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A relaxation approach to optimal control of Volterra integral equations

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Abstract. We consider an optimal control problem for systems defined by nonlinear Volterra integral equations, with state constraints. No convexity assumptions are made on the data, and the problem is transformed into its relaxed form. We prove the existence of an optimal relaxed control and derive necessary conditions for optimality in the form of a relaxed minimum principle of Pontryagin type. We then apply a mixed Frank-Wolfe penalty method which constructs sequences of relaxed controls converging to extremal controls for this problem. A numerical example is given.

Key Words. Optimal control, integral equations, relaxed controls, minimum principle, mixed Frank-Wolfe penalty method.

1. Introduction

It is well known that optimal control problems without any convexity assumptions on the data, have no classical solutions in general. These problems are usually studied by considering their corresponding relaxed formulations. Relaxation in optimal control has been studied by various authors, mainly by Warga [19], Roubíček [14] and Fattorini [7]. It has been introduced, initially, in order to prove existence of optimal controls and then to derive necessary conditions for optimality. Relaxed controls have also been used for studying the convergence of optimization algorithms (see Williamson and Polak [21], Mayne and Polak [10,11], Teo [18], Wilson [22]) and also as a tool for developing optimization methods (Warga [20], Chryssoverghi et al. [6]) and discrete approximation schemes (Chryssoverghi et al. [5], Roubíček [15], Azhmyakov et al. [1]).

In this paper, we consider an optimal control problem for systems defined by nonlinear Volterra integral equations, with terminal equality state constraints and pointwise inequality state constraints. Volterra integral equations are widely used in population biology, physics, engineering, economics, etc. We prove the existence of an optimal relaxed control and derive necessary conditions for optimality in the form of a relaxed minimum principle of Pontryagin type with transversality conditions. Relaxed controls have been applied to integral equations in many papers, among them [12, 23, 16].

Finally, we apply a mixed Frank-Wolfe penalty method (see [6]) which constructs sequences of relaxed controls converging to extremal controls for this problem, i.e. controls which satisfy the strong relaxed necessary conditions for optimality. A numerical example is given showing the efficiency of the method.

The essential contribution of this work is to propose an iterative method for solving