Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-27 (2015) 371–376

# Stability of time-delay reset control systems with time-dependent resetting law [1]

M. A. Davó * F. Gouaisbaut **,*** A. Baños *
S. Tarbouriech **,**** A. Seuret **,****

* Dpto. Informática y Sistemas, University of Murcia, 30071 Murcia, Spain.
** CNRS, LAAS, 7 avenue du Colonel Roche, F-31400 Toulouse, France.
*** Univ de Toulouse, UPS, LAAS, F-31400, Toulouse, France.
**** Univ de Toulouse, LAAS, F-31400, Toulouse, France.

**Abstract:** This work presents results on the stability of time-delay reset control systems under time-dependent resetting conditions. The stability of a reset control system composed by a time-delay process and a proportional and integrative plus Clegg integrator (PI+CI) compensator is tackled by using the framework of sampled-data systems. It leads to sufficient stability conditions expressed in terms of LMIs (Linear Matrix Inequality), that depend explicitly on the reset times. In contrast to previous results, the proposed conditions allow to guarantee the stability of reset systems with unstable base system.



*Keywords:* Reset control, Time-Delay, Impulsive systems, Sampled-data control

## 1. INTRODUCTION

Reset control systems are a special type of hybrid systems, in which the system state (or part of it) is reset at the instants it intersects some reset surface. When the reset is defined as a function of time one can consider the reset control system as an impulsive system. In the past decades, the importance of impulsive systems has been highlighted by many researchers due to the number of potential broad applications in various fields, such as control systems with communication constraints, sampled-data systems, mechanical systems (Haddad et al. (2006); Chen and Zheng (2009b); Hespanha et al. (2008); Naghshtabrizi et al. (2008) and the monographs Bainov and Simeonov (1989) and Lakshmikantham et al. (1989)). In general, impulsive systems may be classified in (i) systems with impulses at fixed instants, (ii) systems with impulses at variable instants, and (iii) autonomous systems with impulse effects. Reset control systems are included in (iii) since they have a reset surface which does not depend on the time. Significant progress on the stability of impulsive dynamical systems has been made during the past 20 years, see Chen and Zheng (2009c,a, 2011); Guan (1999); Khadra et al. (2009); Liu et al. (2007); Wang and Liu (2005); Yang and Xu (2007); Hetel et al. (2013) and references therein. However, most of the research effort has been dedicated to cases (i) and (ii), and then many results are not directly applicable in the case of reset systems.

Recently different results on the stability of time-delay reset control systems have been developed for the zero crossing reset condition. A delay-independent condition was presented in Baños and Barreiro (2009), and extended to delay-dependent condition in Barreiro and Baños (2010); Prieto et al. (2012). Quadratic stability of time-delay reset control systems with reset surface uncertainty was considered in Guo and Xie (2012). More recently the previous results have been extended to input-to-output stability in Mercader et al. (2013b), Mercader et al. (2013a). All these results are based on a generalization of the so-called $H_\beta$ condition (see Beker et al. (2004)) for non delay reset control systems. The main idea is the existence of a Lyapunov-Krasovskii (LK) functional which must always decrease during the flow and must decrease or remain equal during the jumps. In general, this basic result may be conservative. In Davó and Baños (2013a) a less conservative result is obtained by allowing some bounded increments of the functional after the reset instants. Nevertheless, these sufficient conditions are still conservative. The conditions obtained are not able to guarantee asymptotic stability if the base system is not stable. This limitation of the approach comes from the fact that in general the reset action with state-dependent resetting law cannot be guaranteed, which means that the reset control system may evolve without reset action for some initial conditions, implying the stability of the base system.

In this work, we focus on the stability of a reset control system submitted to a single time-delay, and with a time-dependent resetting law. In this case, the time between two consecutive reset instants is considered to be in a given interval. Therefore, the existence of an infinity number of reset actions for any initial condition is guaranteed. In contrast to the previous work, here the stability of the base system is not needed, and then less conservative results are expected. In spite of the fact that most of the results of impulsive systems may be applied, they are limited to a

[1] This work was supported by FEDER (European Union), 'Ministerio de Ciencia e Innovación' of Spain under project DPI2010-20466-C02-02 and the ANR Project LIMICOS 12-BS03-005-01.


Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.11.202

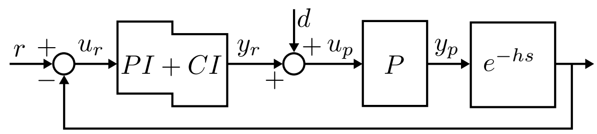


Fig. 1. Reset control system setup.

strict decrement of the LK functional during the impulse actions. Therefore, they cannot be applied to the setup of this paper, since the reset actions cannot affect the states of the process in a reset control system.

The main idea of this paper is the transformation of the reset control system into a sampled-data system, so that the latest stability results in the framework of sampled-data systems can be applied. This transformation can be directly made for a particular reset compensator which is called PI+CI (Baños and Vidal (2012)). This compensator is a simple modification of a PI compensator, which includes a Clegg integrator (CI) in parallel. PI+CI has been shown to be effective in several control experiments of processes with time-delay (Vidal and Baños (2012); Davó and Baños (2013b)). In addition, the stability of the PI+CI has been analyzed in Vidal (2009) for non delay processes. The reset control system composed by the PI+CI compensator can be modeled as a sampled-data system where the controlled process consists of a closed-loop system with internal time-delay. In this way, the results of Seuret (2011, 2012) are extended with a new LK functional (Seuret and Gouaisbaut (2014)), and delay-dependent criterion is developed for the asymptotic stability.

*Notation*: Throughout the article, the sets $\mathbb{R}$, $\mathbb{R}^+$, $\mathbb{R}^n$, $\mathbb{R}^{n\times n}$ and $\mathcal{S}_n$ denote the sets of real numbers, nonnegative real numbers, the n-dimensional Euclidean space, $n \times n$ matrices and symmetric matrices, respectively. A column vector is denoted by $\mathbf{x} \in \mathbb{R}^n$. Given two vectors $\mathbf{x}_1$ and $\mathbf{x}_2$, we write $(\mathbf{x}_1, \mathbf{x}_2)$ to denote $[\mathbf{x}_1^\top, \mathbf{x}_2^\top]^\top$. The notation $|\mathbf{x}|$ is the euclidean norm for $\mathbf{x} \in \mathbb{R}^n$. $\mathcal{C}([a,b],\mathbb{R}^n)$ stands for the set of continuous functions mapping $[a,b]$ to $\mathbb{R}^n$, with the norm $\|\phi\| = max_{\theta\in[a,b]}|\phi(\theta)|$. The identity matrix and the zero matrix of adequate dimensions are denoted by $I$ and 0, respectively. The notation $P \succ 0$ for $P \in \mathcal{S}_n$ means that $P$ is positive definite. The set of positive definite matrices is denoted by $\mathcal{S}_n^+$. For a matrix $A \in \mathbb{R}^{n\times n}$, the notation $He(A)$ refers to $A + A^\top$.

## 2. PRELIMINARIES AND PROBLEM MOTIVATION

Consider a time-delay reset control system as shown in Fig. 1, given by the feedback interconnection of a linear and time invariant (LTI) system P and a PI+CI (both single-input-single-output). P is given by the state-space system

$$P : \begin{cases} \dot{\mathbf{x}}_p(t) = A_p\mathbf{x}_p(t) + B_p u_p(t), \\ y_p(t) = C_p\mathbf{x}_p(t), \end{cases} \tag{1}$$

where $\mathbf{x}_p \in \mathbb{R}^{n_p}$ and $A_p \in \mathbb{R}^{n_p\times n_p}$, $B_p \in \mathbb{R}^{n_p\times 1}$, $C_p \in \mathbb{R}^{1\times n_p}$. The PI+CI compensator is simply a parallel connection of a PI compensator and a Clegg integrator. In the state-space, the PI+CI can be expressed by using a state $\mathbf{x}_r = (x_i, x_{ci}) \in \mathbb{R}^2$, where $x_i$ is the integral term state, and $x_{ci}$ corresponds to the Clegg integrator state. Its state-space realization is given by

$$PI+CI : \begin{cases} \dot{\mathbf{x}}_r(t) = B_r u_r(t), & t \notin \mathcal{T}, \\ \mathbf{x}_r(t^+) = A_\rho\mathbf{x}_r(t), & t \in \mathcal{T}, \\ y_r(t) = C_r\mathbf{x}_r(t) + k_p u_r(t), \end{cases} \tag{2}$$

where $t^+ = t + \epsilon$ with $\epsilon \to 0^+$, and the matrices $B_r$, $C_r$, and $A_\rho$ are given by

$$B_r = \begin{bmatrix} k_i \\ 0 \end{bmatrix}, \quad C_r = [1 \ \ 1], \quad A_\rho = \begin{bmatrix} 1 & 0 \\ -p_r & 0 \end{bmatrix}. \tag{3}$$

In this work, we consider for simplicity that the connection between P and PI+CI is only affected by a time-delay $h$ at the output of system P. The proposed approach can also be applied to obtain similar results when the time-delay is at the input of P. The autonomous closed-loop system (with zero exogenous signals, that is $r = d = 0$) is obtained by making the connections $u_p(t) = y_r(t)$ and $u_r(t) = -y_p(t-h)$:

$$\begin{cases} \dot{\mathbf{x}}(t) = A\mathbf{x}(t) + A_d\mathbf{x}(t-h), & t \notin \mathcal{T}, \\ \mathbf{x}(t^+) = A_R\mathbf{x}(t), & t \in \mathcal{T}, \\ y(t) = C\mathbf{x}(t-h), \\ \mathbf{x}(t) = \phi(t), & t \in [-h, 0], \end{cases} \tag{4}$$

where $\mathbf{x}(t) = (\mathbf{x}_p(t), \mathbf{x}_r(t)) \in \mathbb{R}^n$ with $n = n_p + 2$, $\phi \in \mathcal{C}([-h,0],\mathbb{R}^n)$ is the initial condition function, and matrices $A$, $A_d$, $C$, and $A_R$ are given by

$$A = \begin{bmatrix} A_p & B_pC_r \\ 0 & 0 \end{bmatrix}, \quad A_d = \begin{bmatrix} -k_pB_pC_p & 0 \\ -B_rC_p & 0 \end{bmatrix}, \\ C = [\, C_p \ \ 0 \,], \qquad A_R = \begin{bmatrix} I & 0 \\ 0 & A_\rho \end{bmatrix}. \tag{5}$$

In some previous works (Vidal (2009); Baños and Vidal (2012)), a PI+CI is proposed with a state-dependent resetting law, that is the reset is applied at time $t$ in which $(\mathbf{x}(t), \mathbf{x}(t-h)) \in \mathcal{M}$, for a given reset set $\mathcal{M}$. In this work, the reset is applied at time $t$, which belongs to an infinite and strictly increasing sequence of reset times defined by $\mathcal{T}(\phi) = (t_1, t_2, \cdots)$, which may depend on the initial condition. In addition, we assume that there exist two positive scalars $0 < \mathcal{T}_1 \leq \mathcal{T}_2$ such that $T_k = t_{k+1} - t_k \in [\mathcal{T}_1, \mathcal{T}_2]$ for any $k > 0$.

From the definition of the reset instants, it is clear that there exists a unique solution $\mathbf{x}(t,\phi)$, or simply $\mathbf{x}(t)$, for $t \in [-h, \infty)$ (see Section II of Baños and Barreiro (2012) for a more detailed discussion about existence and uniqueness of solutions).

*Remark 1.* Note that there are not stability results in the literature that can deal with time-delay reset control systems composed by a PI+CI (there are several results for reset control systems without time-delay, see e.g., Loquen et al. (2010); Baños et al. (2011)). The main reason is that the base system of (4)-(5) is not asymptotically stable.

## 3. STABILITY ANALYSIS

In this section, the stability of the time-delay reset control system (4) is analyzed by using the framework of sampled-data systems (see, e.g., Fridman (2014) and the references therein). The proposed approach consists in transforming the reset control system (4) into a linear system interconnected with a sample and hold device as shown in