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Preservation of structural identifiability in expanded systems

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The main result presented by this paper is that the structural identifiability of an original continues-linear time invariant (LTI) dynamic system can be preserved in expanded systems within the inclusion principle when using block structured complementary matrices. This preservation is ensured only for some selections of specific complementary matrices. Overlapping expansions of these systems are then discussed. An original system composed of two overlapped subsystems is used as a general prototype case. An illustrative example is given.

1. Introduction

Choosing a suitable model structure is very important to reach the objectives initially defined. This choice should be performed before any measurement. Testing structural properties (controllability, observability and identifiability) can detect possible defects of the considered model structures even before starting data collection [20]. This paper discusses the transmission of the structural global identifiability from an original controllable, observable and identifiable system to its expansion. Such transmission is very important for identification and present a practical issue for control design. This paper puts also the notions of inclusion and restriction as its principal cases.

Large scale systems are generally composed of subsystems sharing common parts, loosely interconnected among themselves and strongly interconnected through certain dynamics. There are a large variety of these systems such aerial vehicles [6], large scale winding systems [13], freeway traffic regulation systems [11].

For either computational or structural reasons, it is often convenient to decompose the global system by using overlapping decomposition sets. This approach has been introduced by Ikeda and Siljak [10], it is based on the inclusion principle ([1,7,9], etc). This principle consists to define two dynamic systems with different dimensions. The two systems are related through linear transformations (expansions and contractions) that have the freedom to select the complementary matrices. The solutions of the larger system (dimension) include solutions of the smaller system, but it is not usually proved whether structural properties of the original system (with smaller dimensions) are transmitted to its expansion (with larger dimensions). Previous works such as [3] and [2] have proved that there are block structured complementary matrices ensuring partially or always the transmission of controllability and observability. To the author's best knowledge, the problem of transmission of structural identifiability between the original system and its expansion was not studied. Then, this work presents a special selection of structured complementary matrices ensuring the preservation of this structural property.

The paper is organized as follows. Section 2 presents necessary preliminaries on structural identifiability, inclusion principle and overlapping decomposition and expansions method. Section 3 states the studied problem in this paper and the assumptions of the study. The main result on the preservation of structural identifiability in expanded systems, using a prototype case with an original system composed of two overlapped subsystems for a selection of block structured complementary matrices, is presented in Section 4. This result is illustrated by an example. Finally some concluding remarks are given in Section 5.

2. Preliminaries

Consider the linear parametrized system Σ:

$$(\Sigma):\begin{cases}\dot{x} &= A(p)x+B(p)u\\ y &= C(p)x\end{cases} \tag{1}$$

where $x(t)\in\mathcal{R}^n$, $u(t)\in\mathcal{R}^m$, $y(t)\in\mathcal{R}^l$ are the state, input, output of Σ at time t. $p\in\mathcal{P}\subseteq\mathcal{R}^p$ is the vector of the parameters of Σ.

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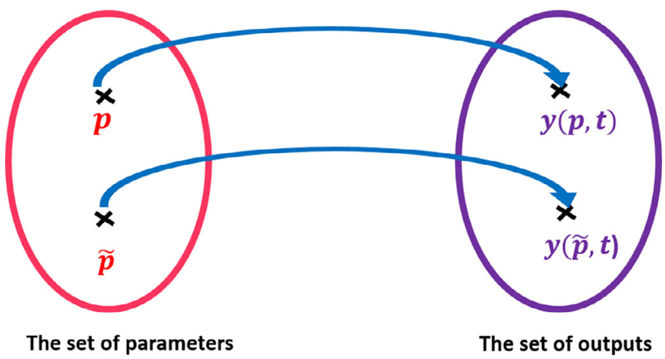

Fig. 1. Algebraic interpretation of the condition of global structural identifiability.

A, B and C are constant matrices of dimensions $n \times n$, $n \times m$, $l \times n$, respectively.

2.1. Structural identifiability

The notion of identifiability addresses the question of whether it is at all possible to obtain unique solutions for unknown parameters of interest in a mathematical model, from collected data. In other words, it consist to verify that output identity for any possible input implies that the estimated parameters of the model are equal to the "true" parameters of the studied process [19].

Literature on structural identifiability is extensive ([4,14], etc.). The reader may be referred to the books [18,20] which treat the subject thoroughly. Its formal definition is recalled in the following definition:

Definition 1 (Global Structural Identifiability [20]). Consider the linear system Σ.

A parameter $p_0 \in \mathcal{P}_0 \subseteq \mathcal{R}$ is said to be *structurally globally identifiable* (s.g.i.) if and only if, for almost any $\tilde{p}_0 \in \mathcal{P}_0$ and an input class $\mathcal{U}$:

$$y(\tilde{p}, t) \equiv y(p, t), \ \ \forall t \in \mathcal{R}^+, \forall u \in \mathcal{U} \implies \tilde{p}_0 = p_0 \tag{2}$$

The system model (Σ) is s.g.i. if and only if all its parameters are s.g.i.

Then, testing the structural identifiability returns to test the injectivity of the input–output behavior of the structure (Fig. 1).

The test of structural global identifiability is performed in idealized framework, i.e., continuous-time and noise free input and output functions.

Notation: For the remainder of this paper, p dependence of all the matrices will be omitted: for instance, $A(p)$ will be denoted with A. For another parametrization $\tilde{p}$, $A(\tilde{p})$ will be denoted by $\tilde{A}$. The n-by-n identity matrix will be denoted by I_n.

There are many methods to test structural identifiability (see for example [15,16,21]). For this paper, similarity transformation approach is used [17]. This approach can be applied to LTI state space systems.

Theorem 1 [17]. *The system (Σ), structurally controllable and observable, is structurally globally identifiable if and only if, given two parameterizations p and $\tilde{p}$ and a non-singular matrix $T \in \mathcal{R}^{n \times n}$, such that:*

$$\begin{cases} TA = \tilde{A}T \\ TB = \tilde{B} \\ \tilde{C}T = C \end{cases} \tag{3}$$

then the solution is $T = \mathrm{I}_n$ and $p = \tilde{p}$.

Example. Consider the linear parametrized system Σ described by these matrices:

$$A = \begin{pmatrix} p_1 & -p_3 \\ -(p_1 + p_2) & p_3 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 \end{pmatrix} \tag{4}$$

Σ is structurally controllable and observable, so the similarity transformation approach applies.

Let

$$T = \begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{pmatrix}$$

Exploit first the structures of the observation and control matrices:

$$\begin{aligned} &\tilde{C}T = C \Rightarrow t_{11} = 1, t_{12} = 0 \\ &TB = \tilde{B} \Rightarrow t_{22} = 1 \end{aligned} \tag{5}$$

so T can be written as:

$$T = \begin{pmatrix} 1 & 0 \\ t_{21} & 1 \end{pmatrix}$$

The set of all possible matrices A satisfies:

$$\begin{aligned} &TA = \tilde{A}T \\ &\begin{pmatrix} p_1 & -p_3 \\ t_{21}p_1 - (p_1 + p_2) & -t_{21}p_3 + p_3 \end{pmatrix} \\ &= \begin{pmatrix} \tilde{p}_1 - \tilde{p}_3 t_{21} & -\tilde{p}_3 \\ -(\tilde{p}_1 + \tilde{p}_2) + \tilde{p}_3 t_{21} & \tilde{p}_3 \end{pmatrix} \end{aligned} \tag{6}$$

then:

$$\begin{aligned} &p_3 = \tilde{p}_3 \\ &p_3(1 - t_{21}) = \tilde{p}_3 = p_3 \\ &\Rightarrow 1 - t_{21} = 1 \Rightarrow t_{21} = 0 \\ &\Rightarrow p_1 = \tilde{p}_1, p_2 = \tilde{p}_2 \end{aligned} \tag{7}$$

then, $T = \mathrm{I}_n$ and $p = \tilde{p}$, which implies that Σ is structurally globally identifiable.

The study of the structural identifiability of systems is more complicated when the size of the system is important (more than 100 state [5]). In some cases, the decomposition of these systems on smaller subsystems can simplify this study [12]. Some decompositions are based on the inclusion/restriction principle.

2.2. Inclusion/restriction

In this part, the inclusion principle of LTI systems is presented.

Consider the linear system $\bar{\Sigma}$:

$$(\bar{\Sigma}) : \begin{cases} \dot{\bar{x}} = \bar{A}\bar{x} + \bar{B}\bar{u} \\ \bar{y} = \bar{C}\bar{x} \end{cases} \tag{8}$$

where $\bar{x}(t) \in \mathcal{R}^{\bar{n}}$, $\bar{u}(t) \in \mathcal{R}^{\bar{m}}$, $\bar{y}(t) \in \mathcal{R}^{\bar{l}}$ are the state, input, output of $\bar{\Sigma}$ at time t. $\bar{A}$, $\bar{B}$ and $\bar{C}$ are constant matrices of dimensions $\bar{n} \times \bar{n}$, $\bar{n} \times \bar{m}$, $\bar{l} \times \bar{n}$, respectively.

Suppose that the dimension of the state, input, output vectors x, u, y of Σ are smaller than those of $\bar{x}$, $\bar{u}$, $\bar{y}$ of $\bar{\Sigma}$.

The systems Σ and $\bar{\Sigma}$ are related by the transformation:

$$\begin{cases} \bar{x} = Vx \Rightarrow x = U\bar{x} \\ \bar{u} = Su \Rightarrow u = Q\bar{u} \\ \bar{y} = Wy \Rightarrow y = E\bar{y} \end{cases} \tag{9}$$

where V, S and W are constant matrices of full column-ranks: $V \in \mathcal{R}^{\bar{n} \times n}$, $S \in \mathcal{R}^{\bar{m} \times m}$ and $W \in \mathcal{R}^{\bar{l} \times l}$.

U, Q and E are constant matrices of appropriate dimensions and full row ranks.