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A Hybrid-Dynamical Model for Passenger-flow in Transportation Systems ⋆

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Abstract: In a network with different transportation modes, or multimodal public transportation system (MPTS), modes are linked among one another not by resources or infrastructure elements - which are not shared, e.g., between different metro lines - , but by the flow of passengers between them. Now, the movements of passengers are steered by the destinations that individual passengers have, and by which they can be grouped into trip profiles. To use the strength of fluid dynamics, we therefore introduce a multiphase hybrid Petri net model, in which the vehicle dynamics is rendered by individual tokens moving in an infrastructure net, while passenger quantities are given as vectors - whose components correspond to trip profiles - and evolve at stations according to fluid dynamics. This model is intended as a building block for obtaining supervisory control, via transport operator actions, to mitigate congestion.

Keywords: Modeling, Networks, Petri nets, Specification, Transportation systems

1. INTRODUCTION

In a multimodal public transportation system (MPTS), different lines with separate infrastructure and belonging to different operators offer fixed-route passenger transportation services. These different *modes* can be assumed not to share their infrastructures or any other resources that would couple their performances together; nonetheless, performance issues such as delays and congestion do propagate from one mode to another via passenger transfers between them. Thus, contrary to the situation in single-mode transportation control where vehicle movements are paramount, see e.g. Ding and Chien (2001), it is here the passenger transfers that have to play a central role in modelling and analyzing perturbations that spread across multimodal networks.

Passengers move according to their *trip profiles*, i.e. their destination and a pre-chosen path through the system toward that destination. At each stop of a vehicle, the movement of all passengers of the same profile will be governed by the same dynamic rule: either all board, or all alight, or all remain where they are, waiting for the right stop before alighting, or waiting for the right train etc before boarding. This may change in case of a traffic perturbation or disruption; imagine e.g. loudspeaker announcements in trains and on platforms advising passengers to prefer alternative routes. In such a situation, all *or part of* the passengers in a trip profile will switch to a different trip profile, and follow its dynamics henceforth until destination, or further changes.

In the literature, several approaches can be found, e.g. in

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- Traffic assignment models as discussed in Fu et al. (2012), where network flow models are used to allocate traffic loads to routes: passengers travel according to efficient paths. These models are static, i.e. do not make vehicle movements explicit; in fact, only load capacities are considered, not the actual transportation performance, let alone its variations.
- the Max Plus-Algebra approach to transportation systems such as in Nait-Sidi-Moh et al. (2002), the focus is on synchronization of vehicle arrivals and departures at local points in the network, with the objective of minimizing, and improving robustness to, operation-related delays. The dynamics induced by passenger movements or congestion are not included, and there seems to be no easy way to add them.
- Multi-agent systems, which offer a fine-grain view of individual actions, are the basis, e.g. of Microsimulation platforms such as MATSim in Balmer (2007), in which agents are moved in a transport network in order to process individual activity plans that comes along with an iterative optimization of the agents' travel behaviours. There also exist discrete, Petri-style models of multi-agent systems such as nets-within-nets in Köhler et al. (2003) or Bednarczyk et al. (2005), and related models. In fact, the presence of passengers inside a moving vehicle is a case of nets within nets: every passenger is both a Petri net reflecting their trip profile and the current state within the intended trajectory, and a token inside the net representing the vehicle's state; whereas the vehicle at the same time moves as a token in the infrastructure net. However, the analysis methods developed thus far for Nets-within-nets-type models focus on reachability and other semantic issues. Our approach focusses on quantities of passengers of the same type, and introduces fluid approximations so as to account for uncertainties in network observation,

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while allowing faster computations of quantitative dynamics.

Our approach can be seen as an extension of (timed) hybrid Petri nets in the sense of David and Alla (2010) and as applied to urban traffic control in Di Febbraro et al. (2004); Dotoli et al. (2008); Júlvez and Boel (2010). State space explosion in such models can be overcome e.g. with integrality relaxation as discussed in Silva and Recalde (2002), and Silva and Recalde (2005). However, the model developed here extends the existing ones in that places are marked with multi-dimensional passenger vectors on places, rather than scalar "liquids"; one may think of these nets as of *coloured* fluid Petri nets. In contrast to Dotoli et al. (2008); Júlvez and Boel (2010) we do not employ a first order approximation of the continuous non-linear dynamics describing the passenger flows so as to obtain an overall piece-wise linear model dynamics. Instead, we comply with Di Febbraro et al. (2004) in that the non-linear transition flows are directly integrated into the firing semantics.

The article is structured as follows. In section 2 we introduce multiphase fluid Petri nets. We use them in section 3 as modelling blocks, in order to capture the passenger arrival and departure processes from / to the outside world of a MPTS; the passenger transfers in the stations; and the passenger flows between the stations and the stopped vehicles. Finally, we provide conclusions and an outlook on future work in section 4.

2. MULTIPHASE FLUID PETRI NETS

In the eyes of fluid dynamics, a place of a classical timed fluid Petri net holds a single phase fluid; the marking of that place defines a quantity of the fluid; and enabling and firing rules define flows of the single phase fluid between the places, i.e. single phase flows. Now, in a multiphase fluid Petri net (mFPN), some places, called multiphase reservoirs (mr) hold a multiphase fluid, i.e. are marked with a *vector* of non-negative real numbers, in which each number refers to the quantity of a particular phase. All other places, called simple reservoirs (sr), are marked with a single non-negative real number that abstracts away from the different phases of the fluid, and refers to a quantity of the multiphase fluid as a whole.

We will now define the structure of mFPNs and the markings of their places, together with balance equations that provide a continuous-time dynamics. Thereby, we relate the marking with the multiphase flows by means of flow transformation matrices. Finally, we take into account capacity-limitations of the network.

Definition 1. A multiphase fluid net (mFN) is a 4-tuple $\mathrm{N} := (P, T, F, \mathrm{c})$, with

- the finite set of places P,
- the finite set of transitions T, in which $P \cap T = \emptyset$,
- the flow relation $F \subseteq (P \times T) \cup (T \times P)$, and
- the colour function $\mathrm{c} : P \to \{sr, mr\}$ that specifies whether a given place is a simple or multiphase reservoir.

Remark 2. Throughout the rest of this article, denote as $P_v := \mathrm{c}^{-1}(\{mr\})$ the set of multiphase reservoirs, and as $P_s := \mathrm{c}^{-1}(\{sr\})$ all simple reservoirs of the considered mFN N. As usual, we note for any place or transition $u \in P \cup T$ the pre- and post-set of u as ${}^\bullet u := \{v \in P \cup T \text{ s.t. } (v, u) \in F\}$ and $u^\bullet := \{v \in P \cup T \text{ s.t. } (u, v) \in F\}$, respectively.

As shown in Fig. 1, we represent multiphase reservoirs as ordinary circles, simple reservoirs as dashed circles, and transitions as boxes. Moreover, we connect an arc from place $p \in P$ to transition $t \in T$ iff $p \in {}^\bullet t$, and from transition t to place p iff $p \in t^\bullet$.

Remark 3. Throughout the rest of this article, $\tau \in \mathbb{R}_{\geq 0}$ denotes a time instant that will be clear from the context, and $X := \{1, 2, \ldots, x\}$ the set of all different phases $x \in \mathbb{N}_{>0}$ of the fluid in the considered mFPN.

We store the *marking* of the simple and multiphase reservoirs of an mFN in two functions and obtain an mFPN.

Definition 4. An x-phased fluid Petri net, with $x \in \mathbb{N}_{>0}$, is a 3-tuple $\mathcal{N} := (\mathrm{N}, \mathrm{M}, \mathrm{m})$, where

- N is a multiphase fluid net,
- $\mathrm{M} : P_v \times \mathbb{R}_{\geq 0} \to (\mathbb{R}_{\geq 0})^x$ the multiphase marking, and
- $\mathrm{m} : P_s \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ the simple reservoirs' marking.

Dynamics. Now, we define how mFPN $\mathcal{N}$'s marking changes as a function of time starting from the initial marking at $\tau = 0$. Assign at $\tau \geq 0$ to every transition $t \in T$ the x-phased flow

$$\phi : T \times \mathbb{R}_{\geq 0} \to (\mathbb{R}_{\geq 0})^x$$
$$(t, \tau) \mapsto \phi(t, \tau),$$

and the $(x \times x)$-dimensional flow transformation matrix

$$\mathrm{R} : T \times \mathbb{R}_{\geq 0} \to (\mathbb{R}_{\geq 0})^{x \times x}$$
$$(t, \tau) \mapsto \mathrm{R}(t, \tau).$$

Next, we set up a balance equation at every multiphase and simple reservoir, in which we integrate both as indicated in Fig 1. Thus, with the above notation, at τ the marking of every multiphase reservoir $v' \in {}^\bullet t \cap P_v$ decreases according to ϕ, and the marking of every simple reservoir $s' \in {}^\bullet t \cap P_s$ according to $1^T \phi$. On the contrary, the marking of every multiphase reservoir $v'' \in t^\bullet \cap P_v$ increases according to $\mathrm{R}\,\phi$, and the marking of every simple reservoir $s'' \in t^\bullet \cap P_s$ according to $1^T \mathrm{R}\,\phi$. We then obtain for every multiphase reservoir $v \in P_v$ the balance equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{M}(v, \tau) := \sum_{t \in {}^\bullet v} \mathrm{R}(t, \tau)\ \phi(t, \tau) - \sum_{t \in v^\bullet} \phi(t, \tau), \quad (1)$$

and for every simple reservoir $s \in P_s$ the balance equation

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \mathrm{m}(s, \tau) := 1^T \sum_{t \in {}^\bullet s} \mathrm{R}(t, \tau)\ \phi(t, \tau) - 1^T \sum_{t \in s^\bullet} \phi(t, \tau). \quad (2)$$

Here, we have used the following notations: Let M be an $m \times n$ matrix with $m, n \in \mathbb{N}_{>0}$, and u a column vector of length m. $M[i, \cdot]$ then denotes the i-th row of matrix M with $i \in \{1, 2, \ldots, m\}$, $M[\cdot, j]$ its j-th column with $j \in \{1, 2, \ldots, n\}$, and M^T its transpose. $u[i]$, on the other hand, denotes the element in the i-th row of vector u. Moreover, 0 denotes a matrix of zeros only, and 1 of ones only. The dimension of such a matrix will be clear from the context.