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Autonomous collision avoidance for wheeled mobile robots using a differential game approach

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ABSTRACT

A multi-agent system consisting of N wheeled mobile robots is considered. The robots are modeled by unicycle dynamics and the *multi-agent collision avoidance problem*, which lies in steering each robot from its initial position to a desired target position while avoiding collisions with obstacles and other agents is considered. The problem is solved in two steps. First, exploiting a differential game formulation, collision-free trajectories are generated for *virtual agents* satisfying single-integrator dynamics. Second, the previous step is used to construct dynamic feedback strategies for the wheeled mobile robots satisfying unicycle dynamics which ensure each of the robots reaches its target without collisions occurring. A numerical study of the proposed methodology is provided through a series of simulations.

1. Introduction

With several important areas of application, such as mobile sensor networks [2,9,10,17,18,36], air traffic management [7,35], power systems [19] and distributed robotics, including collision avoidance and formation flying [15,20,22], multi-agent systems have gained much interest in recent years. Essentially a multi-agent system is a system consisting of several, interacting subsystems, which we refer to as agents. While a system of several simple agents can perform tasks more efficiently and reliably than a single (possibly more complex) agent, often such systems and their applications call for the development of novel control methodologies.

In the context of robot navigation many approaches available in the literature are based on a so-called *navigation function* (see, for instance [3,6,31,33,34]) which is constructed from the geometric information on the considered topology and employed to define *gradient descent* control laws. Recognising that multi-agent systems are ubiquitous in the nature, other approaches are inspired by naturally occurring systems, such as schools of fish, migrating birds and swarms of bees [8,12,25–27,29,32]. In the presence of communication constraints, the communication topology is often represented by a graph and control strategies are designed using notions from graph theory [15,28]. The use of game theory in the context of multi-agent systems has also been explored (see, for instance, [1,13,14,24]).

The *multi-agent collision avoidance problem* for a system of N wheeled mobile robots (WMRs) is considered in this paper, *i.e.* assuming the WMRs satisfy unicycle dynamics we consider the problem of designing feedback controls such that the robots autonomously reach predefined targets while avoiding collisions with obstacles and other robots. A Lyapunov-based approach for avoidance control has been considered in [11], where the authors recognise that the main challenge associated with the approach lies in determining Lyapunov functions. In [22,24] the multi-agent collision avoidance has been considered for agents satisfying single-integrator dynamics. Exploiting the single-integrator dynamics of the agents, feedback strategies which approximate the Nash equilibrium solution of the game are readily constructed, thus providing a systematic manner of constructing Lyapunov functions (see [22,24] for details).

In this paper we extend the results of [24] to WMRs. We also present several interesting and challenging case studies. For instance, we consider situations which include several sources of potential dead-locks which, to the best of our knowledge, are not readily solved using other approaches. The multi-agent collision

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avoidance problem is solved in two steps. First, *virtual agents* satisfying single-integrator dynamics are introduced and the multi-agent collision avoidance problem for the virtual agents is solved using a differential game formulation of the problem and exploiting the main results of [23,24]. Second, the solution of the differential game involving virtual agents is used to design feedback strategies for the WMRs which ensure that the robots reach their targets while avoiding collisions. The novel contributions presented in this paper can be roughly summarised as follows. The introduction of *virtual agents* allows to use the main result of [24] to generate a *trajectory plan* for the WMRs, which can then be tracked with zero error. Moreover, the case studies provide useful insights and validate the theoretical results obtained both in this paper and in [24] through simulations of complex and challenging scenarios as well as a sensitivity analysis with respect to certain control parameters.

The remainder of the paper is structured as follows. The multi-agent collision avoidance problem is introduced in Section 2. A method of generating collision-free trajectories exploiting the virtual agents is presented in Section 3 (see [24] for details). In Section 4 the virtual agents are used to solve the multi-agent collision avoidance problem for the WMRs. A series of illustrative numerical examples are then provided in Section 5 before some concluding remarks and directions for future research are given in Section 6.

Notation: Standard notation is used in the remainder of the paper. The set of real numbers is denoted by $\mathbb{R}$, the set of complex numbers is denoted by $\mathbb{C}$ and $\mathbb{C}^-$ denotes the open left-half plane of $\mathbb{C}$. Given a vector $x \in \mathbb{R}^n$, the gradient of a function $V : \mathbb{R}^n \to \mathbb{R}$ is denoted by $\frac{\partial V}{\partial x}$. The weighted Euclidean norm of a vector is denoted by $\|v\|_A = \sqrt{v^\top A v}$, where $A = A^\top > 0$ and $A \in \mathbb{R}^{n\times n}$. The shorthand $A = \left[A_{ij}\right]$ is used to denote the block matrix $A = \begin{bmatrix} A_{11} & \dots, & A_{1N} \\ \vdots & \ddots & \vdots \\ A_{N1} & \vdots & A_{NN} \end{bmatrix}$. The empty set is denoted by $\emptyset$.

2. Problem formulation

In this section the *multi-agent collision avoidance* problem is introduced, formulated and studied in a *centralised* setting, *i.e.* the positions of each agent are available to the remaining members of the group at all times. We consider a team of N WMRs moving on the ground (Euclidean plane), possibly characterised by the presence of (static) obstacles. In particular, each WMR is described by a *unicycle-like* model with a passive wheel, the dynamics of which are defined (see, for instance, [4,5]) by the equations

$$\begin{aligned} \dot{X}_i &= \cos(\theta_i)v_i, \\ \dot{Y}_i &= \sin(\theta_i)v_i, \\ \dot{\theta}_i &= \omega_i, \end{aligned} \qquad (1)$$

with $i = 1, \dots, N$, where $(X_i, Y_i) \in \mathbb{R}^2$ denote the position on the Euclidean plane of the middle point between the two actuated wheels along the direction of the axle connecting the wheels, θ_i denotes the orientation of the i-th robot and v_i and ω_i represent the longitudinal and the angular velocity controls, respectively. The constant a_i describes the distance between the point (X_i, Y_i) and the centre of mass of the i-th robot $(x_{c,i}, y_{c,i}) \in \mathbb{R}^2$, along the segment connecting (X_i, Y_i) and the (forward) passive wheel (see Fig. 1). The control task may be informally stated as follows: given a target location $(x^*_{c,i}, y^*_{c,i}) \in \mathbb{R}^2$ for the centre of mass of each agent, determine control inputs v_i and ω_i such that the i-th agent is steered towards the desired point while avoiding collisions with other members of the team and with static obstacles.

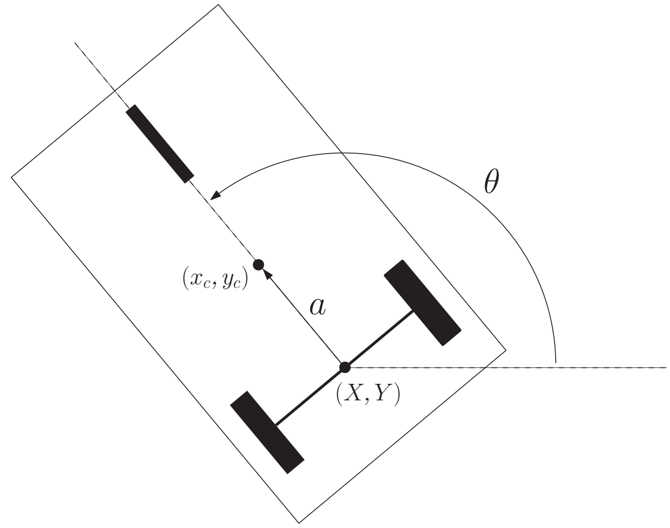

Fig. 1. Diagram of the WMR.

Problem 1. Consider a multi-agent system consisting of N WMRs with dynamics (1), for $i = 1, \dots N$. The *multi-agent collision avoidance problem* consists in determining feedback control laws v_i and ω_i, $i = 1, \dots, N$, that steer each agent from its initial position to a predefined target while avoiding collisions between agents and among agents and static obstacles. ◇

3. Collision-free trajectory planning

Problem 1 is solved in two steps. First, N *virtual agents* (one for each robot) satisfying single-integrator dynamics are introduced and collision-free trajectories are planned for the virtual agents. This result is one of the main contributions of [24] and it is based on a formulation of the multi-agent collision avoidance problem in terms of a nonzero-sum differential game for which a closed-form (approximate) solution is provided. Second, control inputs v_i and ω_i are determined to steer the i-th robot with dynamics (1) along the collision-free trajectories, thus solving Problem 1. The first step, namely *collision-free trajectory planning*, is considered in this section. The reader is referred to [24] for a more detailed discussion on the results presented in this section.

We consider the case in which the movement of each virtual agent is described by single-integrator dynamics, *i.e.*

$$\dot{x}_i = u_i, \qquad (2)$$

$i = 1, \dots N$, where $u_i \in \mathbb{R}^2$ is the control input of the i-th agent and the position of the i-th agent is denoted by $x_i \in \mathbb{R}^2$. Each virtual agent is associated with a desired goal, namely a target position $x^*_i \in \mathbb{R}^2$, $i = 1, \dots, N$. Let $\tilde{x}_i = x_i - x^*_i$, *i.e.* $\tilde{x}_i$ denotes the error variable between the current position of the i-th agent and its corresponding target position. Moreover, each agent i is associated with a parameter $r_i > 0$, which plays the role of *safety radius*, which may differ from one agent to another. These values take into account the fact that the agents are WMRs with dynamics defined by Eqs. (1), rather than point masses with the single-integrator dynamics (2), *i.e.* they account for the physical dimensions of the unicycle.

A static obstacle is represented by its centre of mass $p^c_j \in \mathbb{R}^2$ and the region of the Euclidean plane that it occupies $\mathcal{P}_j \subset \mathbb{R}^2$, with $j = 1, \dots, m$, where $m \geq 0$ denotes the number of static obstacles present. The boundary of the region $\mathcal{P}_j$ is denoted by $\partial\mathcal{P}_j$ and in what follows elliptical obstacles are considered, *i.e.*

$$\partial\mathcal{P}_j = \{x \in \mathbb{R}^2 : \|x - p_j\|^2_{E_j} - \rho^2_j = 0\}, \qquad (3)$$

where $\rho_j > 0$ and $E_j = E^\top_j > 0$. There is a one-to-one relation between the point p_j, ρ_j and E_j, and the physical parameters of the