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Almost-global tracking for a rigid body with internal rotors

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ABSTRACT

Inspired by recent work on a PID-like tracking control law for an interconnected mechanical system on a Lie group, this article presents a novel tracking control law for such mechanical systems, adapted from the one proposed by the authors for systems evolving on a Riemannian manifold setting. The interconnected mechanical system consists of a rigid body with three internal rotors for attitude actuation. The novelty of the control law lies in viewing the rigid body as a simple mechanical system (SMS) by incorporating the additional quadratic velocity terms arising from the interconnection into the control law, and then proceeding with a straightforward stabilizing PID tracking synthesis. The novel algorithm achieves tracking of feasible trajectories from almost all initial conditions.

1. Introduction

Spacecrafts are actuated either through internal or external mechanisms. External mechanisms include gas jet thrusters, while internal mechanisms include spinning rotors and control moment gyros. The attitude of a spacecraft, modeled as a rigid body, evolves on the manifold $SO(3)$. A spacecraft actuated by external mechanisms falls into the class of a fully actuated simple mechanical system (SMS) (defined in Section 2). A spacecraft actuated by internal mechanisms, and operating in a potential-free field, is an underactuated SMS. In the recent past, there has been an increased interest in the design of coordinate-free control laws for SMSs which evolve on Lie groups. Results on stabilization of a rigid body about a desired configuration in $SO(3)$ using proportional plus derivative (PD) control are found in [2,5,8]. Geometric tracking of specific SMSs can be found in [14,15]. Almost-global tracking of a reference trajectory for an SMS on a Lie group implies tracking of the reference from almost all initial conditions in the tangent bundle of the Lie group. A general result on almost-global asymptotic tracking (AGAT) for an SMS on a class of compact Lie groups is found in [9]. In all these results, the rigid body is assumed to be externally actuated and therefore, the control torque is applied through actuators such as gas jets. Almost-global stabilization and tracking of the externally actuated rigid body is, therefore, a sufficiently well studied problem. However, the problem of geometric tracking for an internally actuated rigid body has received much less attention.

In order to define a feasible tracking problem, it is essential to identify a class of reference trajectories. Since the spatial angular momentum of the entire unreduced system – (the rigid body plus rotors) – is conserved; this conserved value of momentum is specified by the momentum map. The evolution on the phase space is then restricted to a level set of the momentum map. This constraint implies that, the attainable angular velocities of the rigid body at any orientation are interlinked with the rotor velocities at that orientation. The reference trajectories which can be tracked by a rigid body with rotors must then evolve on this restricted phase space.

A rigid body plus the internal rotors is an interconnected simple mechanical system. The control torques provided to the rotors gets reflected through the interconnection mechanism to the rigid body. Interconnected mechanical systems have been studied in the context of spherical mobile robots in [6,10,12,17]. The interconnected mechanism leads to the presence of quadratic rotor velocity terms in the rigid body dynamics, which breaks the SMS structure for the rigid body. One of the early papers to study the stabilization of the internally actuated rigid body in the geometric setting is given in [3]. It is shown that any feedback torque on the externally actuated rigid body can be realized with 3 internal rotors attached to the rigid body. This paper illustrates the fact that despite the feedback

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forces acting on an externally actuated rigid body, it is still Hamiltonian and behaves like a heavy rigid body. In other words, if a certain class of feedback torques is applied to the rotors, the rigid body with the rotors behaves like a fully actuated SMS on $SO(3)$. The semi-global tracking problem is considered for a hoop robot in [16] and for a spherical robot in [17] with internal actuation. The uncontrolled hoop and the sphere dynamics in these work do not correspond to a SMS. It is shown that a class of internal actuation configurations exist for which the hoop dynamics can be converted to a fully actuated SMS by feedback torques. This motivates us to study the AGAT problem for a rigid body with rotors in a similar setting.

In [2] and [23], the almost-global asymptotic stabilization (AGAS) problem of a rigid body with 3 internally mounted rotors is solved using proportional plus derivative (PD) control. In [11], any trajectory on $SO(3)$ is followed by employing both external and internal actuation and using a local representation for a rotation matrix. However, to the best of our knowledge the AGAT problem for an internally actuated rigid body has not been investigated.

AGAT of an SMS on a compact Lie group is often achieved by a proportional plus derivative type control [9,22]. A configuration error is chosen on the Lie group with the help of the group action along with a compatible navigation function. A navigation function is a Morse function with a unique minimum [21].

The closed loop error dynamics for a control force proportional to the negative gradient covector field generated by the navigation function plus a dissipative covector field, is then an SMS. This control drives the error dynamics to the lifted minimum of the navigation function on the tangent bundle of the Lie group from all but the stable manifolds of the lifted saddle points and maxima of the navigation function. As the critical points of a Morse function are isolated, this convergence is almost global. The compatibility conditions in [22] ensure that the error function is symmetric, and achieves its minimum when the system configuration coincides with the reference configuration. In [18], the authors propose an ‘integral’ action to the existing PD control law in [9]. The addition of an integral term makes the control law robust to bounded parametric uncertainty and constant disturbances.

The novelty of the contribution in this paper is twofold: 1. Inspired by Maithripala and Berg [18], the article proposes a tracking control law adapted from the one proposed by the authors for systems evolving on a Riemannian manifold setting, to an interconnected mechanical system, and 2. it evokes a separation of the interconnected mechanical system of the rigid body with rotors into a SMS component consisting of the rigid body alone (with the quadratic velocity terms being absorbed in the control law) and synthesizes the tracking control law. Further, the rotor speeds are shown to remain bounded.

The paper is organized as follows – in Section 2, after presenting a few mathematical preliminaries, we review the derivation of the equations for the rigid body with external actuation and with 3 internal rotors. In Section 3, we define the class of feasible reference trajectories for the tracking problem. In Section 4 we append an integral term to the proportional derivative plus feedforward control law for AGAT of an SMS in Theorem 1 of [22]. Further, we propose a control law for AGAT of a feasible reference trajectory for a rigid body with 3 rotors. In Section 5 we present simulation results for the proposed control law.

2. Preliminaries

This section introduces several mathematical notions to describe simple mechanical systems on manifolds. We refer the reader to [1,4,19] for further details. A Riemannian manifold is denoted by the 2-tuple $(Q, \mathbb{G})$, where Q is a smooth connected manifold and $\mathbb{G}$ is a Riemannian metric on Q. Let T_qQ denote the tangent plane to Q at the point $q \in Q$ and T_q^*Q denote the cotangent plane (dual) and and let $\mathfrak{X}(Q)$ denote the set of all smooth vector fields on M.

Definition 1. An affine connection ∇ on a smooth manifold Q is a mapping

$$\nabla : \mathfrak{X}(Q) \times \mathfrak{X}(Q) \to \mathfrak{X}(Q)$$

which is denoted by $\nabla_X Y$ for $X, Y \in \mathfrak{X}(Q)$ that satisfies the following properties:

(i) $\nabla_{fX+gY}Z = f\nabla_X Z + g\nabla_Y Z$.
(ii) $\nabla_X(Y+Z) = \nabla_X Y + \nabla_X Z$.
(iii) $\nabla_X(fY) = f\nabla_X Y + X(f)Y$,

for $Z \in \mathfrak{X}(Q)$ and smooth real valued functions f, g on Q.

Definition 2. A connection ∇ is said to be *compatible* with the metric $\mathbb{G}(\cdot,\cdot) \triangleq \ll \cdot,\cdot \gg$ iff $X \ll Y, Z \gg = \ll \nabla_X Y, Z \gg + \ll Y, \nabla_X Z \gg$, for all X, Y, $Z \in \mathfrak{X}(Q)$.

Definition 3. A connection is said to be symmetric when,

$$\nabla_X Y - \nabla_Y X = [X, Y]$$

where [,] denotes the Jacobi–Lie bracket, for all X, $Y \in \mathfrak{X}(Q)$.

On any Riemannian manifold, there exists a unique affine connection $\overset{\mathbb{G}}{\nabla}$ on Q satisfying the conditions:

(a) ∇ is symmetric.
(b) ∇ is compatible with the Riemannian metric.

The unique connection is called the Levi-Civita (or Riemannian) connection on Q. The flat map $\mathbb{G}^\flat : T_qQ \to T_q^*Q$ is given by $\mathbb{G}(v_1, v_2) = \langle \mathbb{G}^\flat(v_1); v_2 \rangle$ for $v_1, v_2 \in T_qQ$ and the sharp map is its dual $\mathbb{G}^\sharp : T_q^*Q \to T_qQ$, and given by $\mathbb{G}^{-1}(w_1, w_2) = \langle \mathbb{G}^\sharp(w_1); w_2 \rangle$ where w_1, $w_2 \in T_q^*Q$. Therefore if $\{e^i\}$ is a basis for T_q^*Q, $\mathbb{G}^\flat(v_1) = \mathbb{G}_{ij}{v_1}^j e^i$ and $\mathbb{G}^\sharp(w_1) = \mathbb{G}^{ij}w_{1j}e_i$.

2.1. Simple mechanical system on a Riemannian manifold M *embedded in* $\mathbb{R}^m$

A simple mechanical system (or an SMS) on a smooth, connected Riemannian manifold $(M, \mathbb{G})$ is denoted by the 3-tuple $(M, \mathbb{G}, F)$, where $F(q) \in T_q^*Q$ is the generalized resultant external force acting on the system. The governing equations are given by the Euler–Lagrange equations

$$\overset{\mathbb{G}}{\nabla}_{\dot\gamma(t)}\ \dot\gamma(t) = \mathbb{G}^\sharp(F(\dot\gamma(t))) \tag{1}$$

where $\gamma(t)$ is the system trajectory. In order to embed the manifold M in an ambient Euclidean space, we require the following additional notions.

Definition 4. A distribution $\mathcal{D}$ on M is an assignment to each point $x \in M$ a subspace $\mathcal{D}_x$ of a T_xM.

Definition 5. The Euclidean metric $\mathbb{G}_{id}$ on $\mathbb{R}^m$ is the Riemannian metric such that in Cartesian coordinates

$$G_{id} = \delta_{ij} = \begin{cases} 1 & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j. \end{cases} \tag{2}$$

The equations of motion for the SMS $(M, \mathbb{G}, F)$ in (1) can be simplified by embedding M in $\mathbb{R}^m$. The idea behind this approach is that we consider the SMS to evolve on $(\mathbb{R}^m, \mathbb{G}_{id})$ subject to a distribution $\mathcal{D}$ (the velocity constraint) whose integral manifold is M (see Section 4.5 in [4] for more details). By Nash embedding theorem, there exists an isometric embedding $f : M \to \mathbb{R}^m$ for some m depending on the dimension of M such that $\mathbb{G} = f^*\mathbb{G}_{id}$ where $f^*\mathbb{G}_{id}$ is the pull back of $\mathbb{G}_{id}$ (see Definition 3.81 in [4]). Moreover, the

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