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ScienceDirect
IFAC-PapersOnLine 48-27 (2015) 080–085

Controllability of linear systems subject to packet losses ⋆

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Abstract: In this paper we study the controllability properties of discrete-time linear systems subject to packet losses. We tackle the problem from a switching systems perspective in which available information known on the packet loss signal, e.g., there cannot be more than a given maximum number of consecutive losses, is modelled through an automaton. For the resulting constrained switching system, we reformulate the controllability problem into an easier-to-study formulation through an algebraic characterization.
We show that the particular case where the packet loss signal does not contain more than N consecutive dropouts ($N \in \mathbb{N}$) boils down to a similar controllability problem with switching delays previously studied in the literature. For the general case, i.e., for an arbitrary automaton describing the lossy behaviour, we exploit the algebraic characterization and establish that our controllability problem of constrained switching systems is algorithmically solvable. This latter result is obtained by connecting it with the celebrated Skolem Theorem from linear algebra.

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Keywords: Switching Systems, Packet Losses, Networked Control Systems, Controllability, Decidability, Constrained Switching, Automata.

1. INTRODUCTION

In several contexts a real-time control loop is intermittently disrupted by undesired events causing imperfect control updates. These events include packet drops in wireless communication, task deadline misses in shared embedded processors, and outliers in sensor data, which can typically be modeled as data losses. It is obvious that the loss of data can severely influence fundamental properties such as controllability, observability, stabilizability, detectability, etc. of control systems. Due to many important applications suffering from these imperfect control updates, packet losses (or similar phenomena) have attracted much attention recently, see, e.g., Sinopoli et al. (2004); Tabbara et al. (2007); Pajic et al. (2011); D'Innocenzo et al. (2013); Gommans et al. (2013) and the references therein. Most of these works focus on stability and observer/controller design, and only implicitly deal with the fundamental properties such as controllability, observability, stabilizability and detectability under packet losses. In fact, the methods proposed were only providing sufficient conditions for stability or stabilizing controller design, or were assuming that one could fix the switching signal (e.g. to a periodic one) in order to control the plant more easily. However, given that these fundamental properties form cornerstones of modern system theory and indicate the possibilities and impossibilities for controller design, it is important to be able to analyze these properties explicitly when packet losses are an intrinsic feature of the feedback loop.

Therefore, we are interested in this paper in deriving *necessary and sufficient conditions* for controllability of plants subject to data losses (although we envision that our results also apply to other fundamental properties such as observability). In particular, we are interested in effectively deciding whether or not the plant remains controllable despite the possibility of losing data, and despite our inability to control this loss of data. We study a *worst case situation* in the sense that we will investigate whether *there exists one particular* sequence of data losses, which can hamper controllability. Not only is this question relevant in various real-life applications, it also obviously provides sufficient conditions for controllability of a system with probabilistic data loss behavior (which is often considered in the literature), and, in our view, it allows to formally understand from an algebraic point of view the concept of controllability with packet losses.

Our problem setup will lead us to analyze controllability properties of discrete-time linear systems with data losses modelled through automata. The data loss automaton captures the information available on the packet loss behaviour, which typically depends on the networked and

⋆ Raphael is supported by the Communauté francaise de Belgique - Actions de Recherche Concertées, and by the Belgian Programme on Interuniversity Attraction Poles and by the 'Actions de Recherches Concertées' Programme. He is a F.R.S.-FNRS Research Associate. Maurice is supported by the Innovational Research Incentives Scheme under the VICI grant "Wireless control systems: A new frontier in automation" (no. 11382) awarded by NWO (Netherlands Organisation for Scientific Research), and STW (Dutch Science Foundation), and the STW project 12697 "Control based on data-intensive sensing."

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10.1016/j.ifacol.2015.11.156

embedded control architectures used for implementing the control loop. This information could be, for instance, that there cannot be more than a given maximum number of consecutive losses. As we will show below, the overall system can be described as a *constrained switching system* (see Essick et al. (2014); Weiss and Alur (2007); Philippe et al. (2015)), being a switching system in which the admissible switching signals are generated by an automaton. As such, the problem studied in this paper boils down to studying controllability for this class of hybrid systems.

Controllability (and other fundamental system theoretic properties) have received quite some attention in the literature for different classes of hybrid systems, see, e.g., (Lin and Antsaklis, 2009; Blondel and Tsitsiklis, 1999; Bemporad et al., 2000; Ge and Sun, 2005; Babaali and Egerstedt, 2005; Sun and Zheng, 2001; Xie and Wang, 2003; Camlibel et al., 2008) and the references therein. However, it is known that general results for hybrid systems are hard to come by Blondel and Tsitsiklis (1999). This even holds in the context of switched linear systems (even without possible constraints on the switching) showing that analyzing controllability of switching systems is an extremely difficult task.

Although it is relatively easy to construct controllability conditions, which are generalizations of the classical tests for non-switching systems (see, for instance, (Ge and Sun, 2005, Theorem 4.31 p. 137) for such a general statement; our Proposition 1 below can be seen as a similar result for the systems at hand here), verifying them is another story. Indeed, as pointed out in the reference above,

> ...the conditions of (the) theorems are not verifiable in general. The proofs do not provide any information on how to find (...) controllability, reachability, etc.

In fact, these problems are typically *undecidable*, meaning that *there is no algorithm to solve these general problems* – see (Jungers, 2009, p. 29) and references therein for formal statements in this respect.

In this paper, we will exploit the particular algebraic structure of the systems at hand to show that in this case algorithms do exist that solve this problem. This particular algebraic structure comes from the fact that the switching is caused solely by packet losses. This causes that the underlying submodels of the switched systems share the same system (state) matrix and only the input matrix is different for each submodel.

It is worth mentioning that our work is close in philosophy to previous work initiated in Jungers et al. (2012), where controllability algorithms are constructed for other particular classes of switching systems, namely with a *switching delay* in the feedback loop. Below, we make one connection between these two settings: We show that the particular case of our problem mentioned above, where the constraint on the switching formalizes that there cannot be more than a given maximum number of consecutive losses, can be solved with techniques developed in Jungers et al. (2012) [1].

Another work close in nature to ours is Babaali and Egerstedt (2005). In their work, the authors provide sufficient conditions for observability (and controllability) of another class of switching systems which bears some similarities to ours. Similar to our framework, the system (state) matrix is the same for all the submodels, and only the input matrix switches in time. However, there are also differences with our work. For instance, the switching is arbitrary in their case, and there are several different input matrices. Moreover, the controllability conditions in Babaali and Egerstedt (2005) are only sufficient. They require constructing many pairs [2] of the type (A^l, b_i), where A is the plant matrix, l is an integer, and b_i is a possible input vector; and verifying that these pairs are controllable (in the classical sense). The authors prove that, if all these auxiliary pairs are controllable, then the switching system is also controllable.

Outline and contributions. In Section 2, we introduce our problem and start our running example. In Section 3, we provide a characterization of controllability for our systems. In Section 4, we prove our main theorem, leading to an algorithm for verifying controllability. Finally, in Section 5, we show that the controllability of linear systems with variable delays is a particular case of our problem.

2. PROBLEM FORMULATION

We study the controllability of the following discrete-time linear system

$$x(t+1) = \begin{cases} Ax(t) + bu(t), & \text{if } \sigma(t) = 1, \\ Ax(t), & \text{if } \sigma(t) = 0 \end{cases} \tag{1}$$

with $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}$ the state and control input, respectively, at time $t \in \mathbb{N}$. Moreover, $\sigma : \mathbb{N} \to \{0, 1\}$ is the *data loss signal*, which represents the packet dropouts in the sense that $\sigma(t) = 0$ corresponds to the case where the control packet is lost at time t, while $\sigma(t) = 1$ corresponds to the case where the control packet has arrived in good order at time t. Sometimes, we also call this the *actuation signal.* The trajectory generated by the system (1) with initial state $x(0) = x_0$, input sequence $u : \mathbb{N} \to \mathbb{R}$ and actuation signal $\sigma : \mathbb{N} \to \{0, 1\}$ is denoted by $x_{x_0,\sigma,u}$.

In this paper we assume that the data loss signals satisfy certain constraints representing the physics of the shared (wireless) communication network and/or the characteristics of the underlying embedded architecture. In our model, the admissible actuation signals are infinite words accepted by a certain automaton.

Definition 1. An *automaton* is a pair $\mathcal{A} = (M, s) \in \{0, 1\}^{N\times N} \times \{0, 1\}^N$ with N the *number of states*, the *transition matrix* $M \in \{0, 1\}^{N\times N}$, and the *vector of node labels* $s = [s_1\ s_2\ \ldots\ s_N]^\top \in \{0, 1\}^N$.

Automata define the set of data loss signals that are admissible according to the following definition.

Definition 2. A signal $\sigma : \mathbb{N} \to \{0, 1\}$ is said to be *admissible*, if there exists a sequence of states $v : \mathbb{N} \to \{1, \ldots, N\}$

[1] In fact, it can be proved that the *general* setting of Jungers et al. (2012) can be tackled with the tools developed here (though less efficiently, from a computational point of view). We defer this question for further work because of space constraints.

[2] The number of such pairs to test is bounded by making use of the Van Der Waerden Theorem, and is thus highly impractical. For instance, in dimension 3 and with 3 different input matrices, the bound on the number of pairs to test is larger than 10^{14610}.

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