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Rejection of fixed direction disturbances in multivariable electromechanical motion systems

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Abstract: This work discusses rejection of disturbances with known directions in multivariable motion systems. It is shown how frequency domain tradeoffs in multivariable control motivate a design based on disturbance directions. A multivariable design method is presented to design centralized controllers that reject disturbances only in relevant directions. A model of an industrial motion system is used to demonstrate the theory. It is shown how the proposed design method resembles the solution of a competing $\mathcal{H}_\infty$design and offers the ability to interpret $\mathcal{H}_\infty$centralized control solutions, and understand the tradeoffs inherent in such a design.

1. INTRODUCTION

Multivariable control design problems pose complexity issues as gain, phase and directions in different closed loop transfer functions are strongly interrelated. Therefore, interpretation and development of multivariable control strategies is a challenging problem. In many cases, the structure of the plant can be exploited to reduce this complexity. In Freudenberg [1996], Freudenberg [1999] it is discussed how MIMO control problems can reduce to SIMO or MISO control problems in certain frequency regions or as an effect of certain bandwidth limitations. In Hovd [1997] a class of plants is studied where the input and output directions do not vary with frequency, hence decoupling methods can be applied to reduce the MIMO control problem to a set of SISO control problems. However, in some cases, disturbance have specific directional structure that can give rise to full complexity MIMO control issues, even when plant dynamics are decoupled. This was recognized by some authors, Freudenberg [1988], Maciejowski [1989][page. 85] but there has been little discussion on how this may be exploited in multivariable control design.

In many practical applications, the directions of disturbances are constant (fixed) for all frequencies. Typical examples are disturbances that originate from causes that have a fixed location, e.g., floor vibrations, pumps, fans. Also, the control system architecture can give rise to fixed direction disturbances, see, e.g., Skogestad [1997]. In Boerlage [2007b] a method is discussed to identify the direction and cause of such disturbances. A design that exploits this information for a TITO control system was shown in Boerlage [2007a]. It is of great interest to investigate if the structure of fixed direction disturbances allows the development of insightful MIMO design strategies.

In this work, it is shown that exploiting directions of disturbances is motivated from design limitations induced by integral relations such as the Bode sensitivity integral. Using the structure of a class of electromechanical motion systems, a method is proposed to design controllers that reject fixed direction disturbances only in relevant directions. This method allows frequency wise tradeoffs to be made in each disturbance direction independently, leading to transparent design of a centralized MIMO controller. The method is demonstrated on a design problem of an industrial high performance positioning system. A competing controller is designed using $\mathcal{H}_\infty$synthesis. It is shown that the proposed design method can be used to reverse engineer, hence interpret, the MIMO $\mathcal{H}_\infty$controller.

The paper is organized as follows. The next section shows how design limitations motivate the study of disturbance directions in multivariable control design. Section 3 discusses structural properties of electromechanical motion systems that can be exploited in multivariable control design. The proposed design method to reject disturbances in fixed directions is presented in Section 4. The theory is illustrated by application to an industrial control design problem in Section 5. The last section closes with conclusions.

2. DESIGN LIMITATIONS

A feedback control architecture is considered as depicted in Fig. 1. Herein, d_i, d_o, n, denote the input disturbance,

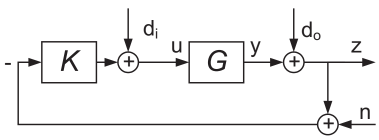

Fig. 1. Control system architecture.

output disturbance and sensor noise respectively. Furthermore, we define the performance variable z, plant input u and the plant output y. It follows that,

$$z = GS_o d_i + S_o d_o - T_o n \tag{1}$$

where $S_o = (I + GK)^{-1}$ is the output sensitivity function and $T_o = GKS_o$ the output complementary sensitivity function. Both are algebraically related as $S_o + T_o = I$. The objective is to keep z as small as possible in presence of d_i, d_o and n. Say that only a disturbance d_o with direction u_d is present at the output of the plant,

$$d_o(s) = u_d v_d(s). \tag{2}$$

And at a single frequency p, the singular value decomposition of $S_o(p)$ is $S_o(p) = V_{So}\Sigma_{So}U_{So}^H$. Then, $z(p)$ is small if the last column of U_{So} is aligned with u_d, so that $v_d(p)$ is amplified with the smallest singular value of S_o. Apart from alignment, high gain feedback helps to reject disturbances in multivariable systems. However, as analytic design limitations hold, use of high gain feedback for all frequencies and in all directions is restricted in certain situations, Freudenberg [1988].

Consider a stable, non-minimum phase, $n \times n$ multivariable output sensitivity function, $S_o(s)$, resulting from a feedback system with stable open loop with relative degree more than two. Then, by pre and post multiplication with vectors $v, u \in \mathbb{C}^n, v^H u \neq 0$ the following scalar transfer function is obtained,

$$S_{vu}(s) = v^H S_o(s) u. \tag{3}$$

The response to a disturbance entering the system in a direction spanned by u is given by $S_o(s)u$. Then, $v^H S_o(s) u$ is the component of that disturbance that appears in the output direction spanned by v.

Theorem 1. From Freudenberg [1985]. For S_{vy} and u, v defined above, the following integral relation applies,

$$\int_0^\infty \log |S_{vu}(j\omega)|\, d\omega = \frac{\pi}{2} \log |v^H u| + \pi \sum_{i=1}^{N_z} z_i \tag{4}$$

with $z_i, i = \{1, \ldots, N_z\}$ the CRHP zeros of S_{vu}.

Proof 1. As S_o is stable, the scalar transfer function S_{vu} is stable, and has at most the relative degree of $S_o(s)$. So that at high frequencies, it holds that,

$$\lim_{s\to\infty} v^H S_o(j\omega) u = |v^H u|. \tag{5}$$

Even if S_o is minimum phase, S_{vu} can become non-minimum phase, hence introduce zeros in the closed right half plane. Hence, scalar integral relations apply, Freudenberg [1985].

This implies that an area where $|S_{vu}(j\omega)|$ is small, must be balanced by an equal area where $|S_{vu}(j\omega)|$ is large.

In practical applications, the bandwidth of the feedback system is limited, e.g., by plant uncertainty at high frequencies. Hence, the frequency range with large $|S_{vu}(j\omega)|$ is constrained. Then, Theorem 1, shows that rejection of disturbances at other frequencies, where $|S_{vu}(j\omega)|$ must be small, has to be compromised. This may imply performance limiting tradeoffs. Specific choices of v, u illustrate how these frequency wise tradeoffs dictate multivariable control design.

Corollary 2. When v, u are chosen to be elementary vectors e_i, $S_{vu}(s) = S_{o,ii}(s)$ equals the i^{th}diagonal element of the sensitivity transfer function matrix. As $e_i^T e_i = 1$, Theorem 1 reduces to the bode integral relation for scalar systems. Hence, for each scalar loop in a multivariable system, the scalar bode integral relation must be satisfied, even if a centralized linear time invariant controller is applied.

Corollary 3. To study non-diagonal terms of the sensitivity transfer function matrix, v, u can be chosen to approach orthogonal elementary vectors, $e_i^H e_j = \epsilon$, $0 < \epsilon \ll 1$, $S_{vu}(s) = e_i^T S_o(s) e_j$. Then,

$$\lim_{\epsilon\to 0} \log(|e_i^H e_j|) = -\infty. \tag{6}$$

This shows that a single non-diagonal term of a sensitivity function matrix can be made arbitrarily small, even in cases where the open loop is non-diagonal. However, as will be shown later, it may be desired to increase non-diagonal terms of a sensitivity function for the sake of disturbance rejection.

Corollary 4. By choosing $u_i = v_j \in \mathbb{R}^n$, $v_i^T v_j = 0$ and $M = Span\{v_1, v_2, \ldots, v_n\}$ an orthogonal transformation $S_{o,M}(s) = M^T S_o(s) M$ can be made. As this is a non-singular input output transformation, $S_{o,M}$ is minimum phase if S_o is minimum phase. Then on each new base Theorem 1 holds with $N_z = 0$.

If the direction of a disturbance is constant for all frequencies, and the performance is defined as a constant linear combination of z, see (1), rejection of that disturbance at one frequency implies that the sensitivity function has to be increased at other frequencies *in that same direction.* Hence, as specifications become tighter, it is crucial for design to reject disturbances only at frequencies and in directions that are relevant. This may imply that centralized controllers (with non-diagonal terms) are to be designed, even in cases where the plant is decoupled.

3. ELECTROMECHANICAL MOTION SYSTEMS

We first show that the multivariable plants we consider have a specific structure. Here, we focus on the control of linear time invariant electromechanical motion systems that have the same number of actuators and sensors as rigid body modes. Typical applications are high performance positioning stages used in semiconductor manufacturing, electron microscopy or component placement machines. The dynamics of such systems are often dominated by the mechanics, which are therefore constructed to be light and stiff, so that resonance modes due to flexible dynamics appear only at high frequencies. Typically, flexible dynamics have low internal damping, so that it is justified to assume proportional damping. Then, the following model can be used to describe the dynamics of the plant, Gawronski [2004],

$$G_m(s) = \sum_{i=1}^{N_{rb}} \frac{c_i b_i^T}{s^2} + \sum_{i=N_{rb}+1}^{N} \frac{c_i b_i^T}{s^2 + 2\zeta_i\omega_i s + \omega_i^2} \tag{7}$$

Herein, N_{rb} denotes the number of rigid body modes. The parameters ζ_i, ω_i are the relative damping and resonance frequency of the flexible modes. The vectors c_i, b_i span the directions of the i^{th} mode shapes and are constant for all frequencies. The resonance frequencies ω_i are high, hence the plant can be approximately decoupled using static input (and/or output) transformations, T_u, T_y respectively so that,