European Journal of Control 000 (2017) 1–10

Contents lists available at ScienceDirect

European Journal of Control

journal homepage: www.elsevier.com/locate/ejcon

Anti-windup design for input-coupled double integrator systems with application to quadrotor UAV's

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ARTICLE INFO

Article history:
Received 24 May 2016
Revised 28 April 2017
Accepted 21 July 2017
Available online xxx

Keywords:
Actuator saturation
Aerospace applications
Antiwindup (AW) compensator design
Nonlinear control
Quadrotor unmanned air vehicles (UAVs)

ABSTRACT

This paper describes the development of an anti-windup scheme for systems which consist of a parallel set of double integrators preceded by a static coupling element and a saturation nonlinearity. A class of anti-windup compensators are proposed which can guarantee global asymptotic stability of the origin of the closed-loop system. Simple linear-like guidelines for choosing the anti-windup compensator parameters are also given. The anti-windup compensator designs are evaluated on a quadrotor unmanned aerial vehicle. Simulation results and flight tests are presented to demonstrate the effectiveness of this approach.

1. Introduction

The double integrator is a fundamental system in control theory and has attracted great interest from control engineers. Double integrators arise in a great many applications, from mechanical systems (satellites, rigid body motion etc.) to behaviour of agents in network controlled systems [22,31].

Saturated double integrators have also been of interest to researchers studying saturated systems since, it transpires, that they can be globally stabilised by linear feedback control; saturated triple integrators cannot. It suffices to say that saturated control of the double integrator has been studied extensively and many techniques have been reported in literature [11,15,23,34]. Typically however, these studies have been devoted to simple double integrator systems; the class of systems covered in this paper have, to the authors knowledge, not been studied

This paper describes the development of anti-windup compensators for systems which comprise a number of parallel double integrators, preceded by a matrix which effectively introduces coupling between the input channels; preceding this matrix is the saturation nonlinearity which models the actuator constraints–see Fig. 1. It is of course possible to use quite standard anti-windup techniques to tackle input saturation in such classes of system and the reader is referred to [3,7,9,10,12,16,24–27,30,32,33] and references therein for details of these. Furthermore, some specific anti-windup techniques for a class of system which includes that considered here have also been recently proposed by the authors [18,20].

Many of these techniques provide stability and performance guarantees for the entire nonlinear system by solving a set of linear matrix inequalities (LMIs) [13,26]. However, for a system of the structure depicted in Fig. 1, one would expect that a simpler method could be used to produce a suitable AW compensator. Furthermore, the use of LMIs, would normally generate one "*optimal*" solution whereas there may exist other solutions which might yield an AW compensator with a satisfactory performance. Also, LMI methods typically focus on the $\mathcal{L}_2$ gain as a performance measure meaning, effectively, that the performance is bounded from above by an affine function of the input energy. However, it may, in fact, not be an adequate measure of the nonlinear system's performance given the fact that the output energy may scale in a nonlinear way with the input energy in the nonlinear system [6].

The aim of this paper is to develop a globally stabilising anti-windup (AW) scheme for the class of systems described in Fig. 1 using the architecture introduced in [30]. The compensators will be parametrised by a state-feedback matrix which is constructed using some intuitive "linear-like" rules and not the $\mathcal{L}_2$ gain conditions which often prevail in anti-windup synthesis. The approach taken uses the method in [28] which uses a Lure–Postnikov Lyapunov function to generate a Popov-like sufficient condition to guarantee closed-loop global stability for the system.

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http://dx.doi.org/10.1016/j.ejcon.2017.07.002
0947-3580/

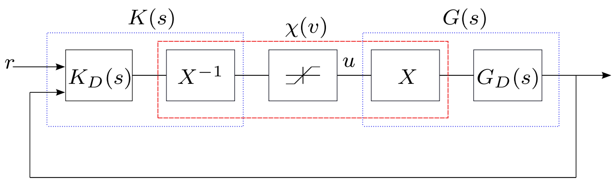

Fig. 1. System under consideration.

This solution provides a very large set of stabilising anti-windup compensators for the constrained input-coupled double integrator system. To choose a particular compensator from this set, we advocate the use of formulae based on a simple linear approximation of the compensator's dynamics. This approach provides a rapid, transparent method for anti-windup design and re-design and the offers a level of simplicity and flexibility that will be highly appreciated in a practical environment.

The paper is organised as follows. Section 2 describes the class of systems under consideration and briefly introduces the anti-windup architecture used. Section 3 describes the main results of the paper, including the anti-windup design approach and the tuning rules. Section 4 introduces the quadrotor platform and presents both simulated and experimental results, and the final section gives a brief conclusion.

Notation: The saturation function is defined as $\text{sat}(.) : \mathbb{R}^m \longmapsto \mathbb{R}^m$ for $u = [u_1, \ldots, u_m]$ and $u_i > 0$, $i \in I[1, m]$ such that

$$\text{sat}(u) = [\text{sat}(u_1), \ldots, \text{sat}(u_m)]^T$$
$$\text{sat}(u_i) = \min\{|u_i|, \bar{u}_i\} \times \text{sign}(u_i)$$

The deadzone function $\text{Dz}(.) : \mathbb{R}^m \longmapsto \mathbb{R}^m$ is simply

$$\text{Dz}(u) = [\text{Dz}(u_1), \ldots, \text{Dz}(u_m)]^T = u - \text{sat}(u) \tag{1}$$

For brevity, we denote $\tilde{u} = \text{Dz}(u)$, the notation $\text{He}\{A\} = A + A^T$.

$\mathbf{P}^m$ set of $m \times m$ symmetric positive-definite matrices.
$\mathbf{N}^m$ set of $m \times m$ symmetric non-negative definite matrices.
$\mathbf{D}$ set of diagonal matrices.

2. Systems under consideration

2.1. The nominal system

The system under configuration is depicted in Fig. 1 where $G(s)$ is the nominal plant and $K(s)$ is the controller. The reference signal is $r(t)$, the measurement $y(t)$ and the controller demand $u(t)$. The plant belongs to the family of input-coupled systems with transfer function matrix

$$G(s) = G_D(s)X \tag{2}$$

where $G_D(s)$ has a block-diagonal structure

$$G_D(s) = \text{blockdiag}(G_1(s), G_2(s), \ldots, G_m(s)) \tag{3}$$

and $X \in \mathbb{R}^{m\times m}$ is a non-singular matrix. Each element in $G_D(s)$ has double integrator dynamics. $G_D(s)$ has the state-space realisation

$$G_D(s) \sim \left[\begin{array}{c|c} A_D & B_D \\ C_D & D_D \end{array}\right] \tag{4}$$

where

$$A_D = \text{blockdiag}(A, A, A, \ldots, A) \in \mathbb{R}^{2m\times 2m} \tag{5}$$

$$B_D = \text{blockdiag}(B, B, B, \ldots, B) \in \mathbb{R}^{2m\times m} \tag{6}$$

$$C_D = \text{blockdiag}(C_1, C_2, C_3, \ldots, C_m)\mathbb{R}^{p\times 2m} \tag{7}$$

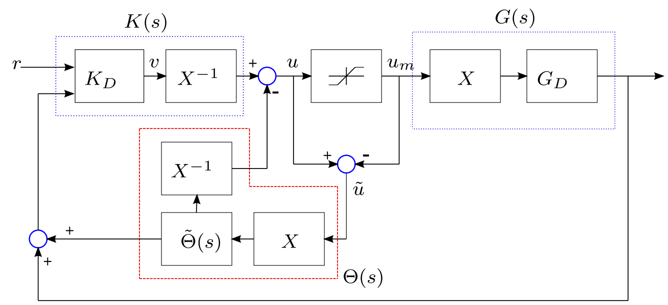

Fig. 2. Input-coupled system with structured anti-windup.

$$D_D = 0 \tag{8}$$

The matrices C_i are not restricted to have a particular structure apart from the fact that (C_i, A) should be detectable for all $i \in \{1, \ldots, m\}$. The controller $K(s)$ has the form

$$K(s) = X^{-1}K_D(s) \tag{9}$$

where $K_D(s)$ has a block diagonal structure compatible with that of $G_D(s)$, viz

$$K_D(s) = \text{blockdiag}(K_1(s), K_2(s), \ldots, K_m(s)) \tag{10}$$

In the absence of saturation, it is assumed that the controller $K(s)$ internally stabilises $G(s)$ and ensures the system exhibits good performance. This is equivalent to $K_D(s)$ internally stabilising $G_D(s)$ and, due to their block diagonal structure, this is equivalent to each $K_i(s)$ interally stabilising $G_i(s)$ for all $i \in \{1, \ldots, m\}$. Thus each $K_i(s)$ can be designed purely on the basis of $G_i(s)$.

When saturation is absent, here is no coupling between the i channels because the nonlinearity (see Fig. 1).

$$\chi(v) = X\text{sat}(X^{-1}v) \tag{11}$$

is simply the identity operator. However, when saturation is present i.e $\chi(v) \neq v$, the saturation element causes some nonlinear coupling between the system's m control loops and, unless X is diagonal, the decoupling offered by the nominal controller (10) is lost. This coupling is a well known trigger for performance deterioration and instability [1].

2.2. Anti-windup compensator architecture

While a number of different anti-windup approaches [9] could be used to tackle the saturation problem described above, these would typically produce an *unstructured* anti-windup compensator. This section introduces a specially structured anti-windup compensator which exploits the structure of the plant (2) and controller (9). The approach used is based on that introduced in [18,20].

Consider the system depicted in Fig. 2 where $K(s)$ and $G(s)$ are the structured plant and controller described in Eqs. (2), (9), (4) and (10). Again, $r(t)$ is the reference, $y(t)$ the output and $u(t)$ the physical control input. Also shown is $v(t)$ which can be considered as the *virtual* control input and the anti-windup compensator $\Theta(s)$. The plant has a state-space realisation

$$G(s) \sim \begin{cases} \dot{x} = A_D x + B_D X u_m \\ y = C_D x \end{cases} \tag{12}$$

where $u_m = sat(u)$ and the controller has the realisation

$$K(s) \sim \begin{cases} \dot{x}_c = A_c x_c + B_{cr} r + B_{cy}(y + y_d) \\ v = C_c x_c + D_{cr} r + D_{cy}(y + y_d) \\ u = X^{-1}(v - v_d) \end{cases} \tag{13}$$