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IFAC-PapersOnLine 48-28 (2015) 562–567

Distributed Weighted Least-Squares Estimation for Power Networks

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Abstract: The paper presents a fully distributed scheme to optimal parameter estimation, with application in multi-area interconnected power systems. In this scheme, each area performs its own local parameter estimation based on low-dimensional local and boundary measurements as well as the estimate of boundary parameters from its neighbors. We show, under certain assumptions, that the distributed estimation scheme provides the same accurate unbiased estimate as the centralized weighted least-squares estimate with finite time convergence. The distributed estimation scheme is robust to communication link failures, delays, and asynchronism of control centers in different areas. A simulation using the IEEE 118-bus system is included to demonstrate the performance.

Keywords: Distributed parameters, parameter estimation, least-squares estimation, power systems, smart power applications.

1. INTRODUCTION

Electric power networks are undergoing profound changes recently and receiving increasing attention from researchers in different fields. By incorporating a communication, computing and control overlay, more efficient and intelligent processes are integrated into the electric power networks. Parameter estimation for the so-called *quasi-steady state* parameters is considered to be one of the key integrating components for the real time energy management system (EMS).

Quasi-steady state parameter estimation (or simply called state estimation) in power networks has been introduced in the early 1970's Schweppe and Wildes (1970). The traditional centralized parameter estimator is typically installed in a control center collecting all measurements over the entire network, and providing the optimal estimate of the parameters for the power network. The data is measured by SCADA systems and the estimation usually takes minutes to get a snapshot of a normal sized power network Ren and Guo (2005); Huang et al. (2007). However, due to the deregulation of energy markets, large amounts of power are transferred over high-rate, long-distance lines spanning multiple areas to form a very large scale network Gomez-Exposito et al. (2011). Also, policy and privacy considerations make a centralized estimation inappropriate for a power network spanned over multiple areas, regional transmission organizations (RTOs), and/or countries. Thus, it leads to the emergence of *hierarchical estimation* Gomez-Exposito and de la Villa Jaen (2009); Korres (2011); Zhao and Abur (2005); Jiang et al. (2007); Patel and Girgis (2007); Jiang et al. (2008); Yang et al. (2012) and *distributed estimation* Lin (1992); Lin and Lin (1994); Falcao et al. (1995); Conejo et al. (2007); Xie et al. (2011); Pasqualetti et al. (2012). A good recent survey on quasi-steady state parameter estimation for power networks refers to Gómez-Expósito et al. (2011).

In principle, a desirable distributed estimator for large-scale power networks should own the following properties: (1) the distributed estimator should be able to deal with the case where local parameters might not be uniquely identifiable due to bad data removal and finer decomposition of control areas; (2) the information exchange between different control centers should be kept as low as possible to reduce the communication load and improve the estimation response time; (3) the resulting estimate should be accurate or close enough to the optimal one obtained from the traditional centralized estimator; (4) the estimation scheme should exhibit a fast convergence rate or even finite time convergence for the purpose of real time monitoring; (5) the convergence to a correct estimate should be robust to link failures and time delays commonly occurred in communication networks, and asynchronism inherent in the distributed setup. However, no estimation scheme in recent development incorporates all the desired properties. For example, most algorithms such as hierarchical estimation of Iwamoto et al. (1989); Zhao and Abur (2005); Korres (2011) and distributed estimation of Lin and Lin (1994); Falcao et al. (1995); Conejo et al. (2007), presume locally topological observability. The locally topological observability assumption is no longer necessary in the distributed estimation schemes of Xie et al. (2011) and Pasqualetti et al. (2012). However, in Xie et al. (2011) and Pasqualetti et al. (2012) each control center has to communicate its own estimate of the entire high-dimensional parameter

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Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.12.188

vector to its neighboring control centers, which scales unfavorably with the size of the power network. Moreover, in Pasqualetti et al. (2012) only an approximate estimate is ensured though an analytical estimation error bound is provided within a finite number of iterations. On the other hand, Xie et al. (2011) shows almost sure convergence towards the centralized parameter estimation result, which is similar to asymptotic convergence of distributed estimation strategies Cattivelli et al. (2008); Schizas et al. (2009) developed in other fields. The performance is not well suited for applications in power networks as the convergence is only asymptotic. Besides, no analytical study is conducted in Xie et al. (2011); Pasqualetti et al. (2012) for the behavior of the estimation schemes in response to communication link failures, transmission delays and asynchronism of distributed control centers.

The objective of this paper is to propose a fully distributed estimation scheme that incorporates all the desired features. To this end, we propose a new distributed estimator for each control center, called *local estimator*, which requires only local (own area) and boundary information. Each local estimator provides an estimate of the parameters of its own area, and the connection with neighboring areas is done by exchanging only small amounts of boundary estimation data. Notice that the physical linkage (tie-lines) between different areas is usually low-dimensional and most measurements taken in one area are not affected by the parameters in other areas. The proposed scheme is based on the distributed method for weighted least squares (WLS) developed in Marelli and Fu (2015). A key property of this method is that it converges to the globally optimal estimate. Moreover, this convergence is achieved in a finite number of iterations, being equal to the diameter of the graph modeling the interconnection of control areas. Finally, we show that the finite-time convergence is also ensured in the presence of link failures and communication delays, as well as asynchronism between distributed control centers.

2. PROBLEM DESCRIPTION

2.1 Measurements in power networks

Consider a power network, for example, the IEEE 118-bus system shown in Fig. 1. Let $x \in \mathbb{R}^n$ be the parameter vector at a certain time instant consisting of the voltage phasors at all buses. The measurements of the whole power network usually take the following form:

$$z = h(x) + \eta, \tag{1}$$

where $z \in \mathbb{R}^m$ is the measurement vector, $h(\cdot)$ is a measurement function, and η is the Gaussian random measurement error vector satisfying $\eta \sim \mathcal{N}(0, R_\eta)$.

The traditional SCADA measurements typically contain voltage magnitudes, power injections at the measured buses, and power flows along the measured transmission lines. In this case, the parameter vector (i.e., quasi-steady state) x in eq. (1) is usually defined in the polar coordinate form. Adopting the approximated estimation model presented in Schweppe and Wildes (1970) which follows from the linearization around an operating point $\tilde{x}$ of eq. (1), the measurements can be expressed as

$$z = Hx + \nu, \tag{2}$$

where $H \in \mathbb{R}^{m\times n}$ is the Jacobbian matrix of $h(\cdot)$ and ν is still assumed to satisfy $\nu \sim \mathcal{N}(0, R_\nu)$. This model is also appropriate if the measurements are made using phasor measurement units (PMU).

2.2 Centralized parameter estimation

For the linear measurement model (2), the centralized WLS estimation (Steven, 1993, Section 8.4) is represented as

$$\hat{x}^{opt} = \arg\min_{\hat{x}} (z - H\hat{x})\, R_\nu^{-1} (z - H\hat{x})\,. \tag{3}$$

If H has full column rank and R_ν is invertible, the WLS estimate $\hat{x}^{opt}$ and the estimation error covariance Σ take the following explicit forms:

$$\hat{x}^{opt} = \Psi^{-1}\alpha; \quad \Sigma = \Psi^{-1}; \tag{4}$$

where

$$\alpha = H^T R_\nu^{-1} z; \quad \Psi = H^T R_\nu^{-1} H. \tag{5}$$

To have the centralized WLS parameter estimation solution, it requires to collect all the measurements distributed in different areas and assume the complete knowledge of the matrix H and R_ν. Thus, both communication and computation burden scales unfavorably with the size of the power networks.

2.3 Partition of power networks

For a practical power network, the measurements are either related to the parameters of one bus (such as power injections and voltage phasor measurements), or indicate the relationship between two adjacent buses (such as power flows and current phasor measurements). These characteristics naturally lead to a sparse measurement matrix H. The measurements can also be classified as *local measurements*, meaning that the measurements are only functions of the parameters of one control area, and *boundary measurements*, consisting of tie-line measurements related to the parameters of more than one control areas. The local measurements are given by

$$y_i = A_i x_i + v_i, \tag{6}$$

where x_i and y_i represent the local parameter vector and the local measurement vector in the control area i, and $v_i \sim \mathcal{N}(0, R_i)$ is the local measurement noise. The boundary measurements linking the parameters in control area i and j are represented as

$$z_{i,j} = D_{i,j} x_i + C_{i,j} x_j + w_{i,j}, \tag{7}$$

where $w_{i,j} \sim \mathcal{N}(0, S_{i,j})$ is the boundary measurement noise. Note that the boundary measurement $z_{i,j}$ is usually of very low dimension and that the boundary measurement (7) is shared by both control center i and j, i.e., $z_{i,j} = z_{j,i}$. All measurement noises are assumed to be uncorrelated.

We use a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ to abstract the partition of a power network, where $\mathcal{V} = \{1, \ldots, I\}$ is a set of nodes with each node corresponding to a control area and $\mathcal{E}$ is an edge set with an edge $(i, j) \in \mathcal{E}$ indicating that there is a boundary measurement $z_{i,j}$ relying on both x_i and x_j. In the following, we use $\mathcal{N}_i$ to represent the neighbor set of i, i.e., $\mathcal{N}_i = \{j : (i, j) \in \mathcal{E}\}$. For a partition made in Fig. 1 for the IEEE 118-bus system, the graph $\mathcal{G}$ is depicted in Fig. 2, for which an edge $(1, 3)$ represents the boundary measurement related to the parameters in areas 1 and 3.