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Dynamic Non-rational Anti-windup for Time-delay Systems with Saturating Inputs

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Abstract: This paper addresses the design of dynamic anti-windup compensators for time-delay systems under amplitude control constraints. Considering that the system is subject to the action of $\mathcal{L}_2$ bounded disturbances, a method for computing a non-rational dynamic anti-windup compensator in order to guarantee both that the trajectories of the system are bounded and a certain $\mathcal{L}_2$ performance level is achieved by the regulated outputs, is proposed. Based on Lyapunov-Krazovskii functionals, the use of a modified sector condition, and a classical change of variables, sufficient LMI conditions, both in local as well as global contexts, are derived to ensure the input-to-state and the internal stability of the closed-loop system. From these conditions, LMI-based optimization problems are proposed in order to minimize the $\mathcal{L}_2$ gain, or in order to maximize the bound on the admissible disturbances for which the trajectories are bounded. The results apply to both stable and unstable open-loop systems and, in particular, for systems presenting delayed states.

Keywords: time-delay, saturation, anti-windup, LMI.

1. INTRODUCTION

It is well-known that the presence of delays in control systems can lead to poor time-domain performances or even to the instability of the closed-loop system (see for instance Niculescu [2001], Richard [2003] and references therein). The difficulty in controlling time-delay systems becomes even greater if the control is forced to be bounded. Unfortunately, this is a practical constraint, which comes from the physical fact that actuator cannot deliver unlimited signals to the controlled plants. As a consequence, input saturation may occur and it can be source of performance degradation and nonlinear behaviors such that limit cycles, multiple equilibria and instability.

Motivated by such problems, many works addressing the problem of stability analysis and stabilization of time-delay systems presenting bounded controls can be found in the literature. We can cite for instance: Oucheriah [1996], Chen et al. [1988], Niculescu et al. [1996], Tissir and Hmamed [1992], Tarbouriech and Gomes da Silva Jr. [2000], Cao et al. [2002] and Fridman et al. [2003]. In those papers, global and local stability (with the characterization of a set of admissible initial conditions) results considering delay-dependent and independent approaches are derived. However, it should be pointed out that all these works deal with state feedback control laws.

On the other hand, the anti-windup approach deals with the actuator saturation problem in a more practical perspective. Considering a pre-computed dynamic output feedback controller, whose design neglected the possibility of input saturation, the idea in this case consists of feeding the controller with the difference between the actuator

input and its output, through a static or dynamic compensator. The aim of the anti-windup compensation is to correct the controller state in order to recover, as much as possible, the nominal performance of the system under saturation, i.e. when there effectively exists a difference between the input and the output of the actuator (see for instance Kapoor et al. [1998], Kothare and Morari [1999], Grimm et al. [2003], Turner and Postlethwaite [2004] and references therein). Considering works dealing with the anti-windup problems for time-delay systems, we can cite for instance Park et al. [2000], Tarbouriech et al. [2004] Zaccarian et al. [2005] and Gomes da Silva Jr. et al. [2006].

In Park et al. [2000] it is considered a plant subject to input delay and saturation. An anti-windup compensator is synthesized for minimizing a cost function, given as the absolute value of the difference between the states of the controller considering a saturation-free actuator, and the controller states when the plant is subject to input saturation and an anti-windup augmentation. It should however to be pointed out that the results apply only to stable open-loop systems and that the approach does not consider systems presenting state delays. In Zaccarian et al. [2005], the optimal controller obtained in Park et al. [2000] is extended in order to cope with disturbances and robustness issues, and output delays.

In Tarbouriech et al. [2004] and Gomes da Silva Jr. et al. [2006], an LMI approach to synthesize stabilizing *static* anti-windup gains have been proposed. Differently from the classical objective of recovering performance, in those works the anti-windup compensation have been used to enlarge the region of attraction of the closed-loop system. In particular, the presence of additive disturbances and closed-loop performance issues were not considered.

In the present work, we focus on the synthesis of *dynamic anti-windup compensators* for systems presenting *delayed states*. Considering that the system is subject to the action of $\mathcal{L}_2$ bounded disturbances, a method for computing the non-rational dynamic anti-windup compensator in order to guarantee both that the trajectories of the system are bounded and a certain $\mathcal{L}_2$ performance level is achieved by the regulated outputs, is proposed. The approach we follow is based on the use of a Lyapunov-Krasovskii functional (see for instance Niculescu [2001]), a modified sector condition, as proposed in Tarbouriech et al. [2004], and the use of non-rational controllers, as proposed in de Oliveira and Geromel [2004]. From these ingredients, LMI conditions to ensure $\mathcal{L}_2$ input-to-state stability as well as internal asymptotic stability of the closed-loop system, are derived both in the local and global contexts. The proposed results apply therefore to both stable and unstable open-loop systems. Considering these conditions, convex optimization problems are proposed in order to address two synthesis objectives: the maximization of the $\mathcal{L}_2$ upper bound on the admissible disturbances, for which the trajectories are bounded (i.e. disturbance tolerance maximization); and, considering a given upper bound on the $\mathcal{L}_2$ norm of the admissible disturbances, the minimization of the $\mathcal{L}_2$ gain between a regulated output and the disturbance (i.e. disturbance rejection maximization).

Notations. For two symmetric matrices, A and B, $A > B$ means that $A - B$ is positive definite. A' denotes the transpose of A. $A_{(i)}$ denotes the ith row of matrix A. $\star$ stands for symmetric blocks; $\bullet$ stands for an element that has no influence on the development. I_m denotes an identity matrix of order m. $\lambda_{max}(P)$ denotes the maximal eigenvalue of matrix P. $\mathcal{C}_\tau = \mathcal{C}([-\tau, 0], \Re^n)$ is the Banach space of continuous vector functions mapping the interval $[-\tau, 0]$ into $\Re^n$ with the norm $\| \phi \|_c = \sup_{-\tau \le t \le 0} \| \phi(t) \|$. $\| \cdot \|$ refers to the Euclidean vector norm. $\mathcal{C}_\tau^v$ is the set defined by $\mathcal{C}_\tau^v = \{\phi \in \mathcal{C}_\tau \; ; \; \| \phi \|_c < v, \; v > 0\}$ For $v \in \Re^m$, $\mathrm{sat}(v) : \Re^m \to \Re^m$ denotes a classical vector-valued saturation function defined as $(\mathrm{sat}(v))_{(i)} = \mathrm{sat}(v_{(i)}) = \mathrm{sign}(v_{(i)})\min\{u_{0(i)}, |v_{(i)}|\}$, for all $i = 1, \dots, m$, where $u_{0(i)} > 0$ denotes the i-th amplitude bound.

2. PROBLEM STATEMENT

Consider the linear continuous-time delay system:
$$\begin{aligned}
\dot{x}(t) &= Ax(t) + A_d x(t-\tau) + Bu(t) + B_w w(t) \\
y(t) &= C_y x(t) + C_{y,d} x(t-\tau) + D_{y,w} w(t) \\
z(t) &= C_z x(t) + C_{z,d} x(t-\tau) + D_z u(t) + D_{z,w} w(t)
\end{aligned} \tag{1}$$
where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $w(t) \in \Re^q$ are the state, the input and the disturbance vectors, respectively. $y(t) \in \Re^p$ corresponds to the measured output and $z(t) \in \Re^r$ is the regulated output. A, A_d, B, B_w, C_y, $C_{y,d}$, C_z, $C_{z,d}$, $D_{y,w}$, D_z and $D_{z,w}$ are real constant matrices of appropriate dimensions. The scalar τ is a constant time delay. The disturbance vector w is assumed to be limited in energy, that is, $w(t) \in \mathcal{L}_2$ and for some scalar δ, $0 \le \frac{1}{\delta} < \infty$, it follows that:
$$\|w\|_2^2 = \int_0^\infty w'(t)w(t)dt \le \frac{1}{\delta} \tag{2}$$
The inputs are supposed to be bounded as follows:
$$-u_{0(i)} \le u_{(i)} \le u_{0(i)}, i = 1, \dots, m \tag{3}$$

For the plant given in (1) a dynamic output feedback controller, possibly non-rational (de Oliveira and Geromel [2004]), is supposed to have been designed, disregarding the saturation, as follows:
$$\begin{aligned}
\dot{x}_c(t) &= A_c x_c(t) + A_{c,d} x_c(t-\tau) + B_c u_c(t) \\
y_c(t) &= C_c x_c(t) + C_{c,d} x_c(t-\tau) + D_c u_c(t)
\end{aligned} \tag{4}$$
where $x_c(t) \in \Re^{n_c}$ is the controller state, $u_c(t) = y(t)$ is the controller input and $y_c(t)$ is the controller output, Matrices A_c, $A_{c,d}$, B_c, C_c, $C_{c,d}$, D_c are of appropriated dimensions.

As the plant input saturates in amplitude, we have:
$$u(t) = sat(y_c(t)) \tag{5}$$

In order to mitigate the possible undesirable effects of saturation, the following non-rational anti-windup dynamic compensator is proposed:
$$\begin{aligned}
\dot{x}_a(t) &= A_a x_a(t) + A_{a,d} x_a(t-\tau) + B_a \psi(y_c(t)) \\
y_a(t) &= C_a x_a(t) + C_{a,d} x_a(t-\tau) + D_a \psi(y_c(t))
\end{aligned} \tag{6}$$
where $x_a(t) \in \Re^{n+n_c}$ is the compensator state vector, $y_a(t)$ is the compensator output, $\psi(y_c(t))$ is the deadzone function defined as $\psi(y_c(t)) = y_c(t) - sat(y_c(t))$. Matrices A_a, $A_{a,d}$, B_a, C_a, $C_{a,d}$ and D_a have appropriated dimensions.

The compensation effect is applied to the system by injecting the signal y_a in the controller (4). Thus, the final controller structure becomes:
$$\begin{aligned}
\dot{x}_c(t) &= A_c x_c(t) + A_{c,d} x_c(t-\tau) + B_c u_c(t) + y_a(t) \\
y_c(t) &= C_c x_c(t) + C_{c,d} x_c(t-\tau) + D_c u_c(t)
\end{aligned} \tag{7}$$

In this work we address the problem of synthesizing the anti-windup compensator with the structure given in (6) in order to ensure that the trajectories of the system are bounded for any disturbance satisfying (2) and, in addition, to ensure an upper bound for the $\mathcal{L}_2$ gain from the disturbance to the regulated output.

3. PRELIMINARIES

Define the following matrices:
$$\mathcal{A} = \begin{bmatrix} \mathbf{A} & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} A + BD_c C_y & BC_c \\ B_c C_y & A_c \end{bmatrix}$$
$$\mathbf{B}_1 = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix}, \mathcal{B}_1 = \begin{bmatrix} 0 & \mathbf{B}_1 \\ I_a & 0 \end{bmatrix}, \mathbf{C} = [\, 0 \; I_a \,]$$
$$\mathcal{A}_d = \begin{bmatrix} \mathbf{A}_d & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{A}_d = \begin{bmatrix} A_d + BD_c C_{y,d} & BC_{c,d} \\ B_c C_{y,d} & A_{c,d} \end{bmatrix}$$
$$\mathbf{K}_1 = \begin{bmatrix} A_a \\ C_a \end{bmatrix}, \mathbf{K}_{1,d} = \begin{bmatrix} A_{a,d} \\ C_{a,d} \end{bmatrix}, \mathbf{K}_2 = \begin{bmatrix} B_a \\ D_a \end{bmatrix}$$
$$\mathcal{B}_\psi = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}$$
$$\mathcal{B}_w = \begin{bmatrix} \mathbf{B}_w \\ 0 \end{bmatrix}, \mathbf{B}_w = \begin{bmatrix} BD_c D_{y,w} + B_w \\ B_c D_{y,w} \end{bmatrix}$$
$$\mathcal{C} = [\, \mathbf{C}_z \; 0 \,], \mathbf{C}_z = [\, C_z + D_z D_c C_y \; D_z C_c \,]$$
$$\mathcal{C}_d = [\, \mathbf{C}_{z,d} \; 0 \,], \mathbf{C}_{z,d} = [\, C_{z,d} + D_z D_c C_{y,d} \; D_z C_{c,d} \,]$$
$$\mathcal{D}_{z,w} = \mathbf{D}_{z,w} = D_{z,w} + D_z D_c D_{y,w}, \mathcal{D}_\psi = \mathbf{D}_\psi = -D_z$$
$$\mathcal{K} = [\, \mathbf{K} \; 0 \,], \mathbf{K} = [\, D_c C_y \; C_c \,], \mathcal{K}_d = [\, \mathbf{K}_d \; 0 \,]$$
$$\mathbf{K}_d = [\, D_c C_{y,d} \; C_{c,d} \,], \mathcal{K}_w = \mathbf{K}_w = D_c D_{y,w}, \mathcal{K}_\psi = 0$$