014) studied the cluster synchronization for network of linear systems under pinning control. For continuous-time systems, Yu et al. (2009) and Yu et al. (2011) presented the sufficient conditions for achieving the average couple-group consensus of fixed and switching topologies.

Inspired all the analysis above, we investigate a group consensus for multi-agent systems under the effect of coupling strength among agents in two groups. The results can be generalized to the case of the networks with more than two groups. Furthermore, we design the dynamics based on the original mathematic model presented in(Yu et al. (2009)) $\dot{x}_i(t) = u_i(t)$, and the ith topology was designed as follows:

$$u_i(t) = \begin{cases} \sum_{v_j \in N_{1i}} a_{ij}(x_j - x_i) + \sum_{v_j \in N_{2i}} a_{ij}x_j, \ \forall i \in \mathscr{F}_1; \\ \sum_{v_j \in N_{1i}} a_{ij}x_j + \sum_{v_j \in N_{2i}} a_{ij}(x_j - x_i), \ \forall i \in \mathscr{F}_2. \end{cases}$$

The remainder of this paper is organized as follows. In Section 2, problem formulation are introduced. Section 3 is the main part focusing on some mathematical models and main results. Section 4 presents several simulation examples which demonstrate the effectiveness of the main results and the effects of the two parameters γ_1, γ_2 on the group consensus. A short conclusion is given in Section 5.

Notation: Throughout this paper, the following notations are used. $\mathbf{I}$ denotes the identity matrix; $\mathbf{1} = [1, 1, \ldots, 1]^T$, $\mathbf{0} = [0, 0, \ldots, 0]^T$ with an appropriate dimension; "$*$" represents ellipsis for the term introduced by symmetric.

2. PROBLEM FORMULATION

2.1 Graphic theory

Here, the interaction among n $(n \geq 2)$ is modeled by a weighted directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$, of order n with the set of nodes $\mathscr{V} = \{v_1, v_2, \ldots, v_n\}$, and $\mathscr{A} = [a_{ij}]$ is a weighted adjacency matrix. The node indexes belong to the infinite index set $\mathscr{F} = \{1, 2, \ldots, n\}$. A directed edge of $\mathscr{G}$ is denoted by $e_{ij} = (v_j, v_i)$. The adjacency elements associated with the edges of the graph are positive, i.e.,$e_{ij} \in \mathscr{E}$ if and only if $a_{ij} \neq 0$. Moreover, we assume $a_{ii} = 0$ for all $i \in \mathscr{F}$. A directed path in $\mathscr{G}$ from i_1 to i_l is a sequence of ordered edges in the form of $(v_{i_k}, v_{i_{k+1}})$, $k = 1, 2, \ldots, l-1$ where $(v_{i_k}, v_{i_{k+1}}) \in \mathscr{E}$. If a directed graph has the property that (v_i, v_j) belongs to $\mathscr{E}$ for any $(v_j, v_i) \in \mathscr{E}$, the directed graph is called undirected. A directed graph is said to be strongly connected, if there is a directed path from every node to every other node. Correspondingly, the Laplacian matrix of directed graph is defined as $\mathscr{L} = [l_{ij}], l_{ij} = -a_{ij}, i \neq j$, and $l_{ii} = \sum_{k=1, k\neq i}^{n+m} a_{ik}$. Moreover, a directed graph is said to have spanning trees, if there exists a node such that there is a directed path from every other node to this node, and this node is called the root node of the spanning tree.

2.2 Mathematical models

In this paper, suppose that the multi-agent system under consideration consists of $n + m(n, m > 1)$ agents with continuous-time dynamics. Each agent is regarded as a node in the communication directed graph $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ and state $x = [x_1, x_2, \ldots, x_{n+m}]$. Denote $\mathscr{F}_1 = \{1, 2, \ldots, n\}$, $\mathscr{F}_2 = \{n+1, n+2, \ldots, n+m\}$, $\mathscr{V}_1 = \{v_1, v_2, \ldots, v_n\}$ $\mathscr{V}_2 = \{v_{n+1}, v_{n+2}, \ldots, v_{n+m}\}$, $\mathscr{V}_1 \cup \mathscr{V}_2 = \mathscr{V}$, $\mathscr{V}_1 \cap \mathscr{V}_2 = \phi$ and $\mathscr{F}_1 \cup \mathscr{F}_2 = \mathscr{F}$, $\mathscr{F}_1 \cap \mathscr{F}_2 = \phi$. Furthermore, let $N_{1i} = \{v_j \in \mathscr{V}_1 : (v_j, v_i) \in \mathscr{E}\}$, $N_{2i} = \{v_j \in \mathscr{V}_2 : (v_j, v_i) \in \mathscr{E}\}$ and $N_i = N_{1i} \cup N_{2i}$.

Suppose the dynamics of ith agent is given by

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n+m, \tag{1}$$

where $x_i(t)$ is the position state, $u_i(t)$ is the control input to be designed.

Here, for (1) we design the following consensus protocol

$$\dot{x}_i(t) = u_i(t) = \begin{cases} \sum_{v_j \in N_{1i}} a_{ij}(x_j - x_i) + \gamma_1 \sum_{v_j \in N_{2i}} a_{ij}x_j, \\ \qquad \forall i \in \mathscr{F}_1; \\ \gamma_2 \sum_{v_j \in N_{1i}} a_{ij}x_j + \sum_{v_j \in N_{2i}} a_{ij}(x_j - x_i), \\ \qquad \forall i \in \mathscr{F}_2, \end{cases} \tag{2}$$

where $a_{ij} \geq 0, \forall i, j \in \mathscr{F}_1$; $a_{ij} \geq 0, \forall i, j \in \mathscr{F}_2$; $a_{ij} \in R, \forall (i, j) \in \Phi = \{(i, j) : i \in \mathscr{F}_1, j \in \mathscr{F}_2\} \cup \{(i, j) : j \in \mathscr{F}_1, i \in \mathscr{F}_2\}$ and $\gamma_1 \cdot \gamma_2 \neq 0$.

Let

$$\bar{a}_{ij} = \begin{cases} \gamma_1 a_{ij}, & \forall i \in \mathscr{F}_1, \forall j \in \mathscr{F}_2, \\ \gamma_2 a_{ij}, & \forall i \in \mathscr{F}_2, j \in \mathscr{F}_1, \\ a_{ij}, & \forall i, j \in \mathscr{F}_1; \forall i, j \in \mathscr{F}_2. \end{cases}$$

Furthermore, denote the weighted adjacency matrix $\bar{\mathscr{A}} = [\bar{a}_{ij}]$ and the directed graph $\bar{\mathscr{G}} = (\mathscr{V}, \mathscr{E}, \bar{\mathscr{A}})$.

Now we give some definitions for later use to end this section.

Definition 1.(Yu et al., 2011) (Group Consensus). Given the dynamic system (1), we say that protocol (2) asymptotically solves the cluster consensus problem, if the states of agents satisfy

$$(a1) \lim_{t \to \infty} [x_i(t) - x_j(t)] = 0, \quad \forall i, j \in \mathscr{F}_1,$$
$$(a2) \lim_{t \to \infty} [x_i(t) - x_j(t)] = 0, \quad \forall i, j \in \mathscr{F}_2.$$

Definition 2.(Yu et al., 2011) (Sub-graph). A network with topology $\mathscr{G}_1 = (\mathscr{V}_1, \mathscr{E}_1, \mathscr{A}_1)$ is said to be sub-network of a network with topology $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ if (i) $\mathscr{V}_1 \subseteq \mathscr{V}$, (ii) $\mathscr{E}_1 \subseteq \mathscr{E}$. and (iii) the weighted adjacency matrix $\mathscr{A}_1$ inherits $\mathscr{A}$. Correspondingly, we call $\mathscr{G}_1$ a sub-graph of $\mathscr{G}$. Furthermore, if the inclusion relations in (i) and (ii) are strict, and $\mathscr{E}_1 = \{(v_i, v_j) : i, j \in \mathscr{V}_1, (v_i, v_j) \in \mathscr{E}\}$, we say that the first network is a proper sub-network of the second one. Correspondingly, we call $\mathscr{G}_1$ a proper sub-graph of $\mathscr{G}$.

Definition 3.(Yu et al., 2011) (Group average consensus). Let $x_i(0)$ be the initial state of agent v_i. Then protocol (2) is said to solve a group average-consensus problem asymptotically if the states of agents satisfy