Available online at www.sciencedirect.com
ScienceDirect
IFAC-PapersOnLine 48-27 (2015) 027–032

Observability and Observer Design of Partially Observed Petri Nets

F. Arichi, * **M. Petreczky,** ** **M. Djemai** *** **and B. Cherki** *

* *Faculté de Technologie, Université de Tlemcen. arichi_fay@hotmail.fr, b.cherki@gmail.com.*
** *Ecole des Mines de Douai 941, rue Charles Bourseul, 59508 Douai, France, mihaly.petreczky@mines-douai.fr.*
*** *LAMIH, CNRS UMR 8201, UVHC, F-59313 Valenciennes, France. mohamed.djemai@univ-valenciennes.fr*
*** *ISIR, CNRS, UMR-7222, Univ. Pierre et Marie CURIE, 4 Place Jussieu, F-75005 Paris, France*

Abstract: This paper investigates observability and observer design of partially observed Petri nets (POPN). By using methods for descriptor systems, two major contributions are presented. First, a novel algebraic condition for POPN observability is presented. Second, an asymptotic observer is constructed for POPNs which satisfy this algebraic condition.

Keywords: Discrete Event Systems, Petri Nets, Observability, Observer design.

1. INTRODUCTION

Observability and state estimation problems for DES in general and for partially observed Petri net (POPN) in particular have been addressed by many researchers. In the framework of DES modeled by POPN, the most well-studied estimation problems are those concerning state estimation. (Giua and Seatzu (2002); Giua et al. (2007); Ru and Hadjicostis (2010); Lefebvre (2014b); Lrivera-rangel et al. (2005); Aguirre-Salas et al. (2002); Basile et al. (2015)). As to problem of estimating transitions, it was investigated in (Li and Hadjicostis (2011); Lefebvre and El Moudni (2001)). The estimation problem for POPN is not limited to state estimation but also to the problem of reconstructing the transition sequence that leads to a given state ((Arichi et al. (2014, 2015)).

In this paper we aim at estimating both the marking and the transition sequences, and we assume that we can measure some places and some transitions. The main idea of this paper is to represent the marking and transition sequences generated by a POPN as solutions of a discrete-time linear descriptor system. Using this correspondence, and by extending some results on linear descriptor systems, we formulate sufficient conditions for observability and causal observability of POPNs. We then present an asymptotic observer for estimating the marking and the transitions of a POPN. The estimates computed by observer are shown to converge to the true values, if the POPN satisfies the sufficient conditions for causal observability and the initial state of the observer is chosen to be the initial marking of the POPN. If the POPN satisfies the sufficient conditions for observability, then the asymptotic observer converges to the true value, regardless of the initial state of the observer.

To the best of our knowledge, the algebraic conditions for observability and causal observability proposed in this paper are new. The idea of representing markings and transitions of POPNs was already used in Koenig and Bourjij (1999). Furthermore, in Koenig and Bourjij (1999) an asymptotic observer was proposed as well. However, Koenig and Bourjij (1999) did not study the question of observability or causal observability of POPNs. In addition, the asymptotic observer proposed in Koenig and Bourjij (1999) is completely different from the one proposed in this paper. Intuitively, in this paper we view the unknown transitions as unknown inputs, while in Koenig and Bourjij (1999) they are viewed as a part of the state.

Notation Throughout the paper a standard notation and terminology are used. In particular, $\mathbb{N}$ denotes the set of natural numbers including zero, and $\mathbb{Z}$ denotes the set of integers. The $n \times n$, $n \in \mathbb{N}, n > 0$ identity matrix will be denoted by I_n, and we denote by $0_{n\times m}$ the $n \times m$, $n, m \in \mathbb{N}, n, m > 0$ matrix whose entries are all zero.

2. BACKGROUND ON PETRI NETS

In this section, we recall the definition of a partially observed Petri nets (Ru and Hadjicostis (2010)), and we recall from Koenig and Bourjij (1999) the representation of such Petri nets by linear descriptor systems.

Definition 1. A partially observed Petri net (abbreviated by POPN) Q is a 6-tuple $(P, T, Pre, Post, P_o, T_o)$, where:

- $P = \{p_1, \dots, p_n\}$ is the set of place,
- $T = \{t_1, \dots, t_m\}$ is the set of transitions,
- $Pre : P \times T \longrightarrow \mathbb{N}$ and $Post : P \times T \longrightarrow \mathbb{N}$ are the so called *incidence functions* which determine the transitions of the Petri net,
- $P_o \subseteq P$ is the set of observable places with cardinality n_1 satisfying $0 \leq n_1 \leq n$;
- $T_o \subseteq T$ is the set of observable transitions with cardinality m_1 satisfying $0 \leq m_1 \leq m$.

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.11.148

In the sequel, without loss of generality we assume that $P = \{1, \ldots, n\}$ and $T = \{1, \ldots, m\}$, $P_o = \{1, \ldots, n_1\}$ and $T_o = \{1, \ldots, m_1\}$, and we will denote by n_2 and m_2 the number of unobservable places and transitions respectively. That is, $n_2 = n - n_1$, $m_2 = m - m_1$.

Intuitively, the semantics of a POPN is as follows. Each place of the POPN is characterized by the number of tokens it contains. The collection of tokens at each place is called marking, i.e. a *marking* (i.e., net state) is a vector $M \in \mathbb{N}^n$. Intuitively, the i^{th} entry of M describes the number of tokens in the place p_i. If $p = i \in P$, then $M(p)$ will denote the i^{th} entry of M. At each step, the Petri net may fire one or several transitions. In the sequel, we identify a set of transitions by a vector $\sigma \in \mathbb{N}^m$ whose entries are either one or zero: σ corresponds to the set S such that $t \in S$ if and only if the tth entry of σ is 1, and $t \notin S$ if and only if the tth entry of σ is zero. We will denote the tth entry of σ by $\sigma(t)$. A set of transitions represented by a vector $\sigma \in \mathbb{N}^m$ is enabled, i.e. it can be fired, at marking M if $\forall p \in P : M(p) \geq \sum_{t=1}^{m} Pre(p,t)\sigma(t)$; this is denoted by $M[\sigma\rangle$. The set of transitions represented by $\sigma \in \mathbb{N}$ may fire, and its firing removes $\sum_{t=1}^{m} Pre(p,t)\sigma(t)$ tokens from each place p and adds $\sum_{t=1}^{m} Post(t,p)\sigma(t)$ tokens to each place p, resulting in new marking M', defined by $M'(p) = M(p) + \sum_{t=1}^{m}(Post(t,p) - Pre(t,p))\sigma(t)$. This will be denoted by $M[\sigma\rangle M'$. The number of tokens $M(p)$ of an observable place $p \in P_o$ can be measured by a sensor (e.g., a vision sensor). In contrast, the number of tokens $M(p)$ of an unobservable place $p \in P_{uo} = P \setminus P_o$ cannot be measured by a sensor. An observable transition $t \in T_o$ indicates the presence of a sensor (e.g., a motion sensor) that indicates when a transition within a given subset of transitions has fired; however, an unobservable transition $t \in T_{uo} = T \setminus T_o$ does not have such a sensor associated with it. Hence, if a set of transitions $\sigma \in \mathbb{N}^m$ is fired, then we can observe only the transitions described by the vector $\sigma^1 \in \mathbb{N}^{m_1}$, formed by the first m_1 entries of σ. Note that here we used the assumption that $P_o = \{1, \ldots, n_1\}$ and $T = \{1, \ldots, m_1\}$.

In this paper, we will view a POPN as a device which generates a sequence of markings and sets of transitions, such that the firing of the $(k+1)$-th set of transitions from the kth marking leads to the $(k+1)$-th marking.

Definition 2. (Marking, transition and output sequences). Consider a POPN Q of the form defined in Definition 1. A sequence of marking vectors and sets of transitions $\{M_k, \sigma_k\}_{k=0}^{\infty}$ is called *an admissible sequence of markings and sets of transitions of Q (admissible sequence for short)*, if for all $k \in \mathbb{N}$, $M_k \in \mathbb{N}^n$, $\sigma_k \in \mathbb{N}^m$, all entries of σ_{k+1} are zeros or ones, $\sigma_0 = 0$ and for all $k \in \mathbb{N}$, $M_k[\sigma_{k+1}\rangle M_{k+1}$ holds. A sequence $\{y(k)\}_{k=0}^{\infty}$ is called *the sequence of outputs of Q corresponding to the admissible sequence* $\{M_k, \sigma_k\}_{k=0}^{\infty}$, if for every $k \in \mathbb{N}$, $y(k) \in \mathbb{N}^{n_1+m_1}$ is defined as follow:

$$y(k) = \begin{bmatrix} M_k^{1^T} & \sigma_k^1 \end{bmatrix}^T \tag{1}$$

where

- $M_k^1 \in \mathbb{N}^{n_1}$ is composed of the first n_1 entries of M_k, i.e.
$$M_k = \begin{bmatrix} M_k^{1^T} & M_k^{2^T} \end{bmatrix}^T \tag{2}$$
for some $M_k^2 \in \mathbb{N}^{n_2}$, and
- $\sigma_k^1 \in \mathbb{N}^{m_1}$ is the vector of the first m_1 components of σ_k, i.e.
$$\sigma_k = \begin{bmatrix} \sigma_k^{1^T} & \sigma_k^{2^T} \end{bmatrix}^T \tag{3}$$
for some $\sigma_k^2 \in \mathbb{N}^{m_2}$.

Let $\{M_k, \sigma_k\}_{k=0}^{\infty}$ be an admissible sequence of Q, and let $\{y(k)\}_{k=0}^{\infty}$ be the output corresponding to $\{M_k, \sigma_k\}_{k=0}^{\infty}$. Define the matrix $W \in \mathbb{Z}^{n \times m}$ such that for each $i = 1, \ldots, n, j = 1, \ldots, m$,

$$W_{i,j} = Post(j,i) - Pre(i,j). \tag{4}$$

Then using the notation of (1)-(3),

$$\begin{aligned} M_{k+1} &= M_k + W\sigma_{k+1} \\ y(k) &= \begin{bmatrix} M_k^1 \\ \sigma^1 \end{bmatrix} = \begin{bmatrix} I_{n_1} & 0 & 0 & 0 \\ 0 & 0 & I_{m_1} & 0 \end{bmatrix} \begin{bmatrix} M_k \\ \sigma_k \end{bmatrix} \end{aligned} \tag{5}$$

for all $k \in \mathbb{N}$. That is, admissible sequences of markings, sets of transitions and outputs satisfy a linear equation (5). This prompts us to represent admissible sequences and outputs of POPNs as state and output trajectories of linear descriptor systems.

To this end, we first recall the notion of a linear descriptor system. A discrete-time descriptor system is a system of the form:

$$\bar{E}x(k+1) = \bar{A}x(k) \tag{6a}$$
$$\psi(k) = \bar{C}x(k) \tag{6b}$$

where $x(k) \in \mathbb{R}^{\alpha}, \psi(k) \in \mathbb{R}^p$ are respectively the state vector and he measured output of the system(6) at time k, and $\bar{E}, \bar{A} \in \mathbb{R}^{\beta \times \alpha}, \bar{C} \in \mathbb{R}^{p \times \alpha}$. A sequence $\{x(k), \psi(k)\}_{k=0}^{\infty}$, where $x(k) \in \mathbb{R}^{\alpha}$, $\psi(k) \in \mathbb{R}^p$ is called a *solution* of the descriptor system, if it satisfies (6a) - (6b). Often, $\{x(k)\}_{k=0}^{\infty}$ is referred to as the state trajectory, and $\{\psi(k)\}_{k=0}^{\infty}$ is referred to as the *output* trajectory. Note that even if we fix the initial state z_0, the descriptor system may admit several state trajectories $\{x(k)\}_{k=0}^{\infty}$ satisfying $\bar{E}x_0 = z_0$, or none at all.

Following the idea of Koenig and Bourjij (1999), consider now the following descriptor system which is constructed from the parameters of a POPN $Q = (P, T, Pre, Post, P_o, T_o)$ of the form as defined in Definition 1:

$$\begin{cases} \mathcal{E}_1\Gamma(k+1) = \mathcal{A}_1\Gamma(k) \\ \bar{y}(k) = \mathcal{C}_1\Gamma(k) \end{cases} \tag{7}$$

$\Gamma(k), \bar{y}(k)$ are respectively the generalized state vector and the output vector defined, and

$$\mathcal{E}_1 = [\, I_n \;\; -W \,], \mathcal{A}_1 = [\, I_n \;\; 0_{n \times m} \,] \text{ and}$$
$$\mathcal{C}_1 = \begin{bmatrix} I_{n_1} & 0_{n_1 \times n_2} & 0_{n_1 \times m_1} & 0_{n_1 \times m_2} \\ 0_{m_1 \times n_1} & 0_{m_1 \times n_2} & I_{m_1} & 0_{m_1 \times m_2} \end{bmatrix}$$

Lemma 1. (Koenig and Bourjij (1999)). If $\{M_k, \sigma_k\}_{k=0}^{\infty}$ is an admissible sequence of markings and transitions of Q and $\{y(k)\}_{k=0}^{\infty}$ is the output corresponding to $\{M_k, \sigma_k\}_{k=0}^{\infty}$, then $\{\Gamma(k), y(k)\}_{k=0}^{\infty}$,

$$\forall k \in \mathbb{N} : \Gamma(k) = \begin{bmatrix} M_k^T & \sigma_k \end{bmatrix}^T \tag{8}$$

is a solution of (7).

3. OBSERVABILITY OF PARTIALLY OBSERVED PETRI NETS

In this section, we will use the correspondence between POPNs and descriptor systems in order to study observability of POPN. To this end, we recall the definition of