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# Comparison of two mathematical programming models for the solution of a convex portfolio-based European day-ahead electricity market

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ABSTRACT

In this paper two mathematical programming models are presented and compared for the solution of the European electricity day-ahead market with adjustable block orders. The concept of "adjustable" products is initially introduced, as products that can be partially cleared; thus, they do not bear the inherent indivisibilities of the classical block orders that are tradable in the European markets. Also, the clearing conditions of these adjustable products are analytically described. Due to their flexibility, the market clearing can adjust their clearing status accordingly, in order to match supply offers with demand bids avoiding non-convexities in the problem solution surface. This leads to the elimination of paradoxically accepted and rejected block orders. These adjustable products are modeled within the context of the European electricity day-ahead market problem, which is formulated as two different mathematical programming models: a Mixed Complementarity Problem (MCP) and a Linear Programming model (LP). The two models are evaluated in terms of computational efficiency using the pan-European zonal power system, by considering an increasing number of adjustable products per product type. The MCP model can formulate more adjustable product types, by efficiently formulating respective non-linear mixed pricing rules, but it is computationally demanding; on the other hand, the LP model is computational efficient, but it is not flexible enough to handle an extended variety of adjustable supply/demand block orders.


## 1. Introduction

Most Power Exchanges (PXs) in Europe allow market participants to submit, in addition to hourly single or portfolio offers/bids, combinatorial products called "block orders" (or "smart orders") that introduce inter-temporal constraints and mimic some of the unit technical (e.g. technical minimum) and operational constraints (e.g. fuel availability, especially for hydro units) and/or multi-period cost structures (start-up cost, shut-down cost, no-load or minimum-load cost). Block orders are "fill-or-kill" (all-or-nothing) orders, namely they are accepted or rejected in their entirety, depending on the hourly market clearing prices (MPs). The block orders that are tradable in the Central Western European (CWE) region [1], in Nord Pool Spot (NPS) day-ahead market [2], and recently in other European PXs, namely EXAA (Austria), OTE (Czech Republic), OKTE (Slovak Republic), HUPX (Hungary) and PMUM (Turkey) [3] comprise fixed and user-defined block orders, profile block orders, linked block orders, block orders belonging to an exclusive group and flexible hourly offers [4].

The presence of block orders complicates the clearing of electricity auctions. In addition to constrained continuous variables for hourly orders, a market clearing problem with blocks requires the inclusion of binary variables, in order to model "all-or-nothing" constraints of block orders. This leads to the formulation of Mixed-Integer Linear Programming (MILP) models. However, such formulations generally lead to inconsistencies between the cleared blocks and their clearing conditions. These cases have been referred many times in the literature [4–7], and are called "paradoxically accepted or rejected blocks" (PABs or PRBs, respectively). These inconsistencies arise from the fact that blocks are indivisible (accepted or rejected in their entirety), so they cannot be marginal in the market clearing. Thus, the clearing of a block introduces a non-convexity in the solution space, and "price jumps" in the market prices (MPs) for the block-related hours, when the block is marginally passing from an "accepted" to a "rejected" status and inversely.

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**Nomenclature**

*Indices and sets*

$t \in T$ set of trading periods within the trading day (typically, the trading period is one hour)

$r_{fpo} \in R_{fpo}, r_{fpb} \in R_{fpb}$ set of instances (time-intervals) within the trading day, in which the flexible profile offers/bids can be cleared, according to the duration of each flexible profile offer/bid; $R_{fpo} = [1, (25 - T_{fpo})]$, $R_{fpb} = [1, (25 - T_{fpb})]$. This set is used here for indexing the possible instances of a flexible profile offer/bid.

$a \in A$ set of bidding areas

$\alpha \in A_{jdp}$ set of bidding areas included in the joint demand profile block bid $jdp$

$a' \in A_a$ set of bidding areas connected to bidding area $a$

$s \in S_a$ set of supply offers submitted at bidding area $a$, $S_a \subseteq S$

$d \in D_a$ set of simple demand bids submitted at bidding area $a$, $D_a \subseteq D$

$po \in PO_a$ set of non-flexible profile orders submitted at bidding area $a$, where $po$ includes the following sub-indices/sets: $sp \in SP_a$ subset of supply profile block offers, $dp \in DP_a$ subset of demand profile block bids, $(jdp, a) \in JDP_a$ double index, indicating the part of joint demand profile block bid $jdp$ that is submitted for area $a$; $PO_a = SP_a \cup DP_a \cup JDP_a$, $PO_a \subseteq PO$

$p \in P_a$ set of all block orders submitted at bidding area $a$, where $p$ includes $po \in PO_a$, i.e. $sp \in SP_a$, $dp \in DP_a$, and $(jdp, a) \in JDP_a$ as well as the following sub-indices/sets $fho \in FHO_a$ subset of flexible hourly supply offers, $fhb \in FHB_a$ subset of flexible hourly demand bids, $fpo \in FPO_a$ subset of flexible profile supply offers, $fpb \in FPB_a$ subset of flexible profile demand bids

*Parameters*

$P_s^t, Q_s^t$ price-quantity pair of the hourly priced energy offer $s$ in trading period $t$, in €/MWh and MWh, respectively

$P_d^t, Q_d^t$ price-quantity pair of the hourly priced demand bid $d$ in trading period $t$, in €/MWh and MWh, respectively

$P_{po}, Q_{po}^t$ price-quantity pair of profile order $po$, in €/MWh and MWh, respectively; the quantity $Q_{po}^t$ for a given profile order $po$ may be different in each trading period $t$

$P_{fho}, Q_{fho}$, $P_{fhb}, Q_{fhb}$ price-quantity pair of flexible hourly order $fho$, $fhb$, in €/MWh and MWh, respectively; the flexible hourly offers/bids include just one quantity

$P_{fpo}, Q_{fpo}^{r,t}$, $P_{fpb}, Q_{fpb}^{r,t}$ price-quantity pair of flexible profile order $fpo$ and $fpb$, in €/MWh and MWh, respectively; each flexible profile offer/bid involves a quantity profile (same for each instance $r$ involved in the offer/bid)

$\underline{T}_{po}(\bar{T}_{po})$ starting (ending) trading period of profile order $po$

$\underline{T}_{fpo}^r(\bar{T}_{fpo}^r)$ starting (ending) trading period of instance $r$ of flexible profile offer $fpo$

$\underline{T}_{fpb}^r(\bar{T}_{fpb}^r)$ starting (ending) trading period of instance $r$ of flexible profile bid $fpb$

$T_{fpo}, T_{fpb}$ duration of flexible profile offer/bid $fpo/fpb$, respectively, in hours

$w_{po}^t$ average price weighting factor of profile order $po \in [SP_a, DP_a]$, in trading period $t$, in MWh

$w_{fpo}^{r,t}, w_{fpb}^{r,t}$ average price weighting factor of flexible profile orders $fpo \in FPO_a, fpb \in FPB_a$ of instance $r$ in trading period $t$, in MWh

$w_{jdp,a}^t$ average price weighting factor of joint demand profile block bid $jdp$ at bidding area $a$ in trading period $t$, in MWh

$FL_{aa'}$ Available Transmission Capacity (ATC) of the interconnection connecting bidding area $a$ to bidding area $a'$, in MW

*Continuous Variables*

$MP_a^t$ marginal clearing price at bidding area $a$ in trading period $t$, in €/MWh

$x_s^t$ cleared quantity of supply offer $s$ in trading period $t$, in MWh

$x_d^t$ cleared quantity of demand offer $d$ in trading period $t$, in MWh

$u_{fho}^t, u_{fhb}^t$ clearing status of flexible hourly offer/bid $fho/fhb$, respectively, in trading period $t$

$u_{fpo}^r, u_{fpb}^r$ clearing status of instance $r$ of flexible profile offer/bid $fpo/fpb$, respectively

$u_{po}$ clearing status of profile order $po$

$flow_{aa'}^t$ exchange in the interconnection connecting bidding area $a$ with $a'$, in MW

*Functions*

$\mathbf{c}_s^t$ cost function of the supply offer $s$ in trading period $t$, in €/h

$\mathbf{c}_d^t$ utility function of the demand bid $d$ in trading period $t$, in €/h

$\mathbf{c}_p^t$ cost/utility function of product $p$ in trading period $t$, in €/h

$\mathbf{v}_p^t$ function denoting cleared quantity of product $p$ in trading period $t$, in MWh

In the literature most research works have historically adopted iterative procedures and empirical simplifying criteria in order to handle PRBs and PABs, reach a solution and obtain the clearing prices, as analytically described in [8]. In [9] the effect of (a) block types, (b) number of blocks and (c) size of blocks is investigated, since western-European PXs impose such restrictions on market participants. The likelihood of finding PRBs is investigated in all cases, resulting in that factors (a) and (c) are directly related to the number of PRBs. The Lagrangian relaxation method is employed in [10] for the market clearing with block orders, for the decomposition of the problem in the separate system zones/areas. The method is tested on a two-area power system. In [11] the modeling of the European day-ahead market as a relaxed Mathematical Programming with Equilibrium Constraints (MPEC) is presented, and two algorithms are proposed for the market clearing: (a) an iterative Bid Cut Heuristic, which is extremely quick but may potentially lead to slightly suboptimal results, and (b) an exact algorithm, Branch-And-Cut Decomposition (using exact bid cuts), which attains optimal solutions but to the detriment of increased computational requirements. The models incorporate all type of orders tradable in European markets and all respective network constrains (ATC-based modeling, flow ramping constraints).

In view of the "target model" that will be enforced in all European markets in conjunction with their forthcoming coupling/integration, a centralized market splitting algorithm is implemented in [12,13], respecting the standard market regulatory framework of PXs and power pools, including (1) block offers/bids, linked block offers/bids, flexible hourly offers/bids and convertible block offers in power exchanges and (2) unit technical/commitment constraints and system operating