Electric Power Systems Research 154 (2018) 528–537

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

A theoretical framework for qualitative problems in power system state estimation

F. Schmidt*, M.C. de Almeida

Department of Systems and Energy, University of Campinas, 400, Albert Einstein Ave., Campinas, Brazil

ARTICLE INFO

Article history:
Received 14 April 2017
Received in revised form 9 July 2017
Accepted 9 August 2017

Keywords:
Fundamental subspaces
Identification of critical measurements and critical sets
Observability analysis
Power system state estimation
Qualitative problems

ABSTRACT

The qualitative problems of observability analysis and identification of critical measurements and critical sets are topics highly addressed in the literature. The analysis of such problems allows one to qualify the available measurements and predicts the extent of the benefits provided by the state estimator. Nevertheless, due to the lack of a concise theoretical approach, important concepts are often overlooked, so one cannot directly relate the existing numerical methods. Thus, based on the Jacobian's fundamental subspaces, this paper presents a unified theoretical framework for qualitative analysis. This approach along with a comprehensive literature review enables a categorization of the existing numerical methods, which are shown to be closely related. Given bases for the subspaces, unified alternative methods are conceived. These methods are independent of the decompositions required to obtain the bases for the subspaces. This paper also proposes a method to estimate the Jacobian's rank, which is used to identify cases prone to be *pathological* avoiding unnecessary iterations required by numerical observability analysis algorithms. An example shows that in 99.8% of cases the iterations can be dispensed with. Case studies with small systems of 6 and 14 buses are used to present the proposed approaches in a didactic way.

1. Introduction

The state estimator (SE) is a major tool in the modern energy management systems, since it provides a real-time network model [1–3]. This real-time model is used for power system static and dynamic security analysis, optimal power flow, etc. The SE can be decomposed into three subfunctions: (i) observability analysis; (ii) state estimation and (iii) bad data processing. State estimation is feasible only if the power system is observable. Power system observability is closely related with how the measurements are distributed throughout the system. If a power system is found to be unobservable, one can restore the observability of the whole system before estimating the state or one can estimate the state of each observable island separately. The observable islands are those islands whose state variables can be uniquely estimated. In order to restore observability, one must allocate a set of adequate pseudo-measurements. Fundamentally, the methods to perform observability analysis can be classified as topological [4], numerical [5–20], hybrid [21–26] or path graph-based [27–29]. Another important problem in this context is the identification of bad data groups [30]. These groups are also known as critical [31] or minimally dependent sets of measurements [32]. Additionally, they play a fundamental role in state estimation, since gross errors (outliers) cannot be detected and identified in critical measurements (Cmeas) and critical sets (Csets), respectively. The critical sets can be identified through methods based on the solution of the estimator [30], by analysing the linear dependence among the measurements [31,33], by hybrid methods [32], by methods based on graph theory [34] or by the statistical correlation among the measurement residuals [35]. Both observability analysis and identification of Cmeas and Csets provide only qualitative results. That is, given the topology and the measurement configuration, a network is either observable or not and measurements are either critical or redundant.

The solution to the aforementioned qualitative problems is a topic highly addressed in the literature. However, a careful and thorough review indicated a lack of a concise theoretical approach to this subject. In fact, based on state of the art knowledge one cannot directly relate the existing methods to solve these problems. This is because most publications only focus on how to implement them, thereby neglecting what is behind the proposed approaches. Additionally, in general these methods deal with qualitative problems in a nonunified way. In this sense, this paper innovates as follows:

* Corresponding author.
E-mail addresses: schmidt@dsee.fee.unicamp.br (F. Schmidt), madson@dsee.fee.unicamp.br (M.C. de Almeida).

http://dx.doi.org/10.1016/j.epsr.2017.08.009
0378-7796/

Nomenclature

$P_{k,m}$	power flow at a branch connecting buses k and m
P_k	power injection at a bus k
$\boldsymbol{\theta}$	bus voltage angles vector
$\boldsymbol{z}$	measurements vector
$\boldsymbol{H}$	Jacobian matrix
n	number of state variables
m	number of measurements
$\mathcal{R}(\boldsymbol{H})$	column space of $\boldsymbol{H}$
$\mathcal{R}(\boldsymbol{H}^T)$	row space of $\boldsymbol{H}$
$\mathcal{N}(\boldsymbol{H})$	null space of $\boldsymbol{H}$
$\mathcal{N}(\boldsymbol{H}^T)$	left null space of $\boldsymbol{H}$
$\boldsymbol{G}$	gain matrix
$\boldsymbol{A}$	Gram matrix
r	rank of $\boldsymbol{H}$
$\boldsymbol{H}_i$	Jacobian matrix of the ith observable island
$\boldsymbol{H}_{IR}$	Jacobian matrix of the irrelevant injections
$\boldsymbol{L}$	branch to node incidence matrix
$\boldsymbol{N}$	basis for $\mathcal{N}(\boldsymbol{H})$
n_{OB}	number of observable buses
n_{OI}	number of observable islands
n_{OI_θ}	number of observable islands containing angle measurements
n_{IB_θ}	number of isolated buses containing angle measurement
n_{IR}	number of irrelevant injections
$\hat{r}$	estimated rank of $\boldsymbol{H}$
$\boldsymbol{H}_c$	Jacobian matrix of the candidate measurements
$\boldsymbol{R}$	basis for $\mathcal{R}(\boldsymbol{H}^T)$
$\boldsymbol{M}$	basis for $\mathcal{N}(\boldsymbol{H}^T)$

- Given bases for the fundamental subspaces of the Jacobian matrix, the paper proposes achievable alternative methods to solve all the qualitative problems (observability checking, identification of observable islands, measurement placement for observability restoration, and identification of Cmeas and Csets) in a unified way;
- The paper proposes solution methods that are independent of the numerical decompositions required to obtain the bases for the fundamental subspaces;
- The paper proposes a unified theoretical framework formalizing and bringing valuable insight to the subject. This theoretical approach along with a comprehensive literature review enable a categorization of the existing numerical methods. Based on this, it is possible to conclude that most of these methods are intimately related.

Another topic often overlooked is the so-called *pathological* cases. Because of them, any numerical algorithm for identifying observable islands needs to be iterative [6,12,19]. Based on the same theoretical background, it is shown that in most cases these additional iterations can be avoided. Thus, this paper also proposes an index that allows the identification of cases prone to be *pathological* such that in the remaining cases the iterations are dispensable. This approach can be straightforwardly applied with any direct numerical method for observability analysis making these algorithms more efficient.

Regarding the current research frontiers of state estimation, modeling measurements from Phasor Measurement Units (PMUs) [36] as well as the need for adequate SEs for distribution systems [37] are important challenges faced nowadays. In both cases, it is fundamental to develop adequate algorithms for observability analysis and identification of Cmeas and Csets. Another current problem is the optimal PMU placement [38,39]. In this case, the qualitative analysis should be used to define the measurement configurations that enable: observability; reliability and bad data processing ability. Therefore, the concepts provided in this paper can be very useful to deal with these challenges.

The rest of this paper is structured as follows: Section 2 deals with measurements modeling. Section 3 gives an overview about important linear algebra concepts. In Sections 4–6 the qualitative problems are dealt with. Discussions on how the proposed theoretical framework can be used to analyze the existing methods are presented. In Section 7 didactic examples are used to highlight the main ideas of the proposed approaches. Conclusions are drawn in Section 8. Appendix A presents a method to obtain bases for the required subspaces.

2. The measurement model

In general, a measurement system is composed of voltage magnitude, power flow and power injection measurements. It is also assumed that power measurements are available in pairs. In such case, it is well known that the active linearized measurement model is sufficient to solve the qualitative problems [2,5]. In this model, only linearized equations of active power flows and injections are taken into account. Furthermore, the state variables are just the bus voltage angles. Therefore, a linearized active power flow at a branch j is written as $P_{k,m} = x_{km}^{-1}\left(\theta_k - \theta_m\right)$, where x_{km} is the branch reactance and k and m are its terminal buses [3,40]. An active power injection at a bus k is modeled as $P_k = \sum_{m \in \Omega} P_{k,m}$, where Ω is the set of buses connected to bus k. This linearized model is still valid if measurements from PMUs are available [38,41]. That is, the real part of a complex current through a branch j can be approximated by $real\left(\vec{I}_{k,m}\right) \approx x_{km}^{-1}\left(\theta_k - \theta_m\right)$. Consequently, for the analysis of qualitative problems, measurements of power flow and phasorial current at branches can be treated indistinctly. The same applies for injection measurements. The modeling of bus voltage angle measurements is trivial. Thus, one can define the following linear system of equations:

$$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{\theta} + \boldsymbol{e} \tag{1}$$

where $\boldsymbol{\theta} \in \mathbb{R}^n$ is the vector of state variables, $\boldsymbol{z} \in \mathbb{R}^m$ is the vector of measurements, $\boldsymbol{e} \in \mathbb{R}^m$ is the vector of measurement errors and $\boldsymbol{H} \in \mathbb{R}^{m\times n}$ is the Jacobian matrix. Notice that, these errors are not considered during the analysis of qualitative problems. Eq. (1) describes a Direct Current (DC) model [3,40].

3. Fundamental concepts of linear algebra

In order to assist the readers, this section presents some fundamental concepts of linear algebra. Given a coefficient matrix $\boldsymbol{H} \in \mathbb{R}^{m\times n}$, the main goals are: (a) to decompose $\boldsymbol{H}$ into its fundamental subspaces and (b) to describe $\boldsymbol{H}$ as a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^m$.

3.1. Subspaces of $\mathbb{R}^n$

A subspace $\mathcal{W}$ of $\mathbb{R}^n$ is a subset of $\mathbb{R}^n$ that satisfies: (i) if $\{\boldsymbol{x}, \boldsymbol{y}\} \in \mathcal{W}$, then $(\boldsymbol{x} + \boldsymbol{y}) \in \mathcal{W}$ and (ii) if $\boldsymbol{x} \in \mathcal{W}$, then $s\boldsymbol{x} \in \mathcal{W}, \forall s \in \mathbb{R}$.

A set of vectors $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_k\}$ belonging to $\mathcal{W}$ span it if any vector $\boldsymbol{x} \in \mathcal{W}$ can be written as a linear combination of them. Furthermore, if they are Linearly Independent (LI), then they form a basis for $\mathcal{W}$. This occurs when the system of equations $t_1\boldsymbol{v}_1 + t_2\boldsymbol{v}_2 + \cdots + t_k\boldsymbol{v}_k = \boldsymbol{0}$ only admits the trivial solution, i.e., $\boldsymbol{t} = \boldsymbol{0}$. In this case k is the dimension of $\mathcal{W}$.