Electric Power Systems Research 158 (2018) 275–283

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

Power flow problems with nested information: An approach based on fuzzy numbers and possibility theory

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ARTICLE INFO

Article history:
Received 7 July 2017
Received in revised form
27 November 2017
Accepted 4 January 2018

Keywords:
Fuzzy number
Power flow
Possibility theory
Uncertainty

ABSTRACT

In this paper, we present a new approach based on possibility theory to deal with the power flow problems in electrical networks. The approach takes into account the available information of a power system that is characterized generally by big data, often uncertain, redundant or insufficient for a correct description of the network status. In particular, we present a method to deal with nested information that can be generated by inaccurate measurements of electrical parameters. In order to solve the power flow problem we define a way to model nested information in power system and formalize an AC fuzzy power flow problem. The power flow results are obtained by an innovative approach based on the solution of a simultaneous nonlinear equations fuzzy system. The effectiveness of the proposed method is proven by applying the proposed approach to two modified IEEE benchmark test systems. Simulation results show accuracy, robustness and good computational cost of the implemented method in the search of the solution.

1. Introduction

Studies of power flow are very important for power systems planning and operation. Deterministic approaches allow the determination of power flow without considering factors of uncertainty, which are increasing as the utility industry undergoes restructuring. In particular, the uncertainty due to load profiles, distributed renewable generation (e.g. wind, photovoltaic), economic growth, line ageing, does not allow an accurate deterministic analysis with an enormous size of calculation to be taken into account [1].

Several studies in the literature deal with the uncertainty in power systems and two methodological approaches are mainly used: methods based on probabilistic theory [2,3], and methods based on alternative arithmetic (e.g., interval, fuzzy, affine arithmetic) [4,5]. Probabilistic load flow (PLF) can be classified into three categories [6]: Monte Carlo simulation (MCS), analytical and approximate methods.

With regard to the probabilistic approach, recent studies are in Refs. [2,7]; in Ref. [2] an adaptive kernel density estimation based on smoothing properties of linear process is presented and the method is validated on transmission system including wind farms; in Ref. [7] a probabilistic power flow considers the correlation among the power generated by power plants, load demands on each bus, and the power injected by wind farms. Analytic methods for solving the PLF, which require linearization procedures and strong assumptions on the independence of the inputs, had large diffusion in the past: some exhaustive examples are in Refs. [8–10] and more recently in Ref. [11]. In these four papers, authors estimate the probability density function of the power generated by a wind turbine by using a quadratic approximation of its power curve; thus, they solve a DC power flow problem. PLF via approximate methods are in Refs. [12,3]; in particular, in Ref. [12] authors utilize the Cornish-Fisher expansion technique to plot the output distribution function, while in Ref. [3] point estimate schemes for solving PLF are proposed.

Alternative approaches to MCS methods have been introduced with interesting results. Methods to solve load flow problem based on the interval arithmetic or the affine arithmetic are in Refs. [4,13,14]. In Ref. [4], authors present a framework based on range arithmetic for solving power flow problems whose input data are specified within real compact intervals. Reliable interval bounds are computed for the power flow problem, which is represented as an optimization model with complementary constraints; in Ref. [13], an affine arithmetic method is proposed for self-validated numerical analysis in which the quantities of interest are rep- resented as affine combinations of certain primitive variables representing the sources of uncertainty in the data or approximations made during the computation. A more recent work is in Ref. [14], where authors present a new solution method based on the linear approximation

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https://doi.org/10.1016/j.epsr.2018.01.008

of the affine arithmetic power flow model and optimal procedures with boundary load flow framework.

Another family of algorithms for load flow calculation under uncertainty is based on the fuzzy set and possibility theory. The main hypothesis is on the uncertainty that is supposed to be originated by a vague or inaccurate information, which is not the case of probabilistic models, highly related to the statistical behavior of a phenomenon [6]. In the literature, the fuzzy theory has been used for power flow analysis with significant results; for instance, in Ref. [15], the application of fuzzy number arithmetic to formulate a DC power flow problem is presented. In Ref. [16], symmetric fuzzy numbers are used to deal with uncertainty in power system analysis: a big attention is put on reducing overestimation due to conservatism inherent in fuzzy arithmetic. Interesting applications are in Refs. [17,18] for radial distribution systems: in Ref. [17] the computation of branch currents and bus voltages are obtained by means of the application of fuzzy set and interval arithmetic; in Ref. [18], a fuzzy approach load flow for balanced and unbalanced radial distribution systems with incorporating load models is proposed. AC power flow solutions based on a fuzzy set approach have been discussed in Ref. [19,5] by using the α-cut approach. In Ref. [20], interesting suggestions for applying fuzzy set theory to planning and operation of power systems are presented.

Alternative approaches based on fuzzy numbers and the fuzzy set are adequately justified by Piegat in a lucid paper where he shows the differences between possibility and probability theory [21]. He states that in the presence of inaccurate (or uncertain), nested or few measurements it is adequate to use possibility distributions. Poor quality of information, in fact, can determinate the difficulty of obtaining accurate probability distributions of the inputs, especially if data are few or do not follow any particular distribution [22,23].

Here, we face the problem of power flow by using this fuzzy approach when information is nested in available data. Even though the literature on uncertainty in power systems is quite extensive, the brief review highlights that there are few studies that deal with power flow problem by using fuzzy numbers and solution algorithms are not very efficient. In particular, fuzzy arithmetic, which would make more efficient the implementation of solution algorithms, is applied properly only for DC power flow [6], or AC power flow on radial networks, where it is possible to use linear dependence. Fuzzy power flows are solved by using a α-cut approach and/or Monte Carlo method with a consistent time-consuming process [16,19,5,21].

Based on these prerequisites, we propose: (i) a new formulation of a fuzzy AC power flow based on an original representation of the uncertainty due to *nested information* of electrical parameter measurements and (ii) the solution method that is based on the division of the power flow problem in two parametric sub-parts. The two sub-problems are solved by the classical Newton–Raphson iterative method that allows the solution of simultaneous nonlinear equations whose parameters are all partially represented by fuzzy numbers and defined according to the possibility theory.

The developed tool allows a power flow analysis under nested data keeping a balance between low computational burden and accuracy of the results. Furthermore, the possibility theory allows the introduction of a useful interpretation of the results for technicians and power network operators. In order to illustrate the proposed method, the fuzzy power flow (FPF) problem based on the possibility theory is formulated, the solution method is tested on modified IEEE 14-bus and IEEE 118-bus benchmark test systems and the results are validated by Monte Carlo's simulations.

This paper is organized as follows. Section 2 briefly presents possibility theory and gives preliminaries on fuzzy numbers. In Section 3, the mathematical formulation of the AC FPF problem is illustrated. In Section 4, the Newton Raphson's method is shown, and in Section 5, the methodology is applied to the case studies and results are discussed. Section 6 concludes the paper.

2. Possibility theory in power systems

In power systems, problems that treat the uncertainty with the classical probabilistic approach can be critical because in many cases there are not enough data to build reliable probabilistic distributions, and practical application suffers the lack of information. These considerations led to formulate a theory of possibility based on a fuzzy set approach [24]. In the past, various interpretations of the concept of possibility have been introduced. The main notions are in Refs. [25–28], but here we consider the interpretation of Dubois and Prade, which can be found in Ref. [26]. This interpretation is dominant and allows the determination of a possibility distribution when we have only uncertain, nested evidence information about a given problem. Here, we recall the main definitions of possibility $\Pi(A)$ and necessity $N(A)$ of the event occurrence $(x \in A)$, which are dual, and the fundamental concepts of the fuzzy sets. We denote $A \subseteq U$ as a fuzzy set of elements, so that, given a possibility distribution $\pi(x)$, which maps the universe U into [0,1], $\Pi(A)$ and $N(A)$ can be expressed as

$$\Pi(A) = 1 - N(A^C) = \sup\left\{\pi(x) | x \in A\right\} \tag{1}$$

$$N(A) = 1 - \Pi(A^C) = inf\left\{1 - \pi(x) | x \notin A\right\} \tag{2}$$

$$\pi(x) = \Pi(\{x\}) \tag{3}$$

where A^c denotes the complement of A.

2.1. Preliminaries on fuzzy numbers

Fuzzy numbers are of great importance for possibility theory. A typical shape of fuzzy number is triangular. The following definitions are useful:

Definition 1. A fuzzy number is a set like u: $\mathbb{R} \rightarrow$ I = [0,1] which satisfied the following conditions: (i) u is upper semicontinuos, (ii) u(x) = 0 is outside some interval [c, d]; (iii) there are real numbers a, b such that $c \le x \le b \le d$ and u(x) is monotonic increasing on [c, a], monotonic decreasing on [b, d], u(x) = 1, $a \le x \le b$.

Definition 2. A Fuzzy number in parametric form is a pair $(\underline{u}, \overline{u})$ of function $\underline{u}(\alpha)$, $\overline{u}(\alpha)$ with $0 \le \alpha \le 1$, which satisfies the following requirements:

i) $\underline{u}(\alpha)$ is a bounded monotonic increasing left continuous function,
ii) $\overline{u}(\alpha)$ is bounded monotonic decreasing right continuous function,
iii) $\underline{u}(\alpha) \le \overline{u}(\alpha)$, $0 \le \alpha \le 1$.

Definition 3. The membership function of a triangular fuzzy number $\tilde{u} = (a,b,c)$ is

$$u(x) = \begin{cases} \dfrac{(x-a)}{(c-a)}, & a \le x \le c \\ \dfrac{(x-b)}{(c-b)}, & c \le x \le b \end{cases} \tag{4}$$

where c ≠ a, c ≠ b, and hence its parametric form is

$$\begin{cases} \underline{u}(\alpha) = a + (c-a)\alpha \\ \overline{u}(\alpha) = b + (c-b)\alpha \end{cases} \tag{5}$$