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Identification for Switched Systems ⋆

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Abstract: For a class of unstable discrete-time multi-variable switched systems with parametric uncertainty, an identification scheme is developed in this paper. Specifically, the identification algorithm is derived to identify the parameter vectors of potential subsystems based on the idea of least geometric mean squares. Furthermore, an neural network classifier is put forward for partition of the identified subsystems. Finally, feasibility and effectiveness of the proposed identification scheme are verified by numerical simulations, which show that the desirable identification performance is guaranteed with the proposed scheme.

Keywords: Identification with multiple models, neural network classifier, least geometric mean squares, unstable discrete-time multi-variable switched systems.

1. INTRODUCTION

Recently, issues relating to identification and control of switched system are of growing concern. The significance of research on switched system and switching control is highlighted in Leith et al. (2003) from several perspectives, including description or identification for many physical systems with switching in dynamics, improvement of control performance for systems with large uncertainty with multiple model switching control Narendra and Balakrishnan (1994), guarantee of robustness in the presence of failure such as reconfigurable flight control in literatures Boskovic et al. (1999); Boskovic and Mehra (2002) and removal of control system constraints Goodwin et al. (2001).

Concerning identification of linear switched systems, four different approaches, namely the algebraic procedure Vidal et al. (2003), the Bayesian procedure Juloski et al. (2005b), the clustering-based procedure Ferrari-Trecate et al. (2003); Nakada et al. (2005) and the bounded-error procedure Bemporad et al. (2005), have been proposed recently. Refer to Juloski et al. (2005a) and Paoletti et al. (2007) for detailed summary and comparison of the above four procedures, from which it will be found out that each approach has its own drawback. In this paper, identification using multiple models introduced in Lai et al. (2011) for single-variable time-varying systems is studied, and an extension of this identification algorithm is derived for multi-variable systems with qualitative analysis.

The main contributions of this paper are highlighted as follows:

(i) The class of systems considered in this paper is unstable linear discrete-time multi-variable systems with parametric uncertainty. Moreover, the unknown parameter vector is a piecewise constant function of time, and additionally the abrupt changes in value of the parameter vector occur at *a prior* unknown time instants.

(ii) For identification of uncertainty, indispensable preliminary work has been discussed and conducted. Furthermore, a parameter identification algorithm is proposed for the aforementioned systems. A set of subsystems is established after parameter identification, and then a neural network classifier is derived from Lai et al. (2011) to implement partition among the subsystems. The training and establishment of the classifier are conducted off-line based on recorded data.

The rest of this paper is organized as follows. Preliminary work is introduced in Section 2. In Section 3, an identification algorithm is brought forward with an neural network classifier proposed for the partition of the identified subsystems. Section 4 presents a simulation example to validate the effectiveness and feasibility of the proposed identification scheme.

2. PRELIMINARY WORK

At first, we introduce the following notations that will be used throughout the paper. The symbol I_n stands for an $n \times n$ identity matrix, and I used sometimes represents an identity matrix of appropriate size. In addition, the superscript T denotes transpose of matrices or vectors. The norm of vector x is defined by $\|x\| \triangleq \sqrt{x^T x}$, which is effectively the equivalent of Euclidean norm in real number space. As for matrices, operator $\|\cdot\|$ amounts to 2 norm. $E_{i,j}$ represents a square matrix of proper dimension whose elements are all zeros except for the i^{th} element in the j^{th} column which is 1 instead. For a matrix $A \in \mathcal{R}^{m\times n}$, a_{ij} is the i^{th} element of the j^{th} column. For a set of matrices, $\{F_1, F_2, \ldots, F_n\}$, with $F_i \in \mathcal{R}^{l\times l}$, for $i = 1, 2, \ldots, n$, $F_{i(b,j)}$ denotes the b^{th} element in the j^{th} column of matrix F_i, for $b, j \in \{1, 2, \ldots, l\}$; The Kronecker product is described by $\otimes$, take for example $P \otimes Q$ with $P = [p_{ij}] \in \mathcal{R}^{m\times n}$ and $Q = [q_{ij}] \in \mathcal{R}^{p\times q}$, it is obtained that $P \otimes Q = [p_{ij}Q] \in \mathcal{R}^{(mp)\times(nq)}$; $col[Y]$ denotes a vector formed by columns of matrix Y, that is, if $Y = [Y_{c1}, Y_{c2}, \ldots, Y_{cn}] \in \mathcal{R}^{m\times n}$, then $col[Y] = [Y_{c1}^T, Y_{c2}^T, \ldots, Y_{cn}^T]^T \in \mathcal{R}^{mn}$; $O_{m\times n}$ represents the matrix whose elements are all zeros, while $I^+_{(n-1)l\times(n-1)l}$ denotes the matrix indicated as follow,

$$I^+_{(n-1)l\times(n-1)l} = \begin{bmatrix} O_{(n-2)l\times l} & I_{(n-2)l} \\ O_{l\times l} & O_{l\times(n-2)l} \end{bmatrix}. \quad (1)$$

In practical terms, there are many systems with abrupt changes in dynamics, and we refer to such time-varying systems as switched systems. Let set $\{t_j | t_j \in \mathcal{R}^+, j \in \mathcal{N}^+\}$

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represents the switching times with $t_j \leq t_{j+1}$. To indicate the characteristics of the switched system at the switching times, two sets are determined as

$\{\bar{t}_{i,k_i}\} = \{t_j + 1|$ when subsystem i is switched on for the k_i times, $i \in \mathcal{P}$, $k_i \in \mathcal{N}^+$ $\}$;

$\{\underline{t}_{m,k_m}\} = \{t_j|$ when subsystem m is switched off for the k_m times, $m \in \mathcal{P}$, $k_m \in \mathcal{N}^+$ $\}$

where subsystem m and subsystem i are accordingly defined as the pre- and post-switching subsystems with respect to the switching time t_j.

As a result, the switching signal $\sigma(t)$ of the switched system is defined as a piecewise constant function of time with $\sigma : [0, \infty) \to \mathcal{P}$, where the index set $\mathcal{P} = \{1, 2, \ldots, N\}$ is finite, and N is the number of subsystems. In addition, $\sigma(t)$ changes with respect to $\{t_j\}$.

Subsequently, given the controlled system in the form of input/output mathematical model, how to acquire its linear regression model and state space representation via equivalent transformation is discussed here, which are the preliminaries for parameter estimation and state feedback control design in the following sections, respectively.

Consider the multi-variable switched systems described by

$$D^{\{\sigma(t)\}}(z^{-1})y(t) = M^{\{\sigma(t)\}}(z^{-1})u(t) \tag{2}$$

where $u(t), y(t) \in \mathcal{R}^l$ are the input and output vectors of the system, respectively. The symbol z is used to denote the time advance operator $zx(t) \triangleq x(t+1)$, while z^{-1} is the backward shift operator, i.e., $z^{-1}x(t+1) \triangleq x(t)$. Here, we define sampling period of the discrete-time system (2) as T_s, which is the interval between two consecutive time instants. Moreover, matrix polynomials $D^{\{\sigma(t)\}}(z^{-1}), M^{\{\sigma(t)\}}(z^{-1}) \in \mathcal{R}^{l\times l}$ are given as

$$\begin{aligned} D^{\{\sigma(t)\}}(z^{-1}) &= I_l + D_1^{\{\sigma(t)\}} z^{-1} + \cdots + D_n^{\{\sigma(t)\}}(t) z^{-n}, \\ M^{\{\sigma(t)\}}(z^{-1}) &= M_1^{\{\sigma(t)\}} z^{-1} + \cdots + M_n^{\{\sigma(t)\}} z^{-n}. \end{aligned} \tag{3}$$

where n represents the order of the system which is supposed to be *a priori* knowledge. With $D_i^{\{\sigma(t)\}}, M_i^{\{\sigma(t)\}} \in \mathcal{R}^{l\times l}$, for $i = 1, 2, \ldots, n$, $\{D_1^{\{\sigma(t)\}}, \ldots, D_n^{\{\sigma(t)\}}\}$ and $\{M_1^{\{\sigma(t)\}}, \ldots, M_n^{\{\sigma(t)\}}\}$ are two sets of coefficient matrices for $D^{\{\sigma(t)\}}(z^{-1})$ and $M^{\{\sigma(t)\}}(z^{-1})$, respectively.

2.1 Linear regression model

From (3), the discrete-time autoregressive moving average (ARMA) model for the linear switched system described in (2) can be derived as

$$y(t) = \Theta_{\sigma(t)}^T \phi(t), \tag{4}$$

with

$$\begin{aligned} \Theta_{\sigma(t)} &= [D_1^{\{\sigma(t)\}}, \ldots, D_n^{\{\sigma(t)\}}, M_1^{\{\sigma(t)\}}, \ldots, M_n^{\{\sigma(t)\}}]^T, \\ \phi(t) &= [-y(t-1)^T, \ldots, -y(t-n)^T, \\ &\quad u(t-1)^T, \ldots u(t-n)^T]^T, \end{aligned}$$

Moreover, there exists parametric uncertainty in matrices $D_i^{\{\sigma(t)\}}, M_i^{\{\sigma(t)\}}$ for $i = 1, \ldots, n$.

For parameter identification, the system (4) can be rewritten as a multi-variable linear regression model, i.e.,

$$y(t) = \Psi(t)\theta_{\sigma(t)} \tag{5}$$

where $\theta_{\sigma(t)} \in \mathcal{R}^{n_\theta}$ is the parameter vector to be identified, and $\Psi(t) \in \mathcal{R}^{l\times n_\theta}$ is the information matrix consisting of the input and output vectors $u(t-i)$, $y(t-i)$.

Exactly, the following two cases are taken into consideration for the explicit formulation of the identification problem for the system (4).

Case 1: If the matrices $D_i^{\{\sigma(t)\}}, M_i^{\{\sigma(t)\}}$ for $i = 1, \ldots, n$ are absolutely unknown, it follows that $\Psi(t)$ and θ in (5) will be written as

$$\begin{aligned} \Psi(t) &= I_l \otimes \phi^T(t), \\ \theta_{\sigma(t)} &= col[\Theta_{\sigma(t)}]. \end{aligned} \tag{6}$$

Case 2: If the matrices $D_i^{\{\sigma(t)\}}, M_i^{\{\sigma(t)\}}$ for $i = 1, \ldots, n$ are partly unknown, namely, only some entries of several matrices are unknown time-varying parameters to be identified, $\Psi(t)$ and θ in (5) will instead be expressed as

$$\begin{aligned} \Psi(t) &= [y_{pk}(t), E_{b,j}y(t-i), \ldots, E_{c,k}u(t-m), \ldots], \\ \theta_{\sigma(t)} &= [1, D_{i(b,j)}^{\{\sigma(t)\}}, \ldots, M_{m(c,k)}^{\{\sigma(t)\}}, \ldots]^T, \end{aligned} \tag{7}$$

where $D_{i(b,j)}^{\{\sigma(t)\}}$ and $M_{m(c,k)}^{\{\sigma(t)\}}$ are the uncertain time-varying parameters, for $i, m \in \{1, \ldots, n\}$, $b, j, c, k \in \{1, \ldots, l\}$, and $y_{pk}(t)$ is calculated as

$$y_{pk}(t) = \bar{\Theta}_{\sigma(t)}^T \phi(t) \tag{8}$$

where $\bar{\Theta}_{\sigma(t)}$ is derived from $\Theta_{\sigma(t)}$ with the only difference between them being that all the uncertain parameters are set to be zeros in $\bar{\Theta}_{\sigma(t)}$.

Remark 1. The above two cases considered in this paper are of great significance from the following two aspects: (i) as for Case 1 where all the elements in $\Theta_{\sigma(t)}$ are of uncertainty, transformation from (4) to (5) with (6) enables identification issue to be solved with least square algorithm, (ii) while in Case 2 conversion of (4) to (5) with (7) simplifies the problem by minimizing the dimension of the parameter vector to be identified. Meanwhile, the true value for the first element of the parameter vector is 1 as presented in (7), which will be one of the criteria for verification of convergence of parameter estimation.

2.2 State space representation

The system (4) developed in Section 2.1 can be rewritten in a state-space form $x(t+1) = f(x(t), u(t))$ by introducing $x(t) = [u^T(t-n+1), \ldots, u^T(t-1), y^T(t-n+1), \ldots, y^T(t-1), y^T(t)]^T \in \mathcal{R}^{(2n-1)l}$ as state vector of the system. The state space representation is consequently given as

$$\begin{aligned} x(t+1) &= A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \\ y(t) &= C_{\sigma(t)}x(t) \end{aligned} \tag{9}$$

with

$$A_{\sigma(t)} = \begin{bmatrix} I^+_{(n-1)l\times(n-1)l} & O_{(n-1)l\times nl} \\ O_{(n-1)l\times(n-1)l} & I'_{(n-1)l\times nl} \\ \bar{M}_{\sigma(t)} & -\bar{D}_{\sigma(t)} \end{bmatrix},$$

$$\begin{aligned} B^T_{\sigma(t)} &= [O_{l\times(n-2)l}, I_l, O_{l\times(n-1)l}, (M_1^{\{\sigma(t)\}})^T], \\ C_{\sigma(t)} &= [O_{l\times 2(n-1)l}, I_l] \end{aligned}$$

and

$$\begin{aligned} \bar{M}_{\sigma(t)} &= [M_n^{\{\sigma(t)\}}, \ldots, M_2^{\{\sigma(t)\}}], \\ \bar{D}_{\sigma(t)} &= [D_n^{\{\sigma(t)\}}, \ldots, D_1^{\{\sigma(t)\}}], \end{aligned} \tag{10}$$

where $I'_{(n-1)l\times nl}$ is the first $(n-1)l$ rows of $I^+_{nl\times nl}$.

Additionally, introduce the definition of the accumulation of tracking error:

$$e_I(t+1) = y(t) - y_r(t) + e_I(t) \tag{11}$$

where $y_r(t) \in \mathcal{R}^l$ is reference signal for the controlled system.