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Parameter Sensitivity and Boundedness of Robotic Hybrid Periodic Orbits ⋆

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Abstract: Model-based nonlinear controllers like feedback linearization and control Lyapunov functions are highly sensitive to the model parameters of the robot. This paper addresses the problem of realizing these controllers in a particular class of hybrid models–systems with impulse effects–through a *parameter sensitivity measure*. This measure quantifies the sensitivity of a given model-based controller to parameter uncertainty along a particular trajectory. By using this measure, output boundedness of the controller (computed torque+PD) will be analyzed. Given outputs that characterize the control objectives, i.e., the goal is to drive these outputs to zero, we consider Lyapunov functions obtained from these outputs. The main result of this paper establishes the ultimate boundedness of the output dynamics in terms of this measure via these Lyapunov functions under the assumption of stable hybrid zero dynamics. This is demonstrated in simulation on a 5-DOF underactuated bipedal robot.

Keywords: Hybrid Zero Dynamics, Parameter Sensitivity Measure, System Identification.

1. INTRODUCTION

Model based controllers like stochastic controllers Byl and Tedrake (2009), feedback linearization Westervelt et al. (2007), the control Lyapunov functions (CLFs) Ames et al. (2014) all require the knowledge of an accurate dynamical model of the system. The advantage of these methods are that they yield sufficient convergence for highly dynamic robotic applications, e.g., quadrotors and bipedal robots, where exponential convergence of control objectives is used to achieve guaranteed stability of the system. This is especially true of bipedal walking robots where *rapid* exponential convergence is used Ames et al. (2014). While these controllers have yielded good results when an accurate dynamical model is known, there is a need for quantifying how accurate the model has to be to realize the desired tracking error bounds. These application domains point to the need for a way to measure parameter uncertainty and a methodology to design controllers for nonlinear hybrid systems, like bipedal robots, that can converge to the control objective under parameter uncertainty.

The goal of this paper is to establish a relationship between parameter uncertainty and the output error bounds on systems with alternating continuous and discrete events, i.e., hybrid systems, while considering a specific example: bipedal walking robots. Inspired by the sensitivity functions utilized for linear systems Zhou et al. (1996), a *parameter sensitivity measure* is defined for continuous systems and the relationship between the boundedness and the measure is established through the use of Lyapunov functions. In the context of hybrid systems, along with defining the measure for the continuous event, an *impact measure* is defined to include the effect of param-

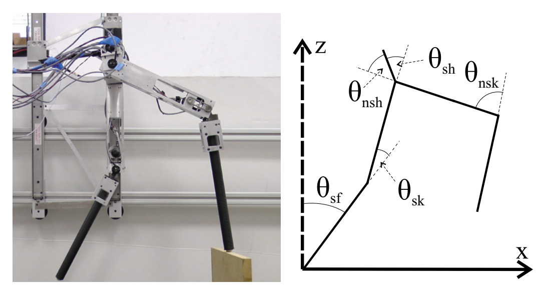

Fig. 1. The biped AMBER (left) and the stick figure of AMBER showing the configuration angles (right).

eter variations in the discrete event. The resulting overall sensitivity measure thus represents how sensitive a given controller is to parameter variations for hybrid systems. When described in terms of Lyapunov functions, which are constructed from the zeroing outputs of the robot, the *parameter sensitivity measure* naturally yields the ultimate bound on the outputs. Considering a 5-DOF bipedal robot, AMBER, shown in Fig. 1, where a stable periodic orbit on the hybrid zero dynamics translates to a stable walking gait on the bipedal robot, the ultimate bound on this periodic orbit will be determined through the use of a particular controller: computed torque+PD.

The paper is structured in the following fashion: Section 2 introduces the robot model and the control methodology used–CLFs through the method of computed torque. Section 3 assesses the controller used for the uncertain model of the robot and establishes the resulting uncertain behavior through Lyapunov functions. In Section 4, the resulting uncertain dynamics exhibited by the robot is measured formally through the construction of *parameter sensitivity measure*, which is the main formulation of this paper on which the formal results will build. It will be shown that there is a direct relationship between the ultimate bound

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on the Lyapunov function and the *parameter sensitivity measure*, which motivates the introduction of an auxiliary controller–computed torque+PD. This will be utilized for establishing bounds for the entire dynamics, under the assumption of a stable limit cycle in the zero dynamics. This method is extended to hybrid systems through the introduction of an impact measure in Section 5. Under the assumption that the hybrid zero dynamics is stable, the computed torque controller appended with the auxiliary input is applied on the model, which results in bounded dynamics of the underactuated hybrid system. The paper concludes with simulation results on a 5-DOF bipedal robot, AMBER, in Section 6.

2. ROBOT DYNAMICS AND CONTROL

A robotic model can be modeled as n-link manipulator. Given the configuration space $\mathbb{Q} \subset \mathbb{R}^n$, with the coordinates $q \in \mathbb{Q}$, and the velocities $\dot{q} \in T_q\mathbb{Q}$, the equation of motion of the n-DOF robot can be defined as:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = B\mathrm{T}, \tag{1}$$

where $D(q) \in \mathbb{R}^{n\times n}$ is the mass inertia matrix of the robot that includes the motor inertia terms, $C(q,\dot{q}) \in \mathbb{R}^{n\times n}$ is the matrix of coriolis and centrifugal forces, $G(q) \in \mathbb{R}^n$ is the gravity vector, $\mathrm{T} \in \mathbb{R}^k$ is the torque input and $B \in \mathbb{R}^{n\times k}$ is the mapping from torque to joints.

AMBER. Considering the 5-DOF underactuated bipedal robot shown in Fig. 1, the configuration can be defined as: $q = (q_{sa}, q_{sk}, q_{sh}, q_{nsh}, q_{nsk})$ corresponding to stance ankle (sa), stance and non-stance knee (sk,nsk), stance and non-stance hip angles (sh,nsh) of the robot. Since the ankle is not actuated, the number of actuators is $k = 4$.

Outputs. We will utilize the method of computed torque since it is widely used in robotic systems. It is also convenient in the context of uncertain models which will be considered in the next section. To realize the controller, *outputs* are picked which are functions of joint angles referred to as actual outputs $y_a : \mathbb{Q} \to \mathbb{R}^k$, which are made to track functions termed the desired outputs $y_d : \mathbb{Q} \to \mathbb{R}^k$. The objective is to drive the error $y(q) = y_a(q) - y_d(q) \to 0$. These outputs are also termed *virtual constraints* in Westervelt et al. (2007). The outputs are picked such that they are relative degree two outputs (see Sastry (1999)). Given the output y:

$$\ddot{y} = \underbrace{\frac{\partial y}{\partial q}}_{J}\ddot{q} + \underbrace{\dot{q}^T\frac{\partial^2 y}{\partial q^2}}_{\dot{J}}\dot{q}. \tag{2}$$

Since, $k < n$, we include $n-k$ rows to J and $\dot{J}$ to make the co-efficient matrix of $\ddot{q}$ full rank. These rows correspond to the configuration which are underactuated resulting in:

$$\begin{bmatrix} 0 \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} D_1 \\ J \end{bmatrix}\ddot{q} + \begin{bmatrix} H_1 \\ \dot{J}\dot{q} \end{bmatrix}, \tag{3}$$

where H_1 is the $n-k$ rows of $H(q,\dot{q}) = C(q,\dot{q})\dot{q} + G(q)$, and D_1 is the $n-k$ rows of the expression, $D(q)$. It should be observed that since the underactuated degrees of freedom have zero torque being applied, the resulting EOM of the robot leads to 0 on the left hand side of (3), and hence the choice of rows. Accordingly, we can define the desired acceleration for the robot to be:

$$\ddot{q}_d = \begin{bmatrix} D_1 \\ J \end{bmatrix}^{-1}\left(\begin{bmatrix} 0 \\ \mu \end{bmatrix} - \begin{bmatrix} H_1 \\ \dot{J}\dot{q} \end{bmatrix}\right), \tag{4}$$

where μ is a linear control input. The resulting torque controller that realizes this desired acceleration in the robot can be defined as:

$$B\mathrm{T} = D(q)\ddot{q}_d + C(q,\dot{q})\dot{q} + G(q). \tag{5}$$

Substituting (5) and (4) in (1) results in linear dynamics: $\ddot{y} = \mu$, with μ chosen through a CLF based controller.

Zero Dynamics and CLF. If we define the vector: $\eta = [y^T, \dot{y}^T]^T$, the dynamics can be reformulated as:

$$\dot{\eta} = \underbrace{\begin{bmatrix} 0_{k\times k} & 1_{k\times k} \\ 0_{k\times k} & 0_{k\times k} \end{bmatrix}}_{F}\eta + \underbrace{\begin{bmatrix} 0_{k\times k} \\ 1_{k\times k} \end{bmatrix}}_{G}\mu, \tag{6}$$

which represent the controllable dynamics of the system. Since, $k < n$ there are states that are not directly controllable which represent the zero dynamics of the system and can be expressed as:

$$\dot{z} = \Psi(\eta, z), \tag{7}$$

where $z \in Z \subseteq \mathbb{R}^{2(n-k)}$ is the zero dynamic coordinates of the system (see Westervelt et al. (2007)).

Consider the Lyapunov Function: $V(\eta) = \eta^T P \eta$, where P is the solution to the continuous-time algebraic Riccati equation (CARE). Taking the derivative yields:

$$\dot{V}(\eta) = \eta^T(F^T P + PF)\eta + 2\eta^T PG\mu. \tag{8}$$

To find a specific value of μ, we can utilize a minimum norm controller (see Freeman and Kokotovic (2008)) which minimizes $\mu^T\mu$ subject to the inequality constraint:

$$\dot{V} = \eta^T(F^T P + PF)\eta + 2\eta^T PG\mu \le -\gamma V, \tag{9}$$

where $\gamma > 0$ is a constant obtained from CARE. Satisfying (9) implies exponential convergence.

We can impose stronger bounds on convergence by constructing a *rapidly exponentially stable control Lyapunov function (RES-CLF)* that can be used to stabilize the output dynamics in a rapidly exponentially fashion (see Ames et al. (2014) for more details). Choosing $\varepsilon > 0$:

$$V_\varepsilon(\eta) := \eta^T \begin{bmatrix} \frac{1}{\varepsilon}I & 0 \\ 0 & I \end{bmatrix} P \begin{bmatrix} \frac{1}{\varepsilon}I & 0 \\ 0 & I \end{bmatrix}\eta =: \eta^T P_\varepsilon \eta.$$

It can be verified that this is a RES-CLF in Ames et al. (2014). Besides, the bounds on RES-CLF can be given as:

$$\alpha_1||\eta||^2 \le V_\varepsilon(\eta) \le \frac{\alpha_2}{\varepsilon^2}||\eta||^2, \tag{10}$$

where $\alpha_1, \alpha_2 > 0$ are the minimum and maximum eigenvalues of P, respectively. Differentiating (10) yields:

$$\dot{V}_\varepsilon(\eta) = L_F V_\varepsilon(\eta) + L_G V_\varepsilon(\eta)\mu, \tag{11}$$

where $L_F V_\varepsilon(\eta) = \eta^T(F^T P_\varepsilon + P_\varepsilon F)\eta$, $L_G V_\varepsilon(\eta) = 2\eta^T P_\varepsilon G$.

We can define a minimum norm controller which minimizes $\mu^T\mu$ subject to the inequality constraint:

$$L_F V_\varepsilon(\eta) + L_G V_\varepsilon(\eta)\mu \le -\frac{\gamma}{\varepsilon}V_\varepsilon(\eta), \tag{12}$$

which when satisfied implies exponential convergence. Therefore, we can define a class of controllers K_ε:

$$K_\varepsilon(\eta) = \{u \in \mathbb{R}^k : L_F V_\varepsilon(\eta) + L_G V_\varepsilon(\eta)u + \frac{\gamma}{\varepsilon}V_\varepsilon(\eta) \le 0\},$$

which yields the set of control values that satisfies the desired convergence rate.