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### $I$ dentification for Switched Systems  $\star$ Qiong Hu <sup>∗</sup> Qing Fei <sup>∗</sup> Hongbin Ma <sup>∗</sup> Qinghe Wu <sup>∗</sup> IFAC-PapersOnLine 48-28 (2015) 514–519<br> **Identification for Switched Systems**  $\star$ Identification for Switched Systems $^{\star}$

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intervalsed by a determinant of the matrix of the matrix of the second and the parameters of the identification algorithm is derived to identify the parameter vectors of potential subsystems based on the idea of least geometric mean squares. Furthermore, an neural network classifier is put forward for partition of the identified subsystems. Finally, reasibility and effectiveness of the proposed identification scheme are verified by numerical simulations, which show that the desirable identification performance is guaranteed with the proposed scheme. Abstract: For a class of unstable discrete-time multi-variable switched systems with parametric idea of least geometric mean squares. Furthermore, an neural network classifier is put forward for partition of the identified subsystems. Finally, feasibility and effectiveness of the proposed algorithm is derived to identify the parameter vectors of potential subsystems based on the parameter of potential substitution of potential substitution of the parameter of potential substitution of  $\alpha$  and  $\alpha$  and  $\alpha$ idea of least geometric mean squares. Furthermore, an neural network classifier is put forward<br>for partition of the identified subsystems. Finally, feasibility and effectiveness of the proposed identification performance is guaranteed with the proposed scheme. for partition of the identified subsystems. Finally, feasibility and effectiveness of the proposed<br>identification scheme are verified by numerical simulations, which show that the desirable  $S_{\rm{max}}$  Institute of Technology, Beijing 100081, P. R. China 100081, P. R. Chin idea of least geometric mean squares. Furthermore, an neural network classifier is put forward<br>for partition of the identified subsystems. Finally, feasibility and effectiveness of the proposed

© 2015, IFAC (International Federation of Automatic Control) Hosting by Elsevier Ltd. All rights reserved.  $\in$  2010, If the (international Federation of Hatomatic Control) Hobting by Ekovici Eta. Hi Hgmor Roch ved.  $\alpha$ identification performance is guaranteed with the proposed scheme. The proposed scheme scheme. The proposed scheme identification scheme are verified by numerical simulations, which show that the desirable  $\heartsuit$  2015, IFAC (International Federation of Automatic Control) Hosting by E

Keywords: Identification with multiple models, neural network classifier, least geometric mean Regwords: Identification with multiple models, neural network classifier, least geometric mean<br>squares, unstable discrete-time multi-variable switched systems. squares, unstable discrete-time multi-variable switched systems.

# 1. INTRODUCTION 1. INTRODUCTION 1. INTRODUCTION 1. <del>INTRODUCTION</del>

Recently, issues relating to identification and control of recently, issues relating to identification and control of<br>switched system are of growing concern. The significance of research on switched system and switching control is highlighted in Leith et al. (2003) from several perspectives, including description or identification for many physical including description or identification for many physical<br>systems with switching in dynamics, improvement of control performance for systems with large uncertainty with trol performance for systems with large trol performance for systems with large uncertainty with<br>multiple model switching control Narendra and Balakrmuttiple model switching control Narendra and Balakr-<br>ishnan (1994), guarantee of robustness in the presence of failure such as reconfigurable flight control in literatures ranure such as reconfigurable hight control in interatures<br>Boskovic et al. (1999); Boskovic and Mehra (2002) and removal of control system constraints  $Goodwin$  et al.  $(2001)$ . of research on switched system and switching control is or research on switched system and switching control is<br>highlighted in Leith et al. (2003) from several perspectives, switched system are of growing concern. The significance ing inguigmed in Lenin et al. (2005) from several perspectives,<br>including description or identification for many physical  $\frac{1}{s}$  systems with switching in dynamics, improvement of con-<br>trol performance for systems with large uncertainty with  $\frac{1}{2}$ ishnan (1994), guarantee of robustness in the presence of  $\frac{1}{2}$ moval of control system constraints Goodwin et al. (2001). moval of control system constraints Goodwin et al. (2001). Boskovic et al. (1999); Boskovic and Mehra (2002) and reof research on switched system and switching control is highlighted in Leith et al. (2003) from several perspectives, switched system are of growing control is of research on switched system and switching control is highlighted in Leith et al. (2003) from several perspectives.

Concerning identification of linear switched systems, four Concerning identification of linear switched systems, four<br>different approaches, namely the algebraic procedure Vidal et al. (2003), the Bayesian procedure Juloski et al.  $(2005b)$ , the clustering-based procedure Ferrari-Trecate  $(2005b)$ , the clustering-based procedure Ferrari-Trecate  $(2005b)$ , the clustering-based procedure Ferrari-Trecate<br>et al.  $(2003)$ ; Nakada et al.  $(2005)$  and the bounded-error procedure Bemporad et al. (2005), have been proposed procedure Bemporad et al. (2006), have been proposed<br>recently. Refer to Juloski et al. (2005a) and Paoletti et al. from Four procedures, from which it all  $(2000a)$  and a concentrate all  $(2007)$  for detailed summary and comparison of the above four procedures, from which it will be found out that each<br>connect health carried that the connect that the connect show that the connection of approach has its own drawback. In this paper, identifica-<br>tion using multiple models introduced in Lei, et al.  $(2011)$  $\epsilon$  contains multiple models introduced in Lai et al. (2011)<br>for single variable time varying systems is studied and  $\frac{1}{2}$  for single-variable time-varying systems is studied, and an extension of this identification algorithm is derived for<br>multi-multi-multi-multi-multi-multi-time analysis multi-variable systems with qualitative analysis. different approaches, namely the algebraic procedure Vi-<br>del et al.  $(2003)$  the Bayesian procedure Juloski et al.  $f(2007)$  for detailed summary and comparison of the above<br>four procedures, from which it will be found out that each

The main contributions of this paper are highlighted as follows: follows:  $\sum_{i=1}^{n}$  The considered in this paper is unstable in this paper is unstable in this paper is unstable in the constant in The main contributions of this paper are highlighted as The main contributions of this paper are highlighted as

 $(i)$  The class of systems considered in this paper is unstable linear discrete-time multi-variable systems with paramet-<br>ric uncertainty. Moreover, the unknown parameter vector ric uncertainty. Moreover, the unknown parameter vector ric uncertainty. Moreover, the unknown parameter vector<br>is a piecewise constant function of time, and additionally the abrupt changes in value of the parameter vector occur at a prior unknown time instants. (i) The class of systems considered in this paper is unstable<br>linear discrete time multi-variable systems with paramet is a piecewise constant function of time, and additionally<br>the abrupt changes in value of the parameter vector occur the abrupt changes in value of the parameter vector occur<br>at *a prior* unknown time instants.

(ii) For identification of uncertainty, indispensable prelim- $\lim_{x \to a}$  work has been discussed and conducted. Further- $\frac{1}{2}$  more, a parameter identification algorithm is proposed for the aforementioned systems. A set of subsystems is  $\overline{a}$ (ii) For identification of uncertainty, indispensable prelim-<br>inexy work has been discussed and conducted. Eurtherfor the aforementioned systems. A set of subsystems is established after parameter identification, and then a neuestablished after parameter identification, and then a neu-<br>ral network classifier is derived from Lai et al. (2011) to<br>implement partition among the subsystems. The training ral network classifier is derived from Eal et al. (2011) to<br>implement partition among the subsystems. The training and establishment of the classifier are conducted off-line based on recorded data. implement partition among the subsystems. The training<br>and establishment of the classifier are conducted off-line based on recorded data. based on recorded data. and establishment of the classifier are conducted off-line

The rest of this paper is organized as follows. Preliminary The rest of this paper is organized as follows. Preliminary<br>work is introduced in Section 2. In Section 3, an identiwork is introduced in section 2. In section 5, an identi-<br>fication algorithm is brought forward with an neural netncation algorithm is brought forward with all heural het-<br>work classifier proposed for the partition of the identified subsystems. Section 4 presents a simulation example to subsystems. Section 4 presents a simulation example to validate the effectiveness and feasibility of the proposed validate the effectiveness and feasibility of the proposed<br>identification scheme. work classifier proposed for the partition of the identified<br>subsystems. Section 4 presents a simulation example to identification scheme.

# 2. PRELIMINARY WORK 2. PRELIMINARY WORK 2. PRELIMINARY WORK 2. PRELIMINARY WORK

At first, we introduce the following notations that will be used throughout the paper. The symbol  $I_n$  stands for an  $n \times n$  identity matrix, and I used sometimes represents an identity matrix of appropriate size. In addition, the superscript T denotes transpose of matrices or vectors. The norm of vector x is defined by  $||x|| \triangleq \sqrt{x^T x}$ , which is effectively the equivalent of Euclidean norm in real number space. As for matrices, operator  $\|\cdot\|$  amounts to 2 norm.<br>  $E_{i,j}$  represents a square matrix of proper dimension whose elements are all zeros except for the  $i^{th}$  element in the  $j^{th}$ column which is 1 instead. For a matrix  $A \in \mathcal{R}^{m \times n}$ ,  $a_{ij}$  is the  $i^{th}$  element of the  $j^{th}$  column. For a set of matrices,  ${F_1, F_2, \ldots, F_n}$ , with  $F_i \in \mathcal{R}^{l \times l}$ , for  $i = 1, 2, \ldots, n$ ,  $F_{i(b,j)}$ denotes the  $b^{th}$  element in the  $j^{th}$  column of matrix  $F_i$ , for  $b, j \in \{1, 2, \ldots, l\}$ ; The Kronecker product is described by ⊗, take for example  $P \otimes Q$  with  $P = [p_{ij}] \in \mathcal{R}^{m \times n}$  and  $Q = [q_{ij}] \in \mathcal{R}^{p \times q}$ , it is obtained that  $P \otimes Q = [p_{ij}Q] \in$  $\mathcal{R}^{(mp)\times(nq)}$ ; col[Y] denotes a vector formed by columns of matrix Y, that is, if  $Y = [Y_{c1}, Y_{c2}, \dots, Y_{cn}] \in \mathbb{R}^{m \times n}$ , then  $col[Y] = [Y_{c1}^T, Y_{c2}^T, \dots, Y_{cn}^T]^T \in \mathcal{R}^{mn}$ ;  $O_{m \times n}$  represents the matrix whose elements are all zeros, while  $I_{(n-1)l \times (n-1)l}^+$ denotes the matrix indicated as follow,<br> $\begin{bmatrix} 0 & I \end{bmatrix}$ used throughout the paper. The symbol  $I_n$  stands for an  $n \times n$  identity matrix, and I used sometimes represents an identity matrix of appropriate size. In addition, the superscript  $T$  denotes transpose of matrices or vectors. The norm of vector x is defined by  $||x|| \triangleq \sqrt{x^T x}$ , which is effectively the equivalent of Euclidean norm in real number space. As for matrices, operator  $\|\cdot\|$  amounts to 2 norm.  $\bar{E}_{i,j}$  represents a square matrix of proper dimension whose elements are all zeros except for the  $i^{th}$  element in the  $j^{th}$ column which is 1 instead. For a matrix  $A \in \mathcal{R}^{m \times n}$ ,  $a_{ij}$  is the *i*<sup>th</sup> element of the *j*<sup>th</sup> column. For a set of matrices,  $\{F_1, F_2, \ldots, F_n\}$ , with  $F_i \in \mathcal{R}^{t \times t}$ , for  $i = 1, 2, \ldots, n$ ,  $F_{i(b, j)}$ denotes the  $b^{th}$  element in the  $j^{th}$  column of matrix  $F_i$ , for  $b, j \in \{1, 2, ..., l\}$ ; The Kronecker product is described by ⊗, take for example  $P \otimes Q$  with  $P = [p_{ij}] \in \mathcal{R}^{m \times n}$  and  $Q = [q_{ij}] \in \mathcal{R}^{p \times q}$ , it is obtained that  $P \otimes Q = [p_{ij}Q] \in \mathcal{R}^{(mn)\times (pq)}$ matrix Y, that is, if  $Y = [Y_{c1}, Y_{c2}, \dots, Y_{cn}] \in \mathcal{R}^{m \times n}$ , then  $col[Y] = [Y_{c1}^T, Y_{c2}^T, \dots, Y_{cn}^T]^T \in \mathcal{R}^{mn}$ ;  $O_{m \times n}$  represents the matrix whose elements are all zeros, while  $I_{(n-1)l \times (n-1)l}^+$ systems assume the system of the contrast of At first, we introduce the following hotations that will be<br>used throughout the paper. The symbol I stands for an used throughout the paper. The symbol  $I_n$  stands for an  $n \times n$  identity matrix, and I used sometimes represents  $n \times n$  identity matrix, and  $T$  used sometimes represents an identity matrix of appropriate size. In addition, the an identity matrix of appropriate size. In addition, the superscript  $T$  denotes transpose of matrices or vectors. superscript T denotes transpose of matrices or vectors.<br>The norm of vector x is defined by  $||x|| \triangleq \sqrt{x^T x}$ , which is effectively the equivalent of Euclidean norm in real number of Euclidean norm of vector x is defined by  $||x|| \triangleq \sqrt{x^T x}$ , which is enectively the equivalent of Euclidean norm in real number<br>space. As for matrices, operator  $\|\cdot\|$  amounts to 2 norm.<br> $E_{i,j}$  represents a square matrix of proper dimension whose  $E_{i,j}$  represents a square matrix of proper unicision whose elements are all zeros except for the  $i^{th}$  element in the  $j^{th}$  column which is 1 instead. For a matrix  $A \in \mathcal{R}^{m \times n}$ ,  $a_{ii}$  is the *i*<sup>th</sup> element of the *i*<sup>th</sup> column. For a set of matrices, the *i*<sup>th</sup> element of the *j*<sup>th</sup> column. For a set of matrices,<br> $\{F_1, F_2, \ldots, F_n\}$ , with  $F_i \in \mathcal{R}^{l \times l}$ , for  $i = 1, 2, \ldots, n$ ,  $F_{i(h,i)}$ denotes the  $b^{th}$  element in the  $j^{th}$  column of matrix  $F_i$ , for  $b, i \in \{1, 2, \ldots, l\}$ : The Kronecker product is described by  $\emptyset, \emptyset \in \{1, 2, \ldots, \ell\}$ ; The Kronecker product is described by  $\emptyset$ , take for example  $P \otimes Q$  with  $P = [p_{ij}] \in \mathbb{R}^{m \times n}$  and ⊗, take for example  $P \otimes Q$  with  $P = [p_{ij}] \in \mathcal{R}^{m \times n}$  and  $Q = [q_{ij}] \in \mathcal{R}^{p \times q}$ , it is obtained that  $P \otimes Q = [p_{ij}] \in$  $\mathcal{N} \longrightarrow \mathcal{N}$ ,  $\omega_1$  | denotes a vector formed by columns of matrix Y, that is, if  $Y = [Y_{c1}, Y_{c2}, \dots, Y_{cn}] \in \mathcal{R}^{m \times n}$ , then matrix 1, that is, if  $I = [I_{c1}, I_{c2}, \ldots, I_{cn}] \in \mathcal{N}$ , then<br>  $col[Y] = [Y_1^T, Y_2^T, \ldots, Y_n^T]^T \in \mathcal{R}^{mn}$ ;  $O_{m \times n}$  represents the  $col[T] = [T_{c1}, T_{c2}, \dots, T_{cn}] \in \mathcal{K}$ ;  $O_{m \times n}$  represents the matrix whose elements are all zeros, while  $I_{(n-1)l \times (n-1)l}^+$  $n \times n$  identity matrix, and I used sometimes represents<br>an identity matrix of appropriate size. In addition, the effectively the equivalent of Euclidean norm in real number column which is 1 instead. For a matrix  $A \in \mathcal{R}^{m \times n}$ ,  $a_{ij}$  is  $\{F_1, F_2, \ldots, F_n\}$ , with  $F_i \in \mathcal{R}^{l \times l}$ , for  $i = 1, 2, \ldots, n$ ,  $F_{i(b,j)}$  $b, j \in \{1, 2, \ldots, l\}$ ; The Kronecker product is described by  $Q = [q_{ij}] \in \mathcal{R}^{p \times q}$ , it is obtained that  $P \otimes Q = [p_{ij}Q] \in$ matrix Y, that is, if  $Y = [Y_{c1}, Y_{c2}, \dots, Y_{cn}] \in \mathcal{R}^{m \times n}$ , then  $col[Y] = [Y_{c1}^T, Y_{c2}^T, \dots, Y_{cn}^T]^T \in \mathcal{R}^{mn}$ ;  $O_{m \times n}$  represents the matrix whose elements are all zeros, while  $I_{(n-1)l\times(n-1)l}^+$ at the first, we introduce the following notations that will be  $n \times n$  identity matrix, and I used sometimes represents The norm of vector x is defined by  $||x|| \triangleq \sqrt{x^T x}$ , which is effectively the equivalent of Euclidean norm in real number  $E_{i,j}$  represents a square matrix or proper dimensions whose elements are all zeros except for the  $t$  element in the j<br>column which is 1 instead. For a matrix  $A \in \mathbb{R}^{m \times n}$  and column which is 1 instead. For a matrix  $A \in \mathcal{R}$ ,  $a_{ij}$  is<br>the  $i^{th}$  element of the  $i^{th}$  column. For a set of matrices,  $\{F_1, F_2, \ldots, F_n\}$  with  $F \in \mathbb{R}^{l \times l}$  for  $i = 1, 2, \ldots, n$ .  $F_{l, l, l}$  ${F_1, F_2, \ldots, F_n}$ , with  ${F_i \in \mathcal{R}}$ , for  $i = 1, 2, \ldots, n, F_i$ ,  $F_i$ ,  $F_i$ , for denotes the  $b^{th}$  element in the  $i^{th}$  column of matrix  $F_i$ , for denotes the b element in the j<sup>th</sup> column of matrix  $F_i$ , for  $b, j \in \{1, 2, ..., l\}$ ; The Kronecker product is described by  $\emptyset$ , take for example  $P \otimes Q$  with  $P = [p_{ij}] \in \mathcal{R}^{m \times n}$  and  $Q = [q_{ij}] \in \mathbb{R}^{p \times q}$ , it is obtained that  $P \otimes Q = [p_{ij}Q] \in$  $\mathcal{R}^n$ ,  $\mathcal{R}^n$ ,  $\mathcal{R}^n$  and  $\mathcal{R}^n$  are set of formed by columns of  $\mathcal{R}^n \times \mathcal{R}^n$ , then matrix T, that is, if  $Y = [Y_{c1}^T, Y_{c2}^T, \ldots, Y_{cn}^T]^T \in \mathcal{R}^{mn}$ ;  $O_{m \times n}$  represents the col[Y ] =  $\begin{bmatrix} 1 & c_1 \\ c_2 & c_3 \\ c_4 & c_5 \end{bmatrix}$  contribute the collection of t matrix whose elements are all zeros, while  $I_{(n-1)l \times (n-1)l}$ <br>denotes the matrix indicated as follow,

$$
I_{(n-1)l \times (n-1)l}^{+} = \begin{bmatrix} O_{(n-2)l \times l} & I_{(n-2)l} \\ O_{l \times l} & O_{l \times (n-2)l} \end{bmatrix} . \tag{1}
$$

In practical terms, there are many systems with abrupt changes in dynamics, and we refer to such time-varying changes in dynamics, and we refer to such time-varying changes in dynamics, and we refer to such time-varying In practical terms, there are many systems with abrupt changes in dynamics, and we refer to such time-varying<br>systems as switched systems. Let set  $\{t_j | t_j \in \mathcal{R}^+, j \in \mathcal{N}^+\}$ systems as switched systems. Let set  $\{t_j | t_j \in \mathcal{R}_+, j \in \mathcal{N}_-\}$ 

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represents the switching times with  $t_j \leq t_{j+1}$ . To indicate the characteristics of the switched system at the switching times, two sets are determined as

 ${\{\overline{t}_{i,k_i}\}} = {\{t_j + 1|\text{ when subsystem }i\text{ is switched on for the}\}}$  $k_i$  times,  $i \in \mathcal{P}, k_i \in \mathcal{N}^+$  };

 $\{\underline{t}_{m,k_m}\} = \{t_j |$  when subsystem m is switched off for the  $k_m$  times,  $m \in \mathcal{P}, k_m \in \mathcal{N}^+$ 

where subsystem  $m$  and subsystem  $i$  are accordingly defined as the pre- and post-switching subsystems with respect to the switching time  $t_i$ .

As a result, the switching signal  $\sigma(t)$  of the switched system is defined as a piecewise constant function of time with  $\sigma : [0, \infty) \to \mathcal{P}$ , where the index set  $\mathcal{P} = \{1, 2, ..., N\}$  is finite, and N is the number of subsystems. In addition,  $\sigma(t)$  changes with respect to  $\{t_j\}$ .

Subsequently, given the controlled system in the form of input/output mathematical model, how to acquire its linear regression model and state space representation via equivalent transformation is discussed here, which are the preliminaries for parameter estimation and state feedback control design in the following sections, respectively.

Consider the multi-variable switched systems described by

$$
D^{\{\sigma(t)\}}(z^{-1})y(t) = M^{\{\sigma(t)\}}(z^{-1})u(t) \tag{2}
$$

where  $u(t), y(t) \in \mathcal{R}^l$  are the input and output vectors of the system, respectively. The symbol z is used to denote the time advance operator  $zx(t) \triangleq x(t + 1)$ , while  $z^{-1}$  is the backward shift operator, i.e.,  $z^{-1}x(t+1) \triangleq$  $x(t)$ . Here, we define sampling period of the discretetime system (2) as  $T_s$ , which is the interval between two consecutive time instants. Moreover, matrix polynomials  $D^{\{\sigma(t)\}}(z^{-1}), M^{\{\sigma(t)\}}(z^{-1}) \in \mathcal{R}^{l \times l}$  are given as

$$
D^{\{\sigma(t)\}}(z^{-1}) = I_l + D_1^{\{\sigma(t)\}} z^{-1} + \dots + D_n^{\{\sigma(t)\}}(t) z^{-n},
$$
  
\n
$$
M^{\{\sigma(t)\}}(z^{-1}) = M_1^{\{\sigma(t)\}} z^{-1} + \dots + M_n^{\{\sigma(t)\}} z^{-n}.
$$

(3) where *n* represents the order of the system which is supposed to be a priori knowledge. With  $D_i^{\{\sigma(t)\}}$ ,  $M_i^{\{\sigma(t)\}}$   $\in$  $\mathcal{R}^{l \times l}$ , for  $i = 1, 2, ..., n$ ,  $\{D_1^{\{\sigma(t)\}}, ..., D_n^{\{\sigma(t)\}}\}$  and  ${M_1^{\{\sigma(t)\}}}, \ldots, M_n^{\{\sigma(t)\}}$  are two sets of coefficient matrices for  $D^{\{\sigma(t)\}}(z^{-1})$  and  $M^{\{\sigma(t)\}}(z^{-1})$ , respectively.

## 2.1 Linear regression model

From (3), the discrete-time autoregressive moving average (ARMA) model for the linear switched system described in (2) can be derived as

with

$$
y(t) = \Theta_{\sigma(t)}^T \phi(t), \tag{4}
$$

$$
\Theta_{\sigma(t)} = [D_1^{\{\sigma(t)\}}, \dots, D_n^{\{\sigma(t)\}}, M_1^{\{\sigma(t)\}}, \dots, M_n^{\{\sigma(t)\}}]^T, \n\phi(t) = [-y(t-1)^T, \dots, -y(t-n)^T, \nu(t-1)^T, \dots, u(t-n)^T]^T,
$$

Moreover, there exists parametric uncertainty in matrices  $D_i^{\{\sigma(t)\}}, M_i^{\{\sigma(t)\}}$  for  $i = 1, ..., n$ .

For parameter identification, the system (4) can be rewritten as a multi-variable linear regression model, i.e.,

$$
y(t) = \Psi(t)\theta_{\sigma(t)}\tag{5}
$$

where  $\theta_{\sigma(t)} \in \mathcal{R}^{n_{\theta}}$  is the parameter vector to be identified, and  $\Psi(t) \in \mathcal{R}^{l \times n_{\theta}}$  is the information matrix consisting of the input and output vectors  $u(t - i)$ ,  $y(t - i)$ .

Exactly, the following two cases are taken into consideration for the explicit formulation of the identification problem for the system (4).

**Case 1:** If the matrices  $D_i^{\{\sigma(t)\}}$ ,  $M_i^{\{\sigma(t)\}}$  for  $i = 1, \ldots, n$ are absolutely unknown, it follows that  $\Psi(t)$  and  $\theta$  in (5) will be written as

$$
\Psi(t) = I_l \otimes \phi^T(t), \n\theta_{\sigma(t)} = col[\Theta_{\sigma(t)}].
$$
\n(6)

**Case 2:** If the matrices  $D_i^{\{\sigma(t)\}}$ ,  $M_i^{\{\sigma(t)\}}$  for  $i = 1, \ldots, n$ are partly unknown, namely, only some entries of several matrices are unknown time-varying parameters to be identified,  $\Psi(t)$  and  $\theta$  in (5) will instead be expressed as

$$
\Psi(t) = [y_{pk}(t), E_{b,j} y(t-i), \dots, E_{c,k} u(t-m), \dots],
$$
\n
$$
\theta_{\sigma(t)} = [1, D_{i(b,j)}^{\{\sigma(t)\}} , \dots, M_{m(c,k)}^{\{\sigma(t)\}} , \dots]^T,
$$
\n(7)

where  $D_{i(b,j)}^{\{\sigma(t)\}}$  and  $M_{m(c,k)}^{\{\sigma(t)\}}$  are the uncertain time-varying parameters, for  $i, m \in \{1, \ldots, n\}, b, j, c, k \in \{1, \ldots, l\}$ , and  $y_{nk}(t)$  is calculated as

$$
y_{pk}(t) = \bar{\Theta}_{\sigma(t)}^T \phi(t)
$$
\n(8)

where  $\bar{\Theta}_{\sigma(t)}$  is derived from  $\Theta_{\sigma(t)}$  with the only difference between them being that all the uncertain parameters are set to be zeros in  $\Theta_{\sigma(t)}$ .

Remark 1. The above two cases considered in this paper are of great significance from the following two aspects: (i) as for Case 1 where all the elements in  $\Theta_{\sigma(t)}$  are of uncertainty, transformation from  $(4)$  to  $(5)$  with  $(6)$ enables identification issue to be solved with least square algorithm, (ii) while in Case 2 conversion of  $(4)$  to  $(5)$  with (7) simplifies the problem by minimizing the dimension of the parameter vector to be identified. Meanwhile, the true value for the first element of the parameter vector is 1 as presented in (7), which will be one of the criteria for verification of convergence of parameter estimation.

### 2.2 State space representation

The system (4) developed in Section 2.1 can be rewritten in a state-space form  $x(t+1) = f(x(t), u(t))$  by introducing  $x(t)=[u^T (t-n+1),\ldots,u^T (t-1), y^T (t-n+1),\ldots,y^T (t-1)]$  $(1), y^T(t)$ ]<sup>T</sup>  $\in \mathcal{R}^{(2n-1)l}$  as state vector of the system. The state space representation is consequently given as

$$
x(t+1) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t),
$$
  
\n
$$
y(t) = C_{\sigma(t)}x(t)
$$
\n(9)

with

$$
A_{\sigma(t)} = \begin{bmatrix} I_{(n-1)l \times (n-1)l}^{+} & O_{(n-1)l \times nl} \\ O_{(n-1)l \times (n-1)l} & I_{(n-1)l \times nl}^{'} \\ M_{\sigma(t)} & -\bar{D}_{\sigma(t)} \\ B_{\sigma(t)}^{T} = [O_{l \times (n-2)l}, I_l, O_{l \times (n-1)l}, (M_1^{\{\sigma(t)\}})^T], \\ C_{\sigma(t)} = [O_{l \times 2(n-1)l}, I_l] \end{bmatrix},
$$

and

$$
\bar{M}_{\sigma(t)} = [M_n^{\{\sigma(t)\}}, \dots, M_2^{\{\sigma(t)\}}],
$$
\n
$$
\bar{D}_{\sigma(t)} = [D_n^{\{\sigma(t)\}}, \dots, D_1^{\{\sigma(t)\}}],
$$
\n(10)

where  $I'_{(n-1)l \times nl}$  is the first  $(n-1)l$  rows of  $I^{+}_{nl \times nl}$ .

Additionally, introduce the definition of the accumulation of tracking error:

$$
e_I(t+1) = y(t) - y_r(t) + e_I(t)
$$
 (11)

where  $y_r(t) \in \mathcal{R}^l$  is reference signal for the controlled system.

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