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System-Centric Minimum-Time Paths for Battery-Powered Vehicles in Networks with Charging Nodes ⋆

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Abstract: We study the routing problem for vehicle flows through a road network that includes both battery-powered Electric Vehicles (EVs) and Non-Electric Vehicles (NEVs). We seek to optimize a system-centric (as opposed to user-centric) objective aiming to minimize the total elapsed time for all vehicles to reach their destinations considering both traveling times and recharging times for EVs when the latter do not have adequate energy for the entire journey. Extending prior work where we considered only EVs entering the network, we formulate the problem by grouping all vehicles into a set of "subflows"and provide solutions based on both a Mixed Integer Non-Linear Programming (MINLP) approach and an alternative flow optimization problem. Since the problem size increases with the number of subflows, its proper selection is essential to render the problem manageable, thus reflecting a trade-off between proximity to optimality and computational effort needed to solve the problem. We propose a criterion and procedure leading to a "good" choice for the number of subflows.

1. INTRODUCTION

The increasing presence of Battery-Powered Vehicles (BPVs), mobile robots and sensors has given rise to novel issues in classical network routing problems (Laporte [1992]). There are four BPV characteristics which are crucial in such problems: limited cruising range, long charge times, sparse coverage of charging stations, and the BPV energy recuperation ability (Artmeier et al. [2010]) which can be exploited. In Khuller et al. [2011], algorithms for several routing problems are proposed, including a single-vehicle routing problem with inhomogeneously priced refueling stations for which a dynamic programming based algorithm is proposed to find a least-cost path from source to destination. The same problem is revisited in Sweda and Klabjan. [2012] for an Electric Vehicle (EV) and extensions may be found in Artmeier et al. [2010], Eisner et al. [2011], Siddiqi et al. [2011]. More recently, an EV Routing Problem with Time Windows and recharging stations (E-VRPTW) was proposed in Schneider et al. [2012], where controlling recharging times is circumvented by simply forcing vehicles to be always fully recharged. In Worley et al. [2012], an integer programming optimization problem was formulated to simultaneously find optimal routes and charging station locations.

All aforementioned work deals with the routing problem for a single EV. This is not easily generalized to a multi-vehicle routing problem. In Wang et al. [2014] and Pourazarm and Cassandras [2014] the problem is first considered from the driver's point of view (the "single-vehicle routing problem") then from the system's point of view (the system-centric "multi-vehicle routing problem".) In the former, the goal is to find an optimal path along with a charging policy for a single EV acting "selfishly" to reach its destination in minimum time; under certain conditions, a Nash equilibrium may then be reached (Roughgarden [2005]). In the latter case, we define a system-wide objective and the goal is to route EVs so that a whole inflow reaches its destination in minimum time, therefore achieving a "social optimum". We studied these problems in networks with both homogeneous and inhomogeneous charging nodes where "inhomogeneity" means that charging rates at different nodes are not identical. In both Wang et al. [2014] and Pourazarm and Cassandras [2014] it was assumed that every arriving vehicle is an EV. In this paper, we relax this assumption by considering both EVs with energy constraints and Non-Electric Vehicles (NEVs) in the inflow to the network. We again seek to optimize a system-centric objective by optimally routing NEVs and EVs along with an optimal policy for charging EVs along the way if needed.

The key to our approach is grouping of EVs and NEVS into "subflows" and then solving the problem by assigning a routing and charging policy to each subflow. We first formulate the problem as a Mixed Integer Non-Linear Programming (MINLP) problem. We exploit some properties of the optimal solution and energy dynamics in order to decompose the problem into (*i*) route selection and (*ii*) recharging amount determination, thus reducing the problem dimensionality. We also provide an alternative flow-based formulation such that each subflow is not required to follow a single end-to-end path, but may be split into an optimally determined set of paths. This Non-Linear Programming (NLP) formulation reduces the computational complexity of the MINLP by orders of magnitude with numerical results showing little loss in optimality. Generally, the problem size and associated computational

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complexity depend on the number of subflows. The second contribution of the paper is to propose a criterion for the selection of the number N of EV subflows (assuming a fixed number of NEV subflows) and a procedure for selecting N so as to trade off computational complexity against proximity to an optimal objective value.

In Section 2, we formulate the multi-vehicle routing problem with both NEVs and EVs. In Section 3 we define a criterion for the selection of the number of EV subflows, N. Numerical examples are included illustrating our approach. Conclusions and further research directions are outlined in Section 4.

2. SYSTEM-CENTRIC MULTI-VEHICLE ROUTING PROBLEM

We assume that a network is defined as a directed graph $G = (\mathcal{N}, \mathcal{A})$ with $\mathcal{N} = \{1, \dots, n\}$ and $|\mathcal{A}| = m$. Node $i \in \mathcal{N}/\{n\}$ represents a charging station and $(i, j) \in \mathcal{A}$ is an arc(link) connecting node i to j (we assume for simplicity that all nodes have a charging capability, although this is not necessary). We also define $I(i)$ and $O(i)$ to be the set of start nodes (respectively, end nodes) of links that are incoming to (respectively, outgoing from) node i, that is, $I(i) = \{j \in \mathcal{N}|(j, i) \in \mathcal{A}\}$ and $O(i) = \{j \in \mathcal{N}|(i, j) \in \mathcal{A}\}$. Nodes 1 and n respectively are defined to be the origin and destination. Extensions to multiple origins and destinations are straightforward. Let all vehicles enter the network at node 1 and let R denote the rate of vehicles arriving at this node. We seek to optimize a *system-centric objective* by routing (both EVs and NEVs) while also charging EVs if needed. Let us assume a fraction P of NEVs in the inflow. Therefore, NEVs and EVs enter the network with flow rates given by RP and $R(1 - P)$ respectively. Relative to the single-vehicle routing problem (where we can assume that all network links operate at fixed congestion levels), the main technical difficulty here is that routing decisions over all vehicles (EVs and NEVs) directly influence traffic congestion.

Clearly, it is extremely hard to make system-wide optimal routing decisions at the individual vehicle level. Thus, the key to our approach is to aggregate vehicles into groups and seek instead to solve a problem where decisions are made at the group level. Viewing all vehicles as defining a *flow*, we divide them into *subflows* associated with each such group; we can then study the effect of the number of subflows selected. The choice of subflow to which a vehicle may be assigned is dictated by similarities in the types and behaviors of vehicles, e.g., large vehicles vs smaller ones or EVs with the same initial energy.

Let us first divide the inflow of NEVs into a fixed number of subflows, M (e.g., the number of distinct paths from the origin to the destination node) and the inflow of EVs into N subflows (we will discuss the effect of N in Section 2.5). Thus, all vehicles in the same subflow follow the same routing and recharging decisions so that we only consider control at the subflow level rather than individual vehicles.

2.1 Mixed Integer Non-Linear Programming Formulation

We begin by extending the system-centric multi-vehicle optimization problem developed in Wang et al. [2014] and Pourazarm and Cassandras [2014] by involving both EV and NEV flows. Our objective is to determine optimal routes for NEV subflows and optimal routes, as well as energy recharging amounts, for each EV subflow so as to minimize the total elapsed time of these subflows from origin to destination. Note that for NEVs, we do not consider the refueling process as part of this optimization problem. The decision variables consist of (i) $x_{ij}^k \in \{0, 1\}$, $k = 1, .., M$ and $y_{ij}^l \in \{0, 1\}$, $l = 1, .., N$, corresponding to the selection of link (i, j) by NEV and EV subflows respectively, and (ii) charging amounts r_i^l for EV subflows for all nodes $i = 1, \dots, n-1$ and subflows $l = 1, \dots, N$. Given traffic congestion effects, the time and energy consumption on each link depends on the values of x_{ij}^k, y_{ij}^l and the fraction of the total flow rate R associated with the kth NEV subflow or the lth EV subflow. The simplest such flow allocation (which we will adopt) is to assign each subflow the same rate, i.e., every NEV subflow $k = 1, .., M$ is assigned a rate RP/M and every EV subflow $l = 1, \dots, N$ is assigned a rate $R(1-P)/N$. Let $\mathbf{x_{ij}} = (x_{ij}^1, \cdots, x_{ij}^M, y_{ij}^1, \cdots, y_{ij}^N)^T$ and $\mathbf{r_i} = (r_i^1, \cdots, r_i^N)^T$ where r_i^l is the amount of charge selected by the lth EV subflow at node i. Then, we denote the traveling time (delay) a vehicle will experience through link (i, j) by some nonlinear function $\tau_{ij}(\mathbf{x_{ij}})$. The corresponding energy consumption for the lth subflow of EVs through (i, j) is a nonlinear function denoted by $e_{ij}^l(\mathbf{x_{ij}})$.

Finally, we define E_i^l to be the residual energy of subflow l of EVs at node i, given by the aggregated residual energy of all EVs in the subflow. If the subflow does not go through node i, then $E_i^k = 0$. We also define g_i as the charging time per unit of energy for a charging node i, i.e., the reciprocal of a charging rate. The optimization problem is formulated as follows:

$$\min_{\mathbf{x_{ij}}, \mathbf{r_i},\, i,j\in\mathcal{N}} \quad \Big[\sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{M} \tau_{ij}(\mathbf{x_{ij}}) x_{ij}^k \frac{RP}{M} + \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{l=1}^{N} \Big(\tau_{ij}(\mathbf{x_{ij}}) y_{ij}^l \frac{R(1-P)}{N} + r_i^l g_i y_{ij}^l\Big)\Big] \tag{1}$$

$$s.t. \text{ for each } k \in \{1, \dots, M\}:$$

$$\sum_{j\in O(i)} x_{ij}^k - \sum_{j\in I(i)} x_{ji}^k = b_i, \quad \text{for each } i \in \mathcal{N} \tag{2}$$

$$b_1 = 1,\, b_n = -1,\, b_i = 0, \text{ for } i \neq 1, n \tag{3}$$

$$\text{for each } l \in \{1, \dots, N\}:$$

$$\sum_{j\in O(i)} y_{ij}^l - \sum_{j\in I(i)} y_{ji}^l = b_i, \quad \text{for each } i \in \mathcal{N} \tag{4}$$

$$b_1 = 1,\, b_n = -1,\, b_i = 0, \text{ for } i \neq 1, n \tag{5}$$

$$E_j^l = \sum_{i\in I(j)} (E_i^l + r_i^l - e_{ij}^l(\mathbf{x_{ij}})) y_{ij}^l, \quad j = 2, \dots, n \tag{6}$$

$$E_1^l \text{ is given}, \quad E_i^l \geq 0, \quad \text{for each } i \in \mathcal{N} \tag{7}$$

$$x_{ij}^k \in \{0, 1\}, \quad y_{ij}^l \in \{0, 1\}, \quad r_i^l \geq 0 \tag{8}$$

In the above formulation, (1) is the objective function which for NEVs is the first sum representing the overall traveling time from origin to destination by adding the link traveling times $\tau_{ij}(\mathbf{x_{ij}})$ when $x_{ij}^k = 1$. For EVs, the second sum includes the charging times $r_i^l g_i$ when $y_{ij}^l = 1$ and an EV subflow selects node l for charging. The constraints (2)-(3) and (4)-(5) represent flow conservation for NEV and EV subflows respectively, while (6)-(7) shows the energy