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Approximating the parameter-space stability boundary considering post-contingency corrective controls ☆

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ABSTRACT

Lately, much work in the area of voltage stability assessment has been focused on finding post-contingency corrective controls. In this article a contribution to this area will be presented where we investigate the surface of maximal loadability while allowing for post-contingency corrective controls. This objective is different from the usual, where the aim is to include the post-contingency controls in a security-constrained optimal power flow. Our aim is rather to find approximations of the post-contingency stability boundary, in pre-contingency parameter space, while including the possibility for post-contingency corrective controls. These approximations can then be used in, for example, a chance-constrained optimal power flow routine.

1. Introduction

With the increasing amount of variable generation that we see in power systems today, knowing the stability margin in a given direction of stress is usually not sufficient for operating the system. To obtain more reliable measures of system security we want to have global information about the stability margins. This global information is expressed through the parameter-space stability boundary Σ, that separates parameters corresponding to stable equilibrium points from those corresponding to unstable or infeasible ones.

Work has already been done in approximating the stability boundary without corrective controls. An approach that will often give conservative estimates is to define the stability margin as the (minimal) distance to the stability boundary. Efficient algorithms for the computation of this distance have been proposed [1–3]. In [4,5], sensitivities of the distance to the stability boundary to changes in system parameters are given for small-signal and voltage stability, respectively. The use of the sensitivities can help the system operator take optimal actions to either steer the system away from instability or make the system return to stability following a contingency. In [6], a formula for a unified sensitivity of the loading margin to changes in system parameters for different types of instabilities is given, and shown to give results that are consistent with the existing sensitivities presented in, for example, [5].

In [7], the stability boundary is approximated by hyperplanes from the inside, so that the approximation is conservative whenever the stability region is convex. Examples are given where the approximations are used for assessing security margins.

In [8], first- and second-order approximations of the small-signal stability boundary are presented. The authors of [8] use the implicit function theorem to express the relationship between the parameters on the stability boundary.

In [9,10] it is suggested that a number of points on the stability boundary should be computed by moving in different directions in parameter space, starting from a given initial point. From these points the entire boundary is then approximated using a neural network.

In [11,12], the normal to and the curvature tensor of the stability boundary are used to express second-order approximations of the voltage stability boundary, thus giving an intuitive geometrical expression of the second-order approximations. In [13] these approximations were extended to small-signal stability as well as thermal stability. A systematic way of finding appropriate points to compute the approximations in was also suggested. A special situation relating to long-term voltage stability was described in [14] and further investigated in [15].

In [16,17] second order approximations of the transient stability boundary were proposed.

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Although many contributions have been made to the subject of approximating parameter-space stability boundaries, none of them consider the important extension of post-contingency corrective controls. If corrective controls are not considered when dispatching the production, the system will in most situations be run in an overly conservative manner.

In [18] the loadability limit surface with corrective controls, Σ^{CC}, was computed in a given direction of stress. The approach considers a quasi-static load recovery and assumes that the post contingency parameter trajectory only intersects the post-contingency stability boundary at one single point, called the *point of intersection*. In a first step, second order approximations of the post-contingency stability boundary are used to get an initial prediction of the intersection point. Then a set of necessary conditions for optimality gives a system of equations that can be solved starting with the prediction obtained in the first step.

In this article we will change view to approximating the entire surface Σ^{CC} by a number of quadratic polynomials. These polynomials will be derived from Taylor's expansions of the surface at points on the different smooth parts that together constitute Σ^{CC}. To find the normal and the curvature of Σ^{CC} we exploit the necessary conditions of optimality proposed in [18].

The approach taken is analogous to the one presented in [13] in the sense that we search for a point on each encountered smooth part of the stability boundary called the *most important point* of that surface. At these points we make series approximations of Σ^{CC} based on local information.

The remainder of this article is organized as follows. First, in Section 2 the notion of a stability boundary is introduced. Then, in Section 3 a summary of stability limits with corrective controls, based on [18], is given. In Section 4, the post-contingency stability boundary with corrective controls, in pre-contingency parameter space, is described and corner points thereof are discussed. In Section 5, second order Taylor's approximations for all different types of smooth parts of Σ^{CC} are derived. Section 6 is devoted to the more practical issue of finding points to base the approximations on and how to make sure that different approximations do not interfere with each other. In Section 7, a numerical example is given. The paper then ends with a discussion of the results of the numerical example, computational complexity of the algorithm and generalizations of the admissible controls, and a summary in Section 8.

2. The stability boundary without corrective controls

To understand the notion of a stability boundary with post-contingency corrective controls we must first define the stability boundary without corrective controls.

2.1. Stability limits

We align the system parameters in the vector $\lambda \in \mathbb{R}^m$. The system parameters can represent quantities such as load demand parameters, or active power production. Later on, we will split the vector λ into a sub-vector $y \in \mathbb{R}^k$ of controllable system parameters, and a vector $P_L \in \mathbb{R}^l$ of non-controllable system parameters.

From an initial parameter vector λ_p, *e.g.* present production and consumption, the stability limit in the direction of stress $d_s \in S^{m-1}$ (the unit sphere in $\mathbb{R}^m$) is given by the solution to the optimization problem

$$\max_{r \in \mathbb{R}} \{r : \text{stable e.p. exists with } \lambda = \lambda_p + r d_s\} \quad (1)$$

This problem can be solved by various methods such as continuation methods [19,20], optimization methods [21], direct methods [22] and quasi-steady state (QSS) simulations [23,24].

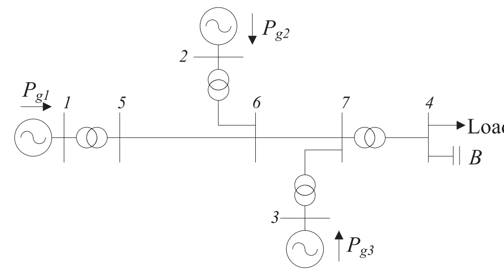

Fig. 1. The power system used in the example.

2.2. The stability boundary

The stability boundary surface Σ is the boundary of the domain wherein the system is small-signal stable. The surface $\Sigma \subset \mathbb{R}^m$ is made up of a number of different smooth manifolds [2]. Due to constraint switching there are two types of feasibility limits and we get the following different types of points on the stability boundary [13]

- *SNB:* A saddle-node bifurcation loadability limit is a loadability limit that may occur when the system Jacobian becomes singular. This type of loadability limit is the most commonly addressed loadability limit in voltage stability assessment VSA.
- *SLL:* Switching loadability limits [25] correspond to cases when the power system becomes immediately unstable when a control variable limit is reached.
- *HB:* Hopf Bifurcation points are points in parameter space where the real part of one pair of complex eigenvalues of the dynamic Jacobian becomes positive as the system parameters change so that the system is no longer small-signal stable.

The stability boundary is not smooth but rather made up of a number of smooth manifolds which intersect at non-smooth points that are referred to as corner points (CPs).

2.3. Example

Consider the system depicted in Fig. 1. This system was analyzed in [25] and consists of three generators and one load. It is assumed that node 1 is the slack node (where all power deviations are balanced) and that the load is of the constant power type with a fixed power factor. The system has three parameters that are allowed to vary; P_{g2}, P_{g3}, and P_{Load}. It is also assumed that each generator has a limited E_f with $E_f^{\text{lim}} = 2.5968$ p.u. for each generator.

In Fig. 2 the stability surface Σ_{pre} corresponding to the original system is plotted together with the stability boundary Σ_{post} with doubled impedance between nodes 5 and 6, when varying P_{g3} and P_{Load}, while keeping P_{g2} fixed at 1.2 p.u.

As can be seen in the figure, at some level of P_{g3}, between 0.5 and 1 p.u., the stability surfaces are non-smooth.

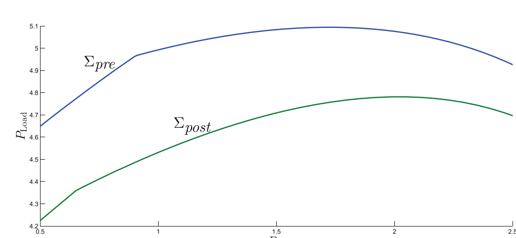

Fig. 2. The pre- and post-contingency stability surfaces in the example.