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Determination of the bifurcation points of the power flow equations through optimisation-based methods

Roberto S. Salgado, Guido R. Moraes, Diego Issicaba*

Department of Electrical Engineering, Federal University of Santa Catarina, Florianópolis, SC, Brazil

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ABSTRACT

This paper describes the determination of quadratic turning points of the non-linear parameterised power flow equations using two optimisation-based approaches. The first is based on the trust region concept combined with sequential quadratic programming. The second uses tensor calculations as an extension of Newton's method. It is shown that the use of these algorithms imparts robustness to the iterative process, providing fast and reliable solutions. Numerical results obtained for problems with thousands of variables are used to illustrate the main features of the approaches, emphasising the robustness of the trust region-based approach.

1. Introduction

From a recent past, much attention has been devoted to the development of computational software to support the operator in the steady state analysis of electrical power systems, particularly the power flow and the optimal power flow programs. The analytical modelling of these problems requires the solution of a set of algebraic equations with a high degree of non-linearity. The conventional iterative methods (like Newton's, for instance) are prone to have difficulties of convergence while dealing with ill-conditioned systems, which mostly occurs when the load level of the power network is too high. Advanced strategies, like those based on the trust region concept, have been proposed to overcome these drawbacks. The preliminary applications of trust region-based algorithms in power system analysis have presented a number of desirable features [1,2], with very promising results in terms of robustness.

A very peculiar class of problems consists in finding the critical loadability of the power flow equations, that is, the load level beyond which there is no solution for the power flow equations. This problem has been associated with the identification of bifurcation points of the power flow equations. Usually, two types of local bifurcations are associated with the steady state aspects of the voltage collapse [3]. The first is the *saddle-node* bifurcation, in which the Jacobian matrix of the algebraic equations representing the steady state power system operation is singular. The second, named *limit-induced* bifurcation, is characterised by changes in the voltage stability condition, as a consequence of a particular variable reaching its limit. The present work focuses on the saddle-node bifurcation points corresponding to the maximum loadability of the steady state power flow equations (which is a static model) [4]. Although interesting from both the theoretical and the practical points of view of power system stability, other types of bifurcation (described in [5–7], for instance) are not focused on here.

Continuation methods have been applied [8] to find a sequence of power flow solutions from a base case to the critical load level. Only a few applications of direct methods to obtain the power flow solution corresponding to the critical load level are found in the literature. Reference [3] presents the basic concepts of saddle-node and limit-induced bifurcation points, as well as the determination of these critical points through the Continuation and Optimisation methods. It also shows the theoretical comparison between the solutions obtained through these methods, and presents numerical results obtained for a small test system through software packages (such as AMPL and KNITRO) to illustrate this comparison. Reference [9] formulates this problem in terms of the *Transversality Conditions* and [10] uses a constrained optimisation model. In both cases, Newton's method is applied to solve a set of non-linear algebraic equations. However, numerical strategies based on first order information are known to be very sensitive to the initial conditions, which is the main drawback for their application.

* Corresponding author.
E-mail addresses: roberto.salgado@ufsc.br (R.S. Salgado), guido-moraes@hotmail.com (G.R. Moraes), diego.issicaba@ufsc.br (D. Issicaba).

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In [11,12], the solution of a static optimisation problem is obtained through Newton's method extended to take advantage of the second order information of the Taylor series expansion. This improves the search for the critical solution and facilitates the treatment of the constraints. The latter proposes the use of a quadratic parameterisation, which reduces the dependence of the critical solution on the initial estimates.

This paper describes and compares two algorithms designed to find turning points of the set of non-linear power flow equations through direct methods. As a contribution, in the first algorithm, the problem is modelled as a constrained optimisation problem, which is solved through a combination of sequential quadratic programming with a trust region strategy [13,14]. The main idea is to adjust the solutions belonging to two orthogonal subspaces, one aimed at satisfying the constraints and the other searching for improving the objective function. The second algorithm takes profit of the second order information (or tensor term) of the Taylor series expansion of the equations representing the optimality conditions, as proposed in [15,16]. Numerical results obtained for problems with thousands of variables are used to illustrate the features of these approaches. In summary, this paper is aimed at showing that the proposed methodology can be numerically robust (because of the Orthogonal matrices), reliable (because of the Trust region) and faster (because of the use of the second order (tensor) term). This brings complementary aspects to the analysis of saddle-node bifurcation points, with respect to what has been presented in the literature.

The remaining of this paper is organised as follows: Section 2 shows the analytical formulation of the optimisation problem that represents the determination of the critical loadability, and the relationship between its Optimality Conditions and the Transversality Conditions; the basic solution methods and the optimisation algorithms are described in Section 3; Section 4 presents numerical results obtained from the computational implementation of the approaches; and the main conclusions are presented in Section 5.

2. Analytical model

2.1. The parameterised power flow equations

Supposing that n_b is the total number of buses of the power system and that rectangular coordinates are used to model the complex bus voltages, the power balance equations of the ith bus are expressed as,

$$\begin{aligned}
\Delta P_i &= P_{g_i} - P_{d_i} - e_i \sum_{j=1}^{n_b} (G_{ij}e_j - B_{ij}f_j) + f_i \sum_{j=1}^{n_b} (G_{ij}f_j + B_{ij}e_j) = 0 \\
\Delta Q_i &= Q_{g_i} - Q_{d_i} - f_i \sum_{j=1}^{n_b} (G_{ij}e_j - B_{ij}f_j) - e_i \sum_{j=1}^{n_b} (G_{ij}f_j + B_{ij}e_j) = 0 \\
\Delta V_i &= (V_i^{sp})^2 - (e_i^2 + f_i^2) = 0
\end{aligned} \tag{1}$$

where i refers to the bus index; ΔP_i, ΔQ_i and ΔV_i are respectively the active power, reactive power and voltage magnitude mismatches; P_{g_i} and Q_{g_i} are the active and reactive power generation; P_{d_i} and Q_{d_i} refer to the previously known active and reactive power demand; e_i and f_i are the real and imaginary components of the complex bus voltage; V_i^{sp} is the pre-specified voltage magnitude; and G_{ij} and B_{ij} are constant components of the bus admittance matrix (which depends only on the topology of the network and the parameters of the transmission lines).

Solving the power flow equations consists in determining the complex bus voltages, such that the mismatch values satisfy a pre-specified tolerance (usually 10^{-3} per unit). Traditionally, there are $(n_b - 1)$ equations related to the active power mismatches. The equations involving ΔV_i correspond to the set of buses with devices (generators, for instance) to control the bus voltage magnitude/reactive power generation. In case a device achieves a reactive power operational limit, the equation related to ΔV_i is interchanged with its reactive power balance equation. Equations related to ΔQ_i correspond to the set of buses without voltage magnitude control devices. As a matter of fact, this procedure is referred to as PV-PQ bus type conversion. The number of equations of ΔQ_i plus ΔV_i is also $(n_b - 1)$. Thus, the power flow problem stated by Eq. (1) involves $n_{eq} = 2n_b - 2$ equations which can be alternatively written as,

$$\begin{aligned}
&P_{g_i} - P_{d_i} - P_i(e, f) = 0 \\
&Q_{g_i} - Q_{d_i} - Q_i(e, f) = 0 \\
&(V_i^{sp})^2 - (e_i^2 + f_i^2) = 0
\end{aligned} \tag{2}$$

where $P_i(e, f)$ and $Q_i(e, f)$ are the bus active and reactive power injections expressed as functions of the real and imaginary terms of the complex voltages. These terms compose a set of $n_{vr} = 2n_b - 2$ variables, since one pair (e_i, f_i) must be pre-specified to balance network losses.

In order to model analytically the determination of the maximum loadability in steady state, the power balance equations of the ith bus are parameterised as suggested in [8], that is,

$$\begin{aligned}
&P_{g_i} - (P_{d_i}^0 + \rho^2 \Delta P_{d_i}) - P_i(e, f) = 0 \\
&Q_{g_i} - (Q_{d_i}^0 + \rho^2 \Delta Q_{d_i}) - Q_i(e, f) = 0
\end{aligned} \tag{3}$$

where $P_{d_i}^0$ and $Q_{d_i}^0$ refer to the active and reactive power demand of the ith bus in a base case, ΔP_{d_i} and ΔQ_{d_i} represent the pre-specified change rate of the active and reactive power load of bus i, ρ is a scalar named *load parameter*, and the other variables have been previously defined. The quadratic parameterisation is equivalent to imposing a non-negativity constraint in the load variation, ensuring that the critical loadability will always be greater than (or at least equal to) a base case loadability. Eq. (3) represents not only a single isolated nonlinear system to be solved, but a family of problems depending on the load parameter ρ. Note that: (1) $\rho = 0$ corresponds to the base case power flow problem, and (2) there is a maximum value for ρ, beyond which there is no power flow solution.

Pre-specified change rates of power load (and generation) can be specified from previously collected data or even from real time load measurements. These values are estimated for each bus, providing a very particular load variation (and eventually generation change) as the load parameter is modified [3]. In the present work, we have specified a change rate of 1% of the active and reactive power of the base case for each bus.

It must be pointed out that since the traditional power flow modelling is adopted, the slack bus compensates the active power balance, such that it has no direct influence on the load increase (except if its capacity is reached). On the other hand, the active power generation of the PV buses can be parameterised; that is, it can be expressed as $P_{g_i} = P_{g_i}^0 + \rho \Delta P_{g_i}$, which requires only the specification of the generation change rate (ΔP_{g_i}) [3,6]. These buses support the largest amount of the load increase, since the attribution of the whole amount of load change to the slack bus could be considered inadequate. Alternatively, the active power generation could be included in the set of optimisation variables. However, this procedure would require the extension of the optimisation process, since in this case the dimension of the problem would be larger, which is a disadvantage in terms of both the additional calculations required and the simplicity of the analytical formulation.

The set of parameterised power flow equations is expressed in a compact form as,

$$\mathbf{g}(\mathbf{x}, \rho) = \mathbf{0} \tag{4}$$