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IFAC-PapersOnLine 48-28 (2015) 544–549

A Grey-Box Distributed Parameter Modeling Approach for a Flexible Manipulator with Nonlinear Dynamics

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Abstract: The flexible manipulator is a spatially distributed mechanical system. An accurate model of the flexible manipulator is essential for the positioning control of the end effector. In this study, a grey-box distributed parameter modeling approach is proposed for the flexible manipulator with unknown nonlinear dynamics. First, a nominal Euler-Bernoulli beam model is derived to describe the linear dynamics. To compensate unknown nonlinear dynamics, a nonlinear term is added in the nominal model. The Galerkin method is used to reduce the infinite-dimensional partial differential equation (PDE) model into a finite-dimensional ordinary differential equation (ODE) model. A neural network is designed to estimate the unknown nonlinearities from the input-output data. The effectiveness of the proposed grey-box distributed parameter modeling approach is verified by the simulations on a flexible manipulator.

Keywords: flexible manipulator, distributed parameter model, model identification, neural networks

1. INTRODUCTION

An accurate model of the flexible manipulator is essential for the positioning control of the end effector. Due to the spatial distribution of the flexible manipulator, the dynamics of the flexible manipulator with respect to the applied external force belong to distributed parameter system (DPS). It should be described by a partial differential equation (PDE). Currently, the modeling of the flexible manipulator can be classified into lumped parameter approach and distributed parameter approach.

For simplicity, the lumped parameter models are often used for modeling the flexible manipulator. The widely used lumped parameter models include transfer function model (Butt, Cappella & Kappl, 2005; Ohler, 2007; Saeidpourazar & Jalili, 2008b) and autoregressive moving average exogenous (ARMAX) model (Khadraoui, Rakotondrabe & Lutz, 2014a). Some parameters in the model are often unknown, thus the parameter estimation from the experimental data is often used. For the transfer function model, the parameter estimation methods are developed (e.g., Stark et al., 2005). Currently, the lumped parameter models are often designed as linear models, while the important nonlinear dynamics are ignored. Though the lumped parameter models are simple for implementation, they always lose some accuracy since the important spatial dynamics of the flexible manipulator are not considered.

To model spatial dynamics, the distributed parameter models should be used, e.g., Euler-Bernoulli beam model (Saeidpourazar & Jalili, 2008a, 2009; Mahdavi, et al., 2008; Rubio-Sierra, Vazquez & Stark, 2006; Butt & Jaschke, 1995; Eslami & Jalili, 2011). Currently, most of Euler-Bernoulli beam models are derived from the first-principle. Though the spatial dynamics are included, most of them only describe linear dynamics under simplified conditions and model parameters are often nominal values. In practice, there are often some unknown uncertainties, e.g., inaccurate parameter or nonlinear dynamics. To improve the model accuracy, it is necessary to compensate these model uncertainties from the data using the system identification techniques.

For other kinds of DPS, the data-based spatio-temporal modeling has been studied, e.g., thermal process and fluid flow. Because the DPS is infinite-dimensional, for implementation model reduction is required to reduce the PDE to a finite-dimensional ordinary differential equation (ODE). The commonly used model reduction methods include finite-difference (Parlitz & Merkwirth, 2000; Guo & Billings, 2007), finite element (Coca & Billings, 2002), spectral methods (Boyd, 2000) and so on. After the model reduction, the identification of ODE model can be performed with traditional modeling methods. When only some parameters of the model are unknown, the parameter estimation methods should be used. When there are some unknown nonlinearities, both model structure and parameters should be identified. For the parabolic PDE with unknown nonlinearities, a spectral method based intelligent modeling is proposed (Deng, Li & Chen, 2005).

In this study, a grey-box distributed parameter modeling approach is proposed for the flexible manipulator with unknown nonlinear dynamics. The nonlinear dynamics can come from several aspects, e.g., the material properties and the geometrical no-uniformity. A nominal Euler-Bernoulli beam model is derived from the first-principle. To compensate uncertain nonlinear dynamics, a nonlinear term is added in the nominal model. The infinite-dimensional PDE model is reduced into a finite-dimensional ODE model using the Galerkin method. Next, a neural network is established to learn the unknown nonlinearities from the input-output data. The proposed grey-box distributed parameter modeling

10.1016/j.ifacol.2015.12.185

approach is verified by the simulations on a typical flexible manipulator.

The rest of this paper is organized as follows. The Euler-Bernoulli beam model is described in section 2. The Galerkin method based model reduction is presented in section 3. The neural network model design is given in section 4. Section 5 reports simulations on a flexible manipulator. Finally, the conclusions are presented in section 6.

2. EULER-BERNOULLI BEAM MODEL FOR THE FLEXIBLE MANIPULATOR

As shown in Figure 1, the dynamics of the flexible manipulator are spatially distributed. Though the Euler-Bernoulli equation can describe the flexible manipulator as a distributed parameter system, it is often derived under ideal conditions and thus it requires parameters of manipulator to be known. In fact, these parameters are often inaccurate or difficult to measure. The simplified modeling will generate the model uncertainties, e.g., the ignored nonlinear dynamics. In practice, the highly accurate positioning needs to compensate the model uncertainties.

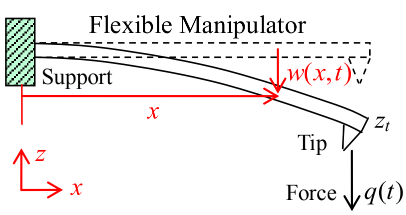

Figure 1: Flexible manipulator

2.1 Nominal Euler-Bernoulli Beam Model

First, a nominal model is derived using the theoretical modeling. As shown in Figure 1, the flexible manipulator is built in at one end, free at the other end. Assume it is a homogeneous beam with a constant rectangular cross section and deforms in the linear elastic range. By neglecting the rotary inertia, shear deformation, axial effects and the tip mass, the flexible manipulator can be described by one-dimensional Euler-Bernoulli equation (Rubio-Sierra, Vazquez & Stark, 2006) as below

$$m\frac{\partial^2 w(x,t)}{\partial t^2}+c\frac{\partial w(x,t)}{\partial t}+EI\frac{\partial^4 w(x,t)}{\partial x^4}=0\,, \tag{1}$$

where $w(x,t)$ is the time-dependent transverse displacement along z-axis relative to its support, $x\in[0,L]$ is the spatial variable along the x-axis, t is the temporal variable, E is Young's modulus of the beam, I is the moment of inertia about the y-axis, m is the constant manipulator mass density, c is the damping factor.

The boundary conditions at the fixed end with respect to the support are zero deflection and zero slope as follows

$$w(0,t)=0, w_x(0,t)=0\,. \tag{2}$$

At the free end assume that there is no torque and a force $q(t)$ is acted perpendicularly to manipulator axis. Then, the boundary conditions at $x=L$ are

$$w_{xx}(L,t)=0, EIw_{xxx}(L,t)=-q(t)\,. \tag{3}$$

Note that the subscripts denote the derivative with respect to the subscripted variable.

2.2 Improved Nonlinear Euler-Bernoulli Beam Model

The model (1) is only a linear approximation of original manipulator since some complex nonlinear dynamics and parameter uncertainties are neglected. Considering these uncertainties, the model is assumed to be

$$m\frac{\partial^2 w(x,t)}{\partial t^2}+c\frac{\partial w(x,t)}{\partial t}+EI\frac{\partial^4 w(x,t)}{\partial x^4}+f(w(x,t))=0\,, \tag{4}$$

where the nonlinear compensation term $f(\cdot)$ is a function of displacement w. For simplicity, the parameter uncertainties on E, I, m and c are integrated into the nonlinearity $f(\cdot)$. More complicated cases, where the nonlinearity $f(\cdot)$ is a function of position x and time t, are ignored for simplicity though the proposed modeling can also be applicable for these cases after minor revisions. The boundary conditions are same as (2) and (3).

3. GALERKIN METHOD FOR MODEL REDUCTION

3.1 Homogenized Boundary Conditions

For implementation, the mode reduction methods are used to transform the infinite-dimensional PDE to finite-dimensional ordinary differential equation (ODE). Spectral method (Deng, Li & Chen, 2005; Boyd, 2000) is used here because it can derive a low-dimensional model. To do so, the boundary conditions (3) need to be homogenized. For this, a new variable $v(x,t)$ is defined as below

$$w(x,t)=v(x,t)+q(t)g(x)\,, \tag{5}$$

where $g(x)$ is a geometrical function to be found later.

Substituting (5) into (2) and (3), the conditions on $g(x)$ to arrive at homogenous boundary conditions in terms of the new variable $v(x,t)$ are

$$\begin{gathered} g(0)=0, g_x(0)=0, \\ g_{xx}(L)=0, g_{xxx}(L)=-1/EI. \end{gathered} \tag{6}$$

Note that $g(x)$ which satisfies conditions (6) is not unique. Here $g(x)$ can be chosen as

$$g(x)=-\frac{1}{18EIL}(2x^4-5Lx^3+3L^2x^2), \tag{7}$$

which satisfies conditions (6). Substituting (5) into the equation (4) and the boundary conditions (2) and (3), the following equation

$$\begin{gathered} m\frac{\partial^2 v(x,t)}{\partial t^2}+c\frac{\partial v(x,t)}{\partial t}+EI\frac{\partial^4 v(x,t)}{\partial x^4}+f(v+q(t)g(x))= \\ -\left(m\frac{\partial^2 q(t)}{\partial t^2}g(x)+c\frac{\partial q(t)}{\partial t}g(x)+EIq(t)\frac{\partial^4 g(x)}{\partial x^4}\right), \end{gathered} \tag{8}$$

and homogenized boundary conditions

$$v(0,t)=0, v_x(0,t)=0, \tag{9}$$