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Fitting the frequency-dependent parameters in the Bergeron line model

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ABSTRACT

A new transmission line modeling – TLM is proposed based on the well-established Bergeron method. The conventional Bergeron model is characterized by the line representation through concentrated longitudinal and transversal parameters, i.e., the line electrical parameters are represented by means of electric circuit elements *R*, *L*, *G* and *C*. The novel approach of this research is the inclusion of the frequency effect in the longitudinal parameters in the Bergeron line representation. This new feature enables to extend the application of the Bergeron method for simulations of transients composed of a wide range of frequencies.

1. Introduction

In general terms, the mathematical modeling of dynamic systems is a simplified and practical method to define initial and boundary conditions for the first step on real projects which are extended since electronic devices, for the most variable applications, up to complex power systems. Thus, the continuous improvement and creation of new computational tools to simulate this "first step" are definitely not outdated.

Initially, a brief review on transmission line modeling – TLM is described emphasizing the current state of the art and the main problems in the area.

There are several transmission line models available in the technical literature to study electromagnetic transients in power transmission systems. Basically, these models may be classified into two general groups: by lumped parameters and by distributed parameters.

In the first group, transmission lines are modeled from the representation by lumped elements, i.e., line is modeled by an equivalent representation by means of resistive, inductive and capacitive circuit elements. These models are developed directly in the time domain and are easily integrated to other time-variable power elements, also modeled in the time domain, such as: capacitors, relays, non-linear loads and many other power components. Since the electrical behavior of most power components are well-known in the time domain, this characteristic represents one of the main advantages in TLM by lumped elements [1].

The line modeling by distributed parameters is developed directly from the frequency-dependent parameters based on the line representation by a two-port circuit in the frequency domain. From this approach, the line modeling and simulations are carried out in the frequency domain and time-domain results are obtained using inverse transforms [2]. The frequency-dependent parameters of the line are accurately represented using frequency-domain models; however, these models have restrictions for inclusion of time-variable elements in the simulation process, since most power components are well known and easily modeled in the time domain [3].

Despite line models by lumped elements are developed in the time domain, the frequency effect on the longitudinal parameters can be included in the line modeling using fitting methods [4]. New frequency-dependent models by lumped parameters have been recently published in the technical literature on power system modeling. These models are developed directly in the time domain from the line representation by cascade of π circuit and the frequency effect on the electrical parameters is modeled by fitting the rational functions $R_{fit}(\omega)$ and $L_{fit}(\omega)$ (resistance and inductance) based on the longitudinal impedance of the line, $Z(\omega)$, properly calculated taken into account the earth-return impedance (soil effect) and the skin effect on the cables. The frequency-dependent line model described in reference [5] shows to be robust and accurate for the most of transient conditions on a conventional power transmission system. However, depending of the transmission system

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characteristics (source, line and load) and transient conditions, the frequency-dependent model based on cascade of lumped elements shows to be costly in computational terms, depending of the quantity of line equivalent segments used in the cascade and total simulation time. Furthermore, some hard unbalanced conditions lead to discrete inaccuracies, because the multi-phase modeling is based on the intrinsic use of a constant and real transformation matrix to calculate each propagation mode of the line. Thus, the line representation by frequency-dependent cascade of π circuits shows to be efficient for several situations; however, some difficulty in the modeling and inaccuracies are observed for specific cases. From this last statement, the current research proposes a new time-domain line model based on the well-known Bergeron method and using the same fitting procedure applied in the line model referred in [5].

The Bergeron method, also known as method of the characteristics, was firstly proposed to solve hydraulic systems and after applied to electrical problems, more specifically, electromagnetic wave propagation along a lossless line [6]. In this case, the line modeling is carried out considering only the longitudinal inductance L and the shunt capacitance C, which means that the line resistance R and transversal conductance G are neglected. Thereafter, Dommel proposed a nodal solution combining the method of the characteristics for transmission lines and the integration method of the trapezoidal rule for lumped parameters. From this development, the electromagnetic transient program – EMTP was created and a new line model for lossless transmission lines was proposed [7].

In reference [8], an extension of the Bergeron's method of characteristics was developed including the line losses. In addition, the inclusion of the frequency effect on the longitudinal parameters was evaluated from the application of inverse transforms, which limits significantly the application of the Bergeron line model for several practical operations in transmission systems, most of them including non-linear and time-variable elements in a single line section. The main contribution of Snelson, in reference [8], is the inclusion of the line losses in the Bergeron model using lumped resistances at both line terminals, concentrating the line losses at the sending and the receiving ends of the line section.

Based on the contributions firstly proposed by Dommel and after by Snelson, this paper proposes a new line model taken into account the inclusion of the frequency effect on the longitudinal parameters in the Bergeron line model, maintaining its characteristic robustness and simplified modeling. The sum of these contributions results in an accurate line model capable to simulate electromagnetic transients composed of a wide range of frequencies, emphasizing that most of line models prior developed by lumped parameters as well as the classical Bergeron model have restrictions for simulations including a wide range of frequencies. Thus, the inclusion of the frequency effect in the Bergeron line model is the major contribution of this research.

This paper is structured into three parts. The first part is an introduction of the classical Bergeron model for lossless lines and for lossy lines using constant parameters. The second part describes the inclusion of the frequency effect in the Bergeron line model using vector fitting. The third part validates the proposed time-domain model comparing with two well-established line models: a frequency-domain model using numerical Laplace transform [2] and a frequency-dependent cascade of π circuits [4,5].

2. The Bergeron line model

The Bergeron's method was firstly applied to lossless transmission lines. This means that only the line inductance per unit of length (p.u.l.) L' and the p.u.l. capacitance C' were included in the model, whereas the longitudinal p.u.l. resistance R' and the p.u.l. conductance G' are neglected. In fact, the method of characteristics could be applied for lossy transmission lines, but the resulting ordinary differential equations could not be directly integrated. Thus, considering a single-phase line with length l, the current and voltage at a point x along the line are related by:

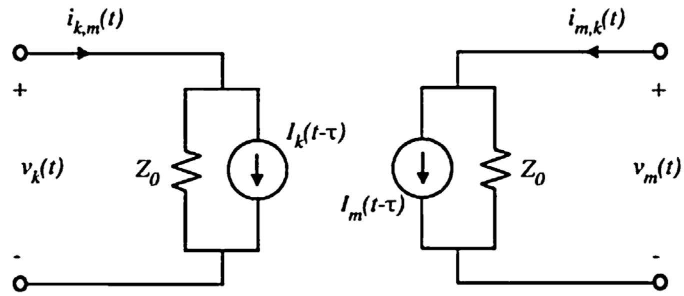

Fig. 1. Equivalent impedance circuit of the Bergeron line model.

$$-\frac{\partial e}{\partial x} = \frac{L'\partial i}{\partial t} \tag{1}$$

$$-\frac{\partial i}{\partial x} = \frac{C'\partial e}{\partial t} \tag{2}$$

Where the first hand of (2) and (3) represents the voltage and the current as a function of space x through the line, i.e., voltage and current wave propagation along the line as a function of the time t.

The general solution of (1) and (2) is expressed [7]:

$$i(x,t) = f_1(x - vt) + f_2(x + vt) \tag{3}$$

$$e(x,t) = Z_0 f_1(x - vt) + Z_0 f_2(x + vt) \tag{4}$$

Where f_1 and f_2 are arbitrary functions of $(x \pm vt)$. Function f_1 represents the forward wave propagation along the line with velocity v (also known as propagation or phase velocity) whereas f_2 represents the wave propagation in a back forward direction. Term Z_0 is the surge impedance, also known in the technical literature as characteristic impedance of the line. Terms Z_0 and v are expressed as follows [7]:

$$Z_0 = \sqrt{\frac{L'}{C'}}; \quad v = \frac{1}{\sqrt{L'C'}} \tag{5}$$

Multiplying (3) by the characteristic impedance Z_0 and adding in (4), the following formulation is obtained [6]:

$$e(x,t) + Z_0 i(x,t) = 2Z_0 f_1(x - vt) \tag{6}$$

$$e(x,t) - Z_0 i(x,t) = -2Z_0 f_2(x + vt) \tag{7}$$

Analyzing (6), $(e + Z_0 i)$ is constant for $(x - vt)$. The same instance is valid for the voltage $(e - Z_0 i)$ in relationship to $(x + vt)$. These constants are intrinsic related to the propagation characteristics and differential equations of a lossless transmission line.

Since $(x \pm vt)$ is constant, the traveling time of an electromagnetic wave from the sending end to the receiving end of the line is also constant and is expressed as follows:

$$\tau = \frac{1}{v} l\sqrt{L'C'} \tag{8}$$

The equivalent circuit for a lossless line is described in Fig. 1.

The forward wave is constant from the node m to the node k at instant $t - \tau$. The same is observed for a back forward wave from the node k to m. From this analysis, the following time-domain expression is given:

$$e_m(t - \tau) + Z_0 i_{m,k}(t - \tau) = e_k(t) + Z_0(-i_{k,m}(t)) \tag{9}$$

From (9), it is possible to verify that the forward wave, which takes τ seconds to reaches the sending end of the line (node k),