Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnline CONFERENCE PAPER ARCHIVE

IFAC-PapersOnLine 48-27 (2015) 161–168

Iterative Robust Stabilization Algorithm for Periodic Orbits of Hybrid Dynamical Systems: Application to Bipedal Running ⋆

Kaveh Akbari Hamed * **Jessy W. Grizzle** **

* *Department of Mechanical Engineering, San Diego State University, San Diego, CA 92182 USA (e-mail: kakbarihamed@mail.sdsu.edu)*
** *Department of Electrical Engineering, University of Michigan, Ann Arbor, MI 48109 USA (e-mail: grizzle@umich.edu)*

Abstract: This paper presents a systematic numerical algorithm to design optimal $\mathcal{H}_\infty$ continuous-time controllers to robustly stabilize periodic orbits for hybrid dynamical systems in the presence of discrete-time uncertainties. A parameterized set of closed-loop hybrid systems is assumed for which there exists a common periodic orbit. The algorithm is created based on an iterative sequence of optimization problems involving Bilinear and Linear Matrix Inequalities (BMIs and LMIs). At each iteration, the optimal $\mathcal{H}_\infty$ problem is translated into a BMI optimization problem which can be easily solved using available software packages. Some sufficient conditions for the convergence of the iterative algorithm are presented. The power of the algorithm is then demonstrated in designing robust stabilizing virtual constraints for running of a highly underactuated bipedal robot with 7 degrees of underactuation in the presence of impact model uncertainties.

Keywords: Hybrid Periodic Orbits, Robust Orbital Stability, Underactuated Bipedal Running

1. INTRODUCTION

The main objective of this paper is to present a systematic numerical algorithm to design optimal $\mathcal{H}_\infty$ continuous-time controllers for robust stabilization of periodic orbits for a class of hybrid dynamical systems arising from bipedal locomotion. The robustness is achieved against uncertainty in the discrete-time dynamics of hybrid systems. Models of bipedal robots are hybrid with ordinary differential equations (ODEs) to describe stance and flight phases and discrete transitions to describe leg toe-off and impact with the ground (Hurmuzlu and Marghitu, 1994; Grizzle et al., 2014, 2001; Westervelt et al., 2007; Chevallereau et al., 2009; Ames et al., 2007; Ames, 2014; Spong and Bullo, 2005; Spong et al., 2007; Manchester et al., 2011; Dai and Tedrake, 2013; Gregg et al., 2012; Gregg and Spong, 2008; Byl and Tedrake, 2008; Akbari Hamed and Grizzle, 2014; Chevallereau et al., 2003; Morris and Grizzle, 2009; Sreenath et al., 2013; Collins et al., 2005; Byl and Tedrake, 2009; Saglam and Byl, 2013).

While the problem of designing optimal $\mathcal{H}_\infty$ controllers for complex systems is well studied in the literature (Gahinet and Apkarian, 1994; Doyle et al., 1991), existing results are tailored for stabilization of *equilibrium points* of ODEs and *not* periodic orbits of hybrid dynamical systems. The most basic tool to investigate the stability of period orbits of hybrid systems is the Poincaré sections method (Grizzle et al., 2001; Haddad et al., 2006; Parker and Chua, 1989; Haddad and Chellaboina, 2008). One of the most serious limitations in employing the Poincaré sections approach to design $\mathcal{H}_\infty$ continuous-time controllers is the lack of closed-form expressions for the Poincaré map and its Jacobian matrices. In particular, they need to be calculated numerically and this becomes a real challenge for hybrid mechanical systems with high degrees of freedom and underactuation.

Previous work in the literature made use of different approaches to stabilize hybrid periodic orbits. One of these approaches employs multi-level hybrid controller structures. In this approach, the stability of the orbit is mainly achieved by higher-level event-based controllers (Grizzle, 2006; Westervelt et al., 2007; Akbari Hamed and Grizzle, 2014; Sreenath et al., 2013). This approach may result in a potentially large delay between the occurrence of a disturbance and the event-based control effort. Other approaches employed nonlinear optimization techniques for the simultaneous design of periodic orbits and stabilizing continuous-time controllers (Chevallereau et al., 2009; Diehl et al., 2009). These approaches minimize the spectral radius of the Jacobian of the Poincaré map or a smoothed version of that and cannot address the optimal $\mathcal{H}_\infty$ control design problems. An alternative approach has been developed based on the moving Poincaré sections analysis and transverse linearization to design time (phase) varying LQR controllers (Manchester et al., 2011; Shiriaev et al., 2010). This latter approach has not been extensively evaluated on legged robots.

The contribution of this paper is to create an iterative optimization algorithm based on Bilinear and Linear Ma-

⋆ The work of K. Akbari Hamed was partially supported by the Center for Sensorimotor Neural Engineering (CSNE) that is an NSF Engineering Research Center. The work of J. W. Grizzle was supported by NSF Grants ECCS-1343720 and ECCS-1231171.

10.1016/j.ifacol.2015.11.169

trix Inequalities (BMIs and LMIs) to design optimal $\mathcal{H}_\infty$ continuous-time controllers for hybrid models of mechanical systems with high degrees of freedom and underactuation. The algorithm acts as a powerful tool to design a general form of robust optimal nonlinear controllers including LQR and virtual constraints. Furthermore, it can be effectively solved with available software packages. Our previous work presented a non-iterative BMI optimization framework for exponential stabilization of periodic walking gaits (Akbari Hamed et al., 2014, 2015). Furthermore, a robustness analysis over two steps during stepping down/up was presented for uneven ground walking. When extending this approach to hybrid models of bipedal running, one would need to apply the BMI optimization in an iterative manner to stabilize the running gait. Furthermore, the running models are more sensitive to impact model uncertainties. This motivates us to present an iterative robust stabilization algorithm to handle $\mathcal{H}_\infty$ robustness against impact model (i.e., discrete-time) uncertainties over an infinite horizon of steps rather than two steps. To do this, the current paper presents a new BMI framework to design optimal $\mathcal{H}_\infty$ controllers. Some sufficient conditions for the convergence of the iterative algorithm in stabilizing the hybrid periodic orbits are also presented. The gait sensitivity norm was introduced in (Hobbelen and Wisse, 2007) as a disturbance rejection measure and demonstrated on a 2 DOF bipedal robot. This paper provides additional results. In regards to feedback design, the current paper presents a systematic $\mathcal{H}_\infty$ algorithm to reduce the sensitivity to impact models. Finally, the power of the algorithm is demonstrated in designing optimal $\mathcal{H}_\infty$ nonlinear controllers for a 2D underactuated bipedal runner with 7 degrees of underactuation.

2. ROBUST STABILIZATION PROBLEM

2.1 A Family of Parameterized Closed-Loop Hybrid Models

We consider a family of parameterized closed-loop hybrid systems with one continuous-time phase as follows

$$\Sigma^{\mathrm{cl}}: \begin{cases} \dot{x} = f^{\mathrm{cl}}(x,\xi), & x^- \notin \mathcal{S} \\ x^+ = \Delta(x^-,\xi) + d, & x^- \in \mathcal{S}, \end{cases} \tag{1}$$

where $x \in \mathcal{X}$ represents the state variables and $\mathcal{X} \subset \mathbb{R}^n$ is the state manifold. The continuous-time portion of the hybrid system is given by the parameterized closed-loop ODE $\dot{x} = f^{\mathrm{cl}}(x,\xi)$, in which $\xi \in \Xi \subset \mathbb{R}^p$ is a vector of adjustable constant parameters. In addition, Ξ represents a *set of admissible parameters* and the superscript "cl" stands for the closed-loop system. Here, $f^{\mathrm{cl}} : \mathcal{X} \times \Xi \to \mathrm{T}\mathcal{X}$ is a smooth (i.e., $\mathcal{C}^\infty$) vector field, in which $\mathrm{T}\mathcal{X}$ is the tangent bundle of the state manifold $\mathcal{X}$. The discrete-time portion of the dynamics is then represented by the parameterized reset law $x^+ = \Delta(x^-,\xi) + d$, where $\Delta : \mathcal{X} \times \Xi \to \mathcal{X}$ denotes a $\mathcal{C}^\infty$ *switching map*. $d \in \mathcal{D} \subset \mathbb{R}^n$ is also an *unknown* and *additive discrete-time disturbance* to represent the uncertainty in the reset model. It is further assumed that $\mathcal{D}$ contains the origin. In our notation, x^- and x^+ denote the state variables just before and after the reset event, respectively. The solutions of the hybrid system (1) undergo an abrupt change according to the reset law on the *switching manifold* $\mathcal{S}$ given by $\mathcal{S} := \{x \in \mathcal{X} \,|\, s(x) = 0\}$, where $s : \mathcal{X} \to \mathbb{R}$ is a $\mathcal{C}^\infty$ real-valued *switching function* satisfying the condition $\frac{\partial s}{\partial x}(x) \neq 0$ for all $x \in \mathcal{S}$. Figure 1 represents a geometric description for the closed-loop hybrid model (1) in the state space. Figure 2 demonstrates that the hybrid model of bipedal running with two continuous-time phases can be represented by an equivalent hybrid system with one continuous-time phase as given in (1).

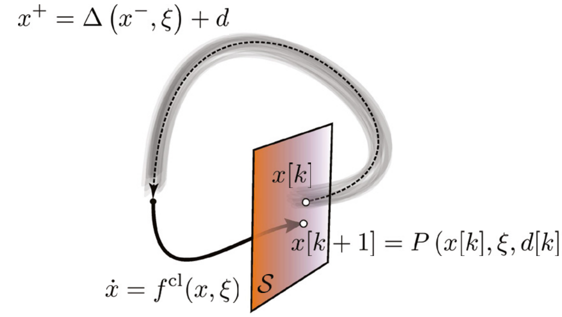

Fig. 1. Illustration of the closed-loop hybrid model (1) with one continuous-time phase. The solid and dashed curves correspond to the flows of the continuous- and discrete-time dynamics $\dot{x} = f^{\mathrm{cl}}(x,\xi)$ and $x^+ = \Delta(x^-,\xi) + d$, respectively. The uncertainty in the discrete dynamics is shown by the cloud around the dashed curve.

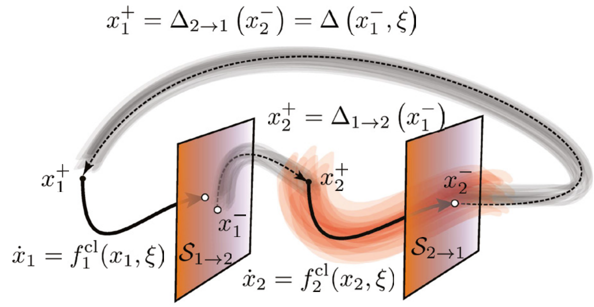

Fig. 2. Illustration of a closed-loop hybrid model with two continuous-time phases for bipedal running. Using (Grizzle et al., 2014, Proposition 4), one can present an equivalent hybrid system with one continuous-time phase as in (1), whose reset map Δ can be expressed as $\Delta := \Delta_{2\to1} \circ \mathcal{F}_2 \circ \Delta_{1\to2}$, where $\mathcal{F}_2$ denotes the flow of $\dot{x}_2 = f_2^{\mathrm{cl}}(x_2,\xi)$ (second phase) and "$\circ$" represents the composition. In this model, the uncertainty d of (1) can arise from uncertainties in $\Delta_{1\to2}$, $\Delta_{2\to1}$ and $\mathcal{F}_2$ as shown by the clouds.

The solution of the parameterized ODE $\dot{x} = f^{\mathrm{cl}}(x,\xi)$ with the initial condition $x(0) = x_0$ is denoted by $\varphi(t,x_0,\xi)$ for all $t \geq 0$ in the maximal interval of existence. The *time-to-reset* function is then defined as $T : \mathcal{X} \times \Xi \to \mathbb{R}_{>0}$ as the first time at which the ODE solution $\varphi(t,x_0,\xi)$ intersects the switching manifold $\mathcal{S}$, i.e.,

$$T(x_0,\xi) := \inf\{t > 0 \,|\, \varphi(t,x_0,\xi) \in \mathcal{S}\}. \tag{2}$$

2.2 Invariant Periodic Orbit

Throughout this paper, we shall assume that the following assumption is satisfied for the family of hybrid systems (1).

Assumption 1. It is assumed that there exists a *common* period-one orbit $\mathcal{O}$ for the family of closed-loop hybrid