Bounded Backstepping Control and Robustness Analysis for Time-Varying Systems under Converging-Input-Converging-State Conditions

Frédéric Mazenc, Michael Malisoff, Laurent Burlion, Jerome Weston

PII: S0947-3580(17)30369-2
DOI: 10.1016/j.ejcon.2018.02.005
Reference: EJCON 257

To appear in: *European Journal of Control*

Received date: 5 October 2017
Revised date: 10 January 2018
Accepted date: 19 February 2018

Please cite this article as: Frédéric Mazenc, Michael Malisoff, Laurent Burlion, Jerome Weston, Bounded Backstepping Control and Robustness Analysis for Time-Varying Systems under Converging-Input-Converging-State Conditions, *European Journal of Control* (2018), doi: 10.1016/j.ejcon.2018.02.005

Bounded Backstepping Control and Robustness Analysis for Time-Varying Systems under Converging-Input-Converging-State Conditions *

Frédéric Mazenc Michael Malisoff Laurent Burlion Jerome Weston

February 22, 2018

Abstract

We provide new bounded backstepping results that ensure global asymptotic stability for a large class of partially linear systems with an arbitrarily large number of integrators. We use a dynamic extension that contains one artificial delay, and a converging-input-converging-state assumption. When the nonlinear subsystem is control affine, we provide sufficient conditions for our converging-input-converging-state assumption to hold. We also show input-to-state stability with respect to a large class of model uncertainties, and robustness to delays in the measurements of the state of the nonlinear subsystem. We illustrate our result in a first example that has a nondifferentiable vector field and so is beyond the scope of classical backstepping, and then in a nonlinear example that illustrates how one can combine Lyapunov and trajectory based methods to check our assumptions.

1 Introduction

Backstepping is probably the most important, most celebrated, and most commonly used technique for constructing controls for nonlinear systems. This paper continues our group's quest (begun in [17], [18], [19], [20], [23], and [24]) for novel backstepping results that help overcome the obstacles to using classical backstepping; see [13] and [15] for traditional backstepping. Classical backstepping entails synthesizing globally asymptotically stabilizing feedback controls, by recursively building globally asymptotically stabilizing controls and corresponding Lyapunov functions for subsystems; see [8], [13], and [16] for improved backstepping theory that includes nonlinearities and uncertainties, and [4], [5], and [6] for backstepping applied to adaptive, aerospace, and robotic systems. However, there are significant instances that call for backstepping where the existing backstepping literature does not apply, e.g., systems with general nonlinear subsystems having bounds on the allowable sup norms of the controls, which produce challenges that we overcome in this work.

In this work, we focus on systems of the form

$$\left\{ \begin{array}{rcl} \dot{x}(t) & = & \mathcal{F}(t, x(t), z(t), \eta(t)) \\ \dot{z}_i(t) & = & z_{i+1}(t), \ \ i \in \{1, \dots, k-1\} \\ \dot{z}_k(t) & = & u(t) + \sum\limits_{j=1}^{k} v_j z_j(t) \end{array} \right. \tag{1}$$

with a scalar valued control u and any number k of integrators, where x is valued in $\mathbb{R}^n$ for any n, $\mathcal{F}$ is known, the v_j's are known real constants, the unknown measurable essentially bounded function η represents

*Key Words: Backstepping, delays, stabilization. Corresponding Author: Frédéric Mazenc, Tel.: + 33 [0] 1 69 85 17 62, Fax: + 33 [0] 1 69 85 17 65. A preliminary version was presented at the 2017 IEEE Conference on Decision and Control; see Section 1 for the differences between the conference version and this paper. Supported by NSF-ECCS Grants 1102348 and 1408295. Mazenc is with EPI DISCO INRIA-Saclay, L2S, CNRS CentraleSupelec, 3 rue Joliot Curie, 91192 Gif-sur-Yvette, France (email: frederic.mazenc@l2s.centralesupelec.fr). Malisoff and Weston are with Department of Mathematics, 303 Lockett Hall, Louisiana State University, Baton Rouge, LA 70803-4918, USA (email: malisoff@lsu.edu, jwesto3@lsu.edu). Burlion is with ONERA, The French Aerospace Lab, 31055 Toulouse, France (email: lburlion@onera.fr).