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Comparing three methods for solving probabilistic optimal power flow

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ABSTRACT

This paper compares three methods for solving probabilistic optimal power flow (P-OPF) problem: Zhao's point estimate method (PEM), Quasi Monte Carlo simulation (QMCS) and Latin hypercube sampling (LHS). With Nataf transformation, P-OPF problem is formulated as a multiple integral over standard normal space. By introducing a differential operator, a mathematical model is developed to compare the performance of QMCS and LHS. Furthermore, a simplified Gaussian mixture model (GMM) is presented to model distributions of P-OPF solutions. Testing on a modified 118-bus system, it is found LHS outperforms PEM and QMCS with a small sample size, but behaves comparably with QMCS for a large sample size. Compared to other statistic models, GMM shows a higher flexibility for data fitting.

1. Introduction

Probabilistic optimal power flow (P-OPF) and probabilistic power flow (PPF) are two important tools for power system planning and operation, especially at a time when the intermittent renewable generations are increasingly integrated into the grid. Regardless of the detailed computational process, PPF and P-OPF may be regarded as a same problem: characterizing the output of an implicit function whose inputs are random variables. From this point of view, the methodologies developed for solving PPF are also applicable to P-OPF problem.

The major difference between P-OPF and PPF lies in the function relationship $F(\cdot)$ of inputs and outputs. In PPF calculation, $F(\cdot)$ is defined by a set of equations; while, in P-OPF problem, these equations are just a set of constraints for an optimization problem, which also includes many inequality constraints. That is to say, the solutions of PPF equations should both satisfy power flow equations and the inequality constraints in optimal power flow (OPF). Therefore, the nonlinearity of $F(\cdot)$ is much more severe for P-OPF than for PPF. Hence, the algorithms for P-OPF computation should be robust for a highly nonlinear function, and can achieve satisfactory accuracy with an acceptable computational burden.

The cumulant method, classified as analytical method, is very efficient for P-OPF computation [1–3]. This algorithm employs a linear function to approximate $F(\cdot)$, and uses it to calculate the cumulants of outputs. While, because of the linearization of $F(\cdot)$, this method shows low accuracy for high-order cumulants. Weighted least square (WLS) method is another analytical method. In Ref. [4], it is combined with Gaussian mixture model (GMM) [5] to solve PPF problem. With each inputs being decomposed into weighted sums of normal variables, PPF problem is solved by executing multiple WLS runs for every possible combinations of decomposed normal components. Although the probability distributions of inputs and outputs are well represented by GMM, the calculation times of WLS increase dramatically with respect to the number of non-normal inputs.

Another set of methods, known as point estimate method (PEM), has been introduced to solve P-OPF problem in recent years [6–8]. In conjunction with normal transformation technique, PEM can handle correlated non-normal input variables. This method chooses representative points and assigns them with associated weights, then, performs deterministic optimal power flow (OPF) for these points, and calculates the statistical moments of outputs. In Refs. [9,10], two algorithms: Hong's PEM and Zhao's PEM are compared for PPF computation, the result shows that Zhao's PEM is more accurate.

The unscented transformation (UT) method may also be classified as PEM, but this method employs a different way to determine points and weights, and can handle correlated input variables directly. In Refs. [11,12], UT method is used to solve PPF problem and P-OPF problem, and gives comparable results with Hong's PEM. Hence, in this paper, Zhao's PEM is adopted as a candidate algorithm for P-OPF computation.

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Table 1
Four statistic models for data fitting.

Model	Basic distribution	RM	PWM	Percentile
Johnson	Normal	Yes [20]	No	Yes [21]
CF	Normal	Yes [22]	No	No
NPNT	Normal	No	Yes [17]	No
GLD	Uniform	Yes [23]	Yes [24]	Yes [25]

Monte Carlo simulation (MCS) is a well known and adaptive method for P-OPF computation [13–16]. With numerous samples generated from probability distributions of inputs, deterministic OPF calculations are carried out. Then, the statistical moments of the output variable can be obtained directly. Unlike analytical method and PEM, MCS does not employ an approximation function of $F(\cdot)$ to get outputs. By increasing the number of samples, the accuracy of MCS can be further improved. Whereas, MCS is not computationally competitive, it requires a large number of samples to yield reliable results.

To improve the efficiency, algorithms are developed to choose samples more effectively, such as Quasi Monte Carlo simulation (QMCS) [17] and Latin hypercube sampling (LHS) [18,19]. By transforming P-OPF problem into independent standard uniform space, these algorithms generate samples which are more uniformly distributed. Compared to MCS, QMCS and LHS can yield more accurate results with less samples.

Another issue of P-OPF is to find a flexible model to represent the probability distributions of outputs. This problem is referred to as data fitting, and several models can serve for this purpose, such as Johnson system, Cornish–Fisher (CF) expansion, Ninth-order polynomial normal transformation (NPNT), and generalized lambda distribution (GLD). All these models are developed by making an elementary transformation of certain basic distribution, such as standard normal distribution and standard uniform distribution. Then, with suitable parameters, various probability distributions can be approximated.

When estimating the parameters of these models, moment matching and percentile matching are two generally used methods. By matching the moments or percentiles of target distribution with those of the model, equations solved for parameters are established. The candidate statistical moments include: standardized central moment, raw moment (RM), cumulant, L-moment and probability weighted moment (PWM). While, the standardized central moment and cumulant can be obtained through an elementary transformation of RM; mathematically, they are actually the same. Similarly, L-moment can also be substituted by PWM. Thus, the moment matching method actually involves two types of moments: RM and PWM.

Table 1 shows algorithms for estimating parameters of these models. For some models, RM, PWM or percentile matching would yield equations too complicated to be solved, and it is denoted as "No" in Table 1.

Although these four models are capable of simulating various unimodal distributions, they fail to handle bimodal or multimodal distributions, which some P-OPF solutions may follow. To accommodate such cases, Gaussian mixture model (GMM) can be employed, but the parameter estimation of GMM involves complicated mathematical calculations [5].

The paper aims to compare three algorithms for solving P-OPF problem: Zhao's PEM, QMCS and LHS. To accommodate correlated non-normal input variables, Nataf transformation is employed, whereby P-OPF is formulated as an integral with respect to independent standard normal variables. With the introduction of a differential operator, the issue of calculating the multiple integral is converted to a multivariate function approximation problem, where QMCS and LHS are compared. To reconstruct the probability distributions of P-OPF solutions, a simplified GMM is put forward, and a percentile matching method is presented to determine its parameters.

The paper is outlined as follows: Section 2 introduces Nataf transformation and P-OPF problem. Section 3 describes three methods for solving P-OPF problem, a theoretical comparison between QMCS and LHS is also given. In Section 4, a simplified GMM model is developed. In Section 5, a case study is performed on a modified IEEE-118 bus system including wind farms. Section 6 provides some conclusions.

2. Problem formulation

2.1. Nataf transformation

Suppose x is a random variable with cumulative distribution function (CDF) $F(x)$, z is a standard normal variable with CDF $\Phi(z)$. The transformation from z to x is denoted as [26,27]:

$$F(x) = \Phi(z) \leftrightarrow x = F^{-1}[\Phi(z)] \tag{1}$$

where $F^{-1}(\cdot)$ is the inverse CDF of x.

Suppose x_i, x_j are two correlated random variables, which are generated from standard normal variables z_i and z_j respectively:

$$\begin{aligned} x_i &= F_i^{-1}[\Phi(z_i)] \\ x_j &= F_j^{-1}[\Phi(z_j)]. \end{aligned} \tag{2}$$

Let ρ_x denote the correlation coefficient between x_i and x_j, let ρ_z denote the one between z_i and z_j. To ensure a desired value of ρ_x, a suitable value of ρ_z should be determined. This issue can be handled by algorithms in Refs. [26,27].

Suppose $\boldsymbol{X} = (x_1, \ldots, x_i, \ldots, x_m)^T$ is an m-dimensional random vector with correlation matrix $\boldsymbol{R_X}$. According to Eq. (1), $\boldsymbol{X}$ can be generated by the standard normal vector $\boldsymbol{Z} = (z_1, \ldots, z_i, \ldots, z_m)^T$ with an appropriate correlation matrix $\boldsymbol{R_Z}$.

For each entry $\rho_x(i, j)\,(i \neq j)$ in $\boldsymbol{R_X}$, calculate the associated $\rho_z(i, j)$, and obtain $\boldsymbol{R_Z}$ in the standard normal space. Then, $\boldsymbol{Z}$ can be obtained by the following linear transformation:

$$\boldsymbol{Z} = \boldsymbol{LU} \tag{3}$$

where $\boldsymbol{U} = (u_1, \ldots, u_i, \ldots, u_m)^T$ is an independent standard normal vector. $\boldsymbol{L}$ represents the lower triangular matrix from Cholesky decomposition of $\boldsymbol{R_Z}$:

$$\boldsymbol{R_Z} = \boldsymbol{LL}^T. \tag{4}$$

In short, Nataf transformation can be expressed as:

$$\begin{pmatrix} u_1 \\ \vdots \\ u_i \\ \vdots \\ u_m \end{pmatrix} \xrightarrow{\substack{\boldsymbol{R_Z} = \boldsymbol{LL^T} \\ \downarrow \\ \boldsymbol{Z} = \boldsymbol{LU}}} \begin{pmatrix} z_1 \\ \vdots \\ z_i \\ \vdots \\ z_m \end{pmatrix} \xrightarrow{x_i = F_i^{-1}[\Phi(z_i)]} \begin{pmatrix} x_1 \\ \vdots \\ x_i \\ \vdots \\ x_m \end{pmatrix}. \tag{5}$$