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Optimistic value model of multidimensional uncertain optimal control with jump

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ABSTRACT

Based on the optimistic value model of uncertain optimal control with jump in one-dimensional case, this paper investigates the optimistic value model of multidimensional uncertain optimal control with jump, which are based on a new uncertainty theory and differs from the stochastic optimal control based on probability theory. The principle of optimality is given and the equation of optimality is obtained. In the end, an example of a portfolio selection is presented to illustrate the effectiveness of the new results.

1. Introduction

Optimal control is a relatively new branch of mathematics developed to find the optimal way to control a dynamic system. In recent decades, it greatly attracted the attention of many mathematicians, engineers and even economists. In the early stage, researchers studied optimal control problem under deterministic environment. Some famous researchers such as Pontryagin, Bellman and Kalman made outstanding contributions for the study of deterministic optimal control theory. Generally speaking, the state evolution of most dynamic systems is often affected by some indeterministic disturbances. These indeterminacy are usually described as randomness, which is studied by applying probability theory. And control system is modelled by a stochastic differential equation. The study of stochastic optimal control was initiated in the late of 1960s and has been made considerable advances both in theory and application, especially for finance. Some studied results can be found in Merton [24], Fleming and Rishel [13], Dixit and Pindyck [12], Zhou and Li [30], Boulier [1], Cairns [2] and the references therein.

Although probability theory has been used to deal with indeterminacy for a long time, it is well known that the basic premise of applying probability theory to describe indeterministic phenomena is that there must be sufficient available sample data to determine probability distribution. However, in many cases in real life, the sample size is too small or even no sample to estimate a probability distribution by means of statistics, such as the price of a new stock, oil field reserves and bridge strength, etc. In this situation, we have no choice but to invite some domain experts to evaluate their belief degree that these indeterministic events will take place. Perhaps some people think that personal belief degree is subjective probability or fuzzy concept. However, Liu [22] showed that it's not appropriate because both probability theory and fuzzy set theory may lead to counterintuitive results in this case. In order to deal with this type of indeterministic phenomena, an uncertainty theory was established by Liu [17] in 2007 and refined in 2010 [21] as a branch of axiomatic mathematics for modeling human uncertainty. Furthermore, in 2008, for describing state evolution of dynamic systems with indeterministic disturbances, uncertain process and canonical process were introduced by Liu [18] as counterparts of stochastic process and Wiener process, respectively. And then, a type of uncertain differential equation driven by canonical process was proposed by Liu [18]. Up to now, uncertainty theory has been well developed and successfully applied to many areas, including uncertain programming [20], uncertain optimal control [31], uncertain statistics [26], uncertain logic [15], uncertain finance [3], etc.

For handling uncertain optimal control problem with uncertain dynamic systems, Zhu [31] presented and studied an expected value model of uncertain optimal control problem without jump

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in 2010. An equation of optimality as a counterpart of Hamilton-Jacobi-Bellman equation was derived by employing dynamic programming method and was applied to an uncertain portfolio selection problem. At present, fruitful results have been achieved on the study of uncertain optimal control problems. For example, by taking advantage of Zhu's equation of optimality, Xu and Zhu [27], Kang and Zhu [14], Yan and Zhu [28] discussed uncertain bang-bang optimal control for continuous time systems, multi-stage systems, and switched systems, respectively. Yao and Qin [29], Chen and Deng [4], Li and Zhu [16] studied uncertain linear quadratic optimal control. And Chen and Zhu [5] proposed an uncertain optimal control problem with time-delay. Besides, Deng and Zhu [6,9–11] investigated a kind of optimal control problems for an uncertain dynamic system with jump. Different from above expected value models, recently, Sheng and Zhu [25] established an uncertain optimal control model under the optimistic value criterion in case without jump. Furthermore, Deng and Chen [7,8] presented the optimistic value model of uncertain optimal control with jump in one-dimensional case.

However, the complexity of the word makes the control systems we face may be multidimensional uncertain systems with jump in many cases because there are usually more than one uncertainty factors. Unlike a one-dimensional control system, in multidimensional control system, various uncertain factors, including diffusion and jump factors, may interact with each other to make optimal control problem becomes more complicated and more difficult to solve. To the best of our knowledge, there is no the research on this issue. Therefore, it is a meaningful work to extend the one-dimensional uncertain optimistic value model with jumps to the multidimensional model.

The objective of this paper is to study the multidimensional uncertain optimal control problem with jumps under optimistic value criterion, where uncertain dynamic systems are modelled by a type of uncertain differential equation driven by canonical process and V jump uncertain process. The contributions of this paper are: (1) to derive the equation of optimality for the proposed model; (2) to give the estimation of optimistic value of the variables as the form of $a\eta + b\eta^2$, where η is an increment of V-jump uncertain process. Compared with stochastic optimal control, the main advantages of our results is that it can be apply to the cases where no enough samples are available to estimate the probability distribution.

The remainder of this paper is organized as follows. In next Section, we review some basic concepts and theorems in uncertainty theory. In Section 3, an optimistic value model of multidimensional uncertain optimal control with jumps is established. The principle of optimality is proposed and the equation of optimality are obtained for the presented model. An application of obtained results in the field of portfolio selection is given in Section 4. Section 5 gives a conclusion. The last Section is a Appendix, in which α-optimistic value estimation of some uncertain variables are derived.

2. Preliminary

For convenience, in this section, we will introduce some useful definitions and and theorems in uncertainty theory [17].

Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element Λ in $\mathcal{L}$ is called an event.

A set function $\mathcal{M}$ defined on the σ-algebra $\mathcal{L}$ over Γ is called an *uncertain measure* if it satisfies the following three axioms.

Axiom 1 (*normality axiom*). $\mathcal{M}\{\Gamma\} = 1$ for the universal set Γ;

Axiom 2 (*duality axiom*). $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event $\Lambda \in \mathcal{L}$;

Axiom 3 (*subadditivity axiom*) $\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \le \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$ for every countable sequence of events $\{\Lambda_i\}$.

The triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is called an *uncertainty space*. Furthermore, Liu [19] defined a product uncertain measure by the product axiom.

Axiom 4 (*product axiom*) Let $(\Gamma_i, \mathcal{L}_i, \mathcal{M}_i)$ be uncertainty spaces for $i = 1, 2, \ldots$. Then, the *product uncertain measure* $\mathcal{M}$ on the product σ-algebra satisfies

$$\mathcal{M}\{\prod_{i=1}^{\infty} \Lambda_i\} = \bigwedge_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\},$$

where $\{\Lambda_i\}$ are arbitrarily chosen events from $\mathcal{L}_i$ for $i = 1, 2, \ldots,$ respectively.

An *uncertain variable* is a function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers such that for any Borel set of real numbers, the set $\{\xi \in B\} = \{\gamma \in \Gamma \mid \xi(\gamma) \in B\}$ is an event. In addition, an uncertain variable ξ may be described by its *uncertainty distribution function* Φ: $\Re \to [0, 1]$ which is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}.$$

The *expected value* of uncertain variable ξ is defined by

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \ge r\}dr - \int_{-\infty}^{0} \mathcal{M}\{\xi \le r\}dr$$

provided that at least one of the two integrals is finite. And the *variance* of ξ is

$$V[\xi] = E[(\xi - E[\xi])^2].$$

The uncertain variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be *independent* if

$$\mathcal{M}\{\bigcap_{i=1}^{m}\{\xi_i \in B_i\}\} = \min_{1\le i\le m} \mathcal{M}\{\xi_i \in B_i\},$$

for any Borel set $B_1, B_2, \ldots, B_m$ of real numbers.

Definition 2.1. (Liu [17]) Let ξ be an uncertain variable, and $\alpha \in (0, 1]$. Then $\xi_{\sup}(\alpha) = \sup\{r \mid \mathcal{M}\{\xi \ge r\} \ge \alpha\}$ is called the α-optimistic value to ξ; and $\xi_{\inf}(\alpha) = \inf\{r \mid \mathcal{M}\{\xi \le r\} \ge \alpha\}$ is called the α-pessimistic value to ξ.

Theorem 2.1. *(Liu [17, 21]) Let ξ and η be are independent uncertain variables and $\alpha \in (0, 1]$. Then we have*

(i) *if $c \ge 0$, then $(c\xi)_{\sup}(\alpha) = c\xi_{\sup}(\alpha)$ and $(c\xi)_{\inf}(\alpha) = c\xi_{\inf}(\alpha)$;*
(ii) *if $c < 0$, then $(c\xi)_{\sup}(\alpha) = c\xi_{\inf}(\alpha)$ and $(c\xi)_{\inf}(\alpha) = c\xi_{\sup}(\alpha)$;*
(iii) *$(\xi + \eta)_{\sup}(\alpha) = \xi_{\sup}(\alpha) + \eta_{\sup}(\alpha)$, $(\xi + \eta)_{\inf}(\alpha) = \xi_{\inf}(\alpha) + \eta_{\inf}(\alpha)$.*

Definition 2.2. (Deng and Zhu [10]) An uncertain process V_t is said to be a V jump process with parameters r_1 and r_2 $(0 < r_1 < r_2 < 1)$ for $t \ge 0$ if (i) $V_0 = 0$, (ii) V_t has stationary and independent increments, (iii) every increment $V_{s+t} - V_s$ is a Z jump uncertain variable $Z(r_1, r_2, t)$.

Let V_t be a V jump uncertain process, and $\eta = \Delta V_t = V_{t+\Delta t} - V_t$. Then for any $\alpha \in (0, 1)$, it follows from the definition of α-optimistic value and α-pessimistic value that

$$\eta_{\sup}(\alpha) = \begin{cases} \left(1 - \dfrac{\alpha}{2(1-r_2)}\right)\Delta t, & \text{if } 0 < \alpha < 1 - r_2 \\ \dfrac{\Delta t}{2}, & \text{if } 1 - r_2 \le \alpha < 1 - r_1 \\ \dfrac{1-\alpha}{2r_1}\Delta t, & \text{if } 1 - r_1 \le \alpha < 1, \end{cases} \tag{2.1}$$