Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-27 (2015) 147–154

Safety-Critical Control for Dynamical Bipedal Walking with Precise Footstep Placement

Quan Nguyen * **Koushil Sreenath** **

* *Dept. of Mechanical Engineering, Carnegie Mellon University, Pittsburgh, PA 15213 USA (e-mail: qtn@ andrew.cmu.edu).*
** *Depts. of Mechanical Engineering, Robotics Institute, Electrical & Computer Engineering, Carnegie Mellon University, Pittsburgh, PA 15213 USA (e-mail: koushils@cmu.edu).*

Abstract: This paper presents a novel methodology to achieve dynamic walking for underactuated and hybrid dynamical bipedal robots subject to safety-critical position-based constraints. The proposed controller is based on the combination of control Barrier functions and control Lyapunov functions implemented as a state-based online quadratic program to achieve stability under input and state constraints, while simultaneously enforcing safety. The main contribution of this paper is the control design to enable stable dynamical bipedal walking subject to strict safety constraints that arise due to walking over a terrain with randomly generated discrete footholds and overhead obstacles. Evaluation of our proposed control design is presented on a model of RABBIT, a five-link planar underacted bipedal robot with point feet.

Keywords: Safety-critical; Nonlinear control; Lyapunov function; Quadratic programming.

1. INTRODUCTION

Cyber physical systems (CPSs) have strict safety constraints and designing controllers that provide formal guarantees of enforcing these safety constraints is critical. In this paper we consider a specific CPS, a bipedal robot. Bipedal walking is subject to several safety constraints. One that is most visual is that of walking over discrete footholds, requiring critical guarantees of precise foot placements for ensuring the safety of the bipedal robot. Here we consider the problem of dynamic walking, and develop a controller that provides guarantees on stability and safety. The proposed methodology is also applicable to other CPSs whose dynamics evolve on complex nonlinear manifolds as illustrated in Wu and Sreenath (2015).

Footstep placement for fully actuated legged robots has been carried out by several researchers Kajita et al. (2003); Kuffner et al. (2001); Chestnutt et al. (2005). Impressive results in footstep planning and placements in obstacle filled environments with vision-based sensing is carried out in Michel et al. (2005); Chestnutt et al. (2003). However, these methods essentially rely on quasi-static walking using the ZMP criterion which only enables slow walking with small steps. Moreover, for a point-foot legged robot, the above methods are not feasible. The DARPA Robotics Challenge has inspired several new methods, some based on mixed-integer quadratic programs Deits and Tedrake (2014). However, as mentioned in (Deits, 2014, Chap. 4), the proposed method of mixed-integer based footstep planning does not offer dynamic feasibility even for a simplified model. On the other hand, footstep placement for dynamic locomotion typically rely on simple models with massless legs, see Desai and Geyer (2012); Rutschmann et al. (2012); Wu and Geyer (2013). Although these results are impressive, extending them to non-trivial bipedal models is hard. Recent results of using centroidal dynamics for whole-body dynamic motion including arm and foot contacts in Dai et al. (2014) is impressive, however stability guarantees of the hybrid locomotion is unclear.

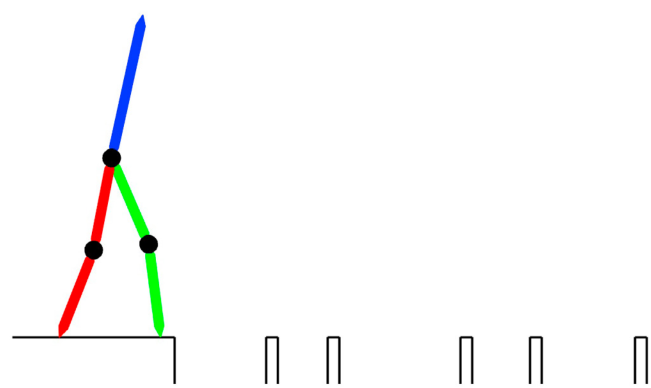

Fig. 1. The problem of dynamically walking over a randomly generated set of discrete footholds. The discrete footholds serve as strict safety constraints that need to be enforced with formal guarantees for the safety of the bipedal robot. Although the proposed safety-critical control is developed in this paper for this particular problem, it's also applicable to more general CPSs.

The proposed approach in this paper deviates from these prior results by starting off with a periodic dynamic walk-

This work is partially supported through funding from the Google Faculty Award and NSF Grant IIS-1464337.

2405-8963
Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.11.167

ing gait with formal stability guarantees and enforcing safety-critical constraints for achieving precise footstep placement. The proposed controller can also enforce various input and state-based constraints. It must be noted that precise footstep placement for dynamic walking on underactued bipedal robots with nonlinear and hybrid dynamics with provable stability and safety is challenging. Our research develops on recent work on control Lyapunov function based quadratic programs for bipedal robots and control Barrier function based quadratic programs for autonomous cruise control in cars, see Galloway et al. (2015); Ames et al. (2014a). The proposed methodology enables guaranteeing the trajectory of the swing foot to be within certain constraints, so as to result in a given step length when the swing foot hits the ground. This results in placing the foot precisely on the discrete foothold.

In particular, we employ the method of Hybrid Zero Dynamics (HZD) and virtual constraints, Westervelt et al. (2003, 2007), which has been successful in dealing with the hybrid and underactuated dynamics of legged locomotion. Experimental implementations of the HZD method using input-output linearization with PD control for dynamic walking has been carried out in Sreenath et al. (2011) and dynamic running in Sreenath et al. (2013) on MABEL. Recent work on control Lyapunov function (CLF)-based controllers has resulted in HZD-based stable walking, Ames et al. (2014b). The flexible control design using CLFs has been carried out through online quadratic programs (QPs), facilitating incorporating additional constraints into the control computation, Galloway et al. (2015), and L_1 adaptive control with model uncertainty in Nguyen and Sreenath (2015a).

Although the HZD based control design for dynamic legged locomotion is powerful, these types of controllers cannot directly help us to enforce safety-critical constraints, such as walking over discrete footholds and walking under obstacles. One way of enforcing the safety constraints is to recompute the walking gait by changing the virtual constraints used for walking. Virtual constraints are a set of output functions that need to be regulated by the controller to achieve periodic walking. However, this involves a complex nonlinear constrained optimization which is time-consuming and intractable for online implementation in a feedback controller.

The main contribution of this paper is a novel safety-critical control strategy that can guarantee precise footstep placement for dynamic walking of a hybrid, nonlinear, underactuated bipedal system. We do this by combining control Lyapunov function based quadratic programs (CLF-QPs) Galloway et al. (2015) for tracking the nominal virtual constraints while respecting the saturation limits of control inputs, and control Barrier function based quadratic programs (CBF-QPs) Ames et al. (2014a) to guarantee state dependent safety constraints. The goal of this paper is to relax the tracking behavior of the nominal gait by enforcing a set of state dependent safety constraints, governed by control Barrier functions, that guide the swing foot trajectory to the discrete foothold and bend the torso to avoid overhead obstacles. Our method enables dealing with a large range of desired foothold separations with precise placement of footsteps on small footholds that are less than 5% of the leg length. As noted earlier, this work is generalizable to other CPSs (see Wu and Sreenath (2015).)

The rest of the paper is organized as follows. Section 2 revisits control Lyapunov function-based quadratic programs (CLF-QPs). Section 3 revisits Control Barrier Function and its combination with CLF-QP. Section 4 presents the proposed CBF-CLF-QP based feedback controller for enforcing safety constraints. Here we will first present its application to the simpler problem of avoiding overhead obstacles before developing for the main problem of dynamic bipedal walking over a terrain with discrete footholds (see Fig. 1). Section 5 then presents numerical validation of the controller on the model of RABBIT, a five-link planar bipedal robot. Finally, Section 6 provides concluding remarks.

2. CONTROL LYAPUNOV FUNCTION BASED QUADRATIC PROGRAMS REVISITED

In this section we start by introducing a hybrid dynamical model that captures the dynamics of a bipedal robot. We then review recent innovations on control Lyapunov functions for hybrid systems and control Lyapunov function based quadratic programs, introduced in Ames et al. (2014b) and Galloway et al. (2015) respectively.

2.1 Model

This paper will focus on the specific problem of walking of bipedal robots such as RABBIT (described in Chevallereau et al. (2003)), which is characterized by single-support continuous-time dynamics, when one foot is assumed to be in contact with the ground, and double-support discrete-time impact dynamics, when the swing foot undergoes an instantaneous impact with the ground. Such a hybrid model is obtained as,

$$\mathcal{H} = \begin{cases} \begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = f(q,\dot{q}) + g(q,\dot{q})u, & (q^-,\dot{q}^-) \notin S, \\ \\ \begin{bmatrix} q^+ \\ \dot{q}^+ \end{bmatrix} = \Delta(q^-,\dot{q}^-), & (q^-,\dot{q}^-) \in S, \end{cases} \tag{1}$$

where $q \in \mathcal{Q}$ is the robot's configuration variables, $u \in \mathbb{R}^m$ is the control inputs, representing the motor torques, $(q^-, \dot{q}^-)$ represents the state before impact and $(q^+, \dot{q}^+)$ represents the state after impact, S represents the switching surface when the swing leg contacts the ground, and Δ represents the discrete-time impact map.

We also define output functions $y \in \mathbb{R}^m$ of the form

$$y(q) := H_0 q - y_d(\theta(q)), \tag{2}$$

where $\theta(q)$ is a strictly monotonic function of the configuration variable q, H_0 is an appropriately-sized matrix prescribing linear combinations of state variables to be controlled, and $y_d(\cdot)$ prescribes the desired evolution of these quantities (see Sreenath et al. (2011) for details.) The method of Hybrid Zero Dynamics (HZD) aims to drive these output functions (and their first derivatives) to zero, thereby imposing "virtual constraints" such that the system evolves on the lower-dimensional zero dynamics manifold, given by

$$Z = \{(q,\dot{q}) \in T\mathcal{Q} \mid y(q) = 0,\ L_f y(q,\dot{q}) = 0\}, \tag{3}$$