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Control Barrier Certificates for Safe Swarm Behavior

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Abstract: Multi-agent robotics involves the coordination of large numbers of robots, which leads to significant challenges in terms of collision avoidance. This paper generates provably collision free swarm behaviours by constructing swarm safety control barrier certificates. The safety barrier, implemented via an optimization-based controller, serves as a low level safety controller formally ensuring the forward invariance of the safe operating set. In addition, the proposed method naturally combines the goals of collision avoidance and interference with the coordination laws in a unified and computationally efficient manner. The centralized version of safety barrier certificate is designed on double integrator dynamic model, and then a decentralized formulation is proposed as a less computationally intensive and more scalable solution. The safety barrier certificate is validated in simulation and implemented experimentally on multiple mobile robots; the proposed optimization-based controller successfully generates collision free control commands with minimal overall impact on the coordination control laws.

1. INTRODUCTION

The literature on multi-robot coordination strategies traditionally focuses on the design of localized coordination rules with provable, global properties such as achieving and maintaining formation, covering areas, or tracking boundaries, e.g., Bullo et al. (2009); Mesbahi and Egerstedt (2010). However, what is actually deployed on teams of robots must also be safe in the sense that collisions are avoided, which typically calls for a secondary, low-level collision-avoidance controller that takes over the operation of the robots as they get too close to each other, e.g., Arkin (1998). The consequence of this construction is that what is tested is, in reality, a combination of the "formally" designed algorithm in conjunction with the "hand-crafted" collision-avoidance controller. Furthermore, as the number of robots increases, the "robot density" increases as well, with the result that the collision-avoidance controller starts to dominate the behavior of the robot team which means that the desired, global properties can no longer be ensured, e.g., Roumeliotis and Mataric (2000).

One solution to the problem of avoiding collisions is to make collision-avoidance an integral part of the coordinated control design. However, this significantly increases the complexity of the design-task and, more importantly, makes the many proposed design tools (see the textbooks Bullo et al. (2009); Mesbahi and Egerstedt (2010); Ren and Beard (2008) for a representative sample of these tools and techniques) no longer applicable. A remedy to this problem is to let the coordinated control design proceed without taking collisions into account and then *ensure that the safety controllers are minimally invasive* in the sense that they do as little as possible unless collisions are absolutely imminent. This idea was pursued in Tomlin et al. (1998) for pairs of kinematic aircraft and, based on global optimal control, hybrid control laws were developed that dictated when the aircraft needed to switch from their current mode of operations to an evasive maneuver in order to avoid collisions. Although elegant, the computational costs associated with solving the full-fledged Hamilton-Jacobi-Bellman Equations quickly become prohibitive when scaling up from two to more agents. Moreover, even for two agents, the problem cannot be solved in real-time; instead, the solution is viewed as a precomputed evasive maneuver that is stored and, subsequently, deployed by the aircraft.

The goal of this paper is to develop controllers that respect desired coordinated control laws as much as possible (in a least-squares sense) while simultaneously guaranteeing collision-free behavior. The key tool for producing these safety-critical controllers us to utilize control barrier certifications to prevent the robots from entering unsafe sets—naturally expressed in a minimally invasive fashion through the use of optimization-based controllers. Building upon the notion of barrier certificates proposed by Prajna et al. (2007), and adopting the control barrier function analogue recently proposed by Ames et al. (2014), safety constraints that prevent collision yield an inequality constraint affine in the control input. A given control law for coordination can then be implemented in the cost of an quadratic program (QP) based control law with constraints given by the safety barrier certificate that enforces collision free behavior. The resulting provably safe algorithm is applied to arbitrarily large teams of mobile robots in both centralized and decentralized representations.

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The outline of this paper is as follows: In Section 2 we briefly recall the control barrier certificate construction from Ames et al. (2014) and show that the enabling feature is the inclusion of the barrier as a constraint in an optimization-based controller (as opposed to inclusion in the cost, as is traditionally done; see Panagou et al. (2013)). In Section 3, we show how the barrier certificates can be designed in a centralized manner, i.e., by an external computational unit that has access to the states of all robots in the swarm. In order to reduce the computational burden, it is shown that it is enough to consider robots that are sufficiently close together, i.e., barriers must be considered only between a small subset of agents. This observation is what leads to a decentralized formulation in Section 4, where the individual robots themselves compute their own barrier certificates and corresponding, safe control actions based solely on locally available information. In Section 5, the control laws are experimentally implemented on a team of mobile robots, and the concluding remarks and future directions are the topics of Section 6.

2. BACKGROUND: BARRIER CERTIFICATES

Consider a nonlinear system of the form

$$\dot{x} = f(x) + g(x)u \tag{1}$$

for $x \in \mathbb{R}^n$ and $u \in U \subset \mathbb{R}^m$, with f and g assumed to be locally Lipschitz. For a given set of $\mathcal{C} \subset \mathbb{R}^n$, the goal is to generate a controller that ensures invariance of the set $\mathcal{C}$, i.e., solutions to (1) that start in $\mathcal{C}$ stay in $\mathcal{C}$ for all time. Establishing invariance of $\mathcal{C}$ can be done through the use of a *barrier function* $B : \mathcal{C} \to \mathbb{R}$ (or barrier certificate; see Prajna et al. (2007)). In particular, if B satisfies the properties:

$$\inf_{x \in \mathrm{Int}(\mathcal{C})} B(x) \geq 0, \qquad \lim_{x \to \partial\mathcal{C}} B(x) = \infty \tag{2}$$

then the question becomes: how does one constraint the behavior of $\dot{B}(x, u)$ to ensure invariance of $\mathcal{C}$?

Conventional design of barrier functions assumed invariant level sets of $\mathcal{C}$, i.e. $\dot{B} \leq 0$ (Tee et al., 2009). Yet this condition is unnecessarily strict, restricting the availability of control inputs to (1). To address this, Ames et al. (2014) recently presented a novel formulation that relaxes the conditions on the change in B to only require that:

$$\dot{B} \leq \frac{\gamma}{B} \tag{3}$$

with $\gamma > 0$. It was shown that this condition still ensures invariance of $\mathcal{C}$, since B is allowed to grow at a rate proportional to the distance of the system from the boundary of $\mathcal{C}$, the set of available control inputs that keep the system safe is greatly increased. The set $\mathcal{C}$ is given by

$$\begin{aligned} \mathcal{C} &= \{x \in \mathbb{R}^n : h(x) \geq 0\}, \\ \partial\mathcal{C} &= \{x \in \mathbb{R}^n : h(x) = 0\}, \\ \mathrm{Int}(\mathcal{C}) &= \{x \in \mathbb{R}^n : h(x) > 0\}, \end{aligned} \tag{4}$$

for a smooth function $h : \mathbb{R}^n \to \mathbb{R}$. Then the condition (3) naturally leads to a notion of a control barrier function:

Definition 1: For the dynamical system (1), a function $B : \mathcal{C} \to \mathbb{R}$ is a control barrier function (CBF) for the set $\mathcal{C}$ defined by (4) for a continuously differentiable function $h : \mathbb{R}^n \to \mathbb{R}$, if there exist locally Lipschitz class $\mathcal{K}$ functions α_1, α_2 such that, for all $x \in \mathrm{Int}(\mathcal{C})$,

$$\frac{1}{\alpha_1(h(x))} \leq B(x) \leq \frac{1}{\alpha_2(h(x))} \tag{5}$$

$$\inf_{u \in U} \left[L_f B(x) + L_g B(x) u - \frac{\gamma}{B(x)} \right] \leq 0 \tag{6}$$

Given a CBF B, consider the set:

$$K_{cbf}(x) = \left\{ u \in U \ : \ L_f B(x) + L_g B(x) u - \frac{\gamma}{B(x)} \leq 0 \right\}$$

wherein it was shown in Ames et al. (2014):

Theorem [Ames et al. (2014)]. Given a set $\mathcal{C} \subset \mathbb{R}^n$ defined by (4) with associated control barrier function B, any Lipschitz continuous controller $u(x) \in K_{cbf}(x)$ for the system (1) renders the set $\mathcal{C}$ forward invariant.

Note that in Ames et al. (2014), control barrier functions were only constructed in the case when h has relative degree 1 (this was extended to higher relative degrees in Hsu et al. (2015)), and applied to adaptive cruise control. In this paper we want to explore the application of CBFs in a multi-agent environment, where each agent is constrained by their own set of CBFs.

3. CENTRALIZED SAFETY BARRIER CERTIFICATES

This section focuses on developing centralized safety barrier certificates that are less intrusive to the nominal controller, but at an expense of central coordination. Then the safety barrier certificates will be decentralized in Section 4 requiring only local information, which leads to a scalable but more conservative solution.

3.1 Problem Formulation

Let $\mathcal{M} = \{1, 2, \dots, N\}$ be the set of N mobile agents. The dynamics of agent i in the robot swarm is given by

$$\begin{bmatrix} \dot{\mathbf{p}}_i \\ \dot{\mathbf{v}}_i \end{bmatrix} = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{p}_i \\ \mathbf{v}_i \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} \mathbf{u}_i, \tag{7}$$

where $\mathbf{p}_i \in \mathbb{R}^2$, $\mathbf{v}_i \in \mathbb{R}^2$, and $\mathbf{u}_i \in \mathbb{R}^2$ are the position, velocity, and acceleration of agent i respectively. The velocity and acceleration limits are $\|\mathbf{v}_i\|_p \leq v_{max}$ and $\|\mathbf{u}_i\|_p \leq a_{max}$, where $\|\cdot\|_p$ is vector $p-$norm determined by actual robot model. The relative position between agent i and j is denoted as $\Delta\mathbf{p}_{ij} = \mathbf{p}_i - \mathbf{p}_j$, relative velocity is $\Delta\mathbf{v}_{ij} = \mathbf{v}_i - \mathbf{v}_j$.
The safety constraint of the robot swarm requires that all agents should always keep safety distance D_s from each other. A pairwise safety constraint on relative velocity and relative position:

$$-\frac{\Delta\mathbf{p}_{ij}^T}{\|\Delta\mathbf{p}_{ij}\|} \Delta\mathbf{v}_{ij} \leq \sqrt{2\Delta a_{max}(\|\Delta\mathbf{p}_{ij}\| - D_s)}, \forall i \neq j \tag{8}$$

is proposed to regulate the dynamics of all robot agents within admissible range. This pairwise safety constraint is inspired by the idea of always keeping safety distance while applying the maximum braking force until relative velocity equals zero, which is adopted by many classic collision avoidance literature (see Fox et al. (1997), Ogren and Leonard (2005)). The pairwise robot agent collision avoidance case is a variation of single agent case, where