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Output feedback stabilization for multi-dimensional Kirchhoff plate with general corrupted boundary observation ☆

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ABSTRACT

We consider boundary output feedback stabilization for a multi-dimensional Kirchhoff plate with boundary observation suffered from a general external disturbance. We adopt for the first time the active disturbance rejection control approach to stabilization of multi-dimensional partial differential equations under corrupted output feedback. In terms of this approach, the disturbance is estimated by a relatively independent estimator, based on (possibly) an infinite number of ordinary differential equations reduced from the original PDEs by infinitely many time-dependent test functions. This gives a state observer, an additional result via this approach. The disturbance is compensated in the feedback-loop. As a result, the control law can be designed almost as the same as that for the system without disturbance. We show that with a time varying gain properly designed, the observer driven by the disturbance estimator is convergent; and that all subsystems in the closed-loop are asymptotically stable. We also provide numerical simulations which demonstrate the convergence results and underline the effect of the time varying high gain estimator.

1. Introduction

In the past three decades, many control approaches have been developed to cope with uncertainty or external disturbance in control systems. These include the internal model principle for output regulation to deal with external disturbance; the robust control for systems with uncertainties; the sliding mode control for system with internal and/or external disturbance; and the adaptive control for systems with unknown parameters, to name just a few. Most of these approaches are based on the idea of "worst case concern", and the control strategies designed are usually over-conservative. This results in excessive control efforts (or using very large control energy), which is often unnecessary for particular systems.

The active disturbance rejection control (ADRC) proposed by Han, however, adopts a quite different way, see, for instance, [12] for a nice survey. One of the remarkable features of ADRC is that the disturbance is first estimated in real time through an extended state observer [9], and it is then compensated (canceled) in the feedback loop. Because of this estimation/ cancelation nature, the control energy can be significantly reduced [25] in the closed-loop system. Furthermore, it has been proved that the ADRC is capable of dealing with very complicated uncertainties and disturbances; including coupling of the external disturbance, the system unmodeled dynamics, and the superadded unknown part of control input. The convergence of ADRC for general nonlinear lumped parameter systems is available recently in [9,10]. The generalization of ADRC to the systems described by partial differential equations (PDEs) are reported in our recent works [6–8]. Note that the disturbance considered in [6–8] is only found in the control channel. Other interesting studies on stabilization of unstable systems or even anti-stable PDEs were previously investigated in [20] where it introduces a non-adaptive design for wave equation with boundary anti-damping; [21] where it introduces the design for wave equation with in-domain anti-damping; and [15] where it introduces adaptive design for the first time for handling the parabolic PDEs with disturbance and anti-damping. In recent study [2], the idea of ADRC is applied to an observer based output feedback stabilization for a one-dimensional wave equation considered in [5] but with general boundary external disturbance. However, since the system in [2] is one-dimensional, the disturbance depends on time only.

In this paper, we generalize [2] to a multi-dimensional PDE. We consider boundary stabilization of the following multi-dimensional Kirchhoff equation with Neumann boundary control

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and observation, where the observation is suffered from an external disturbance:

$$\begin{cases} w_{tt}(x,t)-\gamma\Delta w_{tt}(x,t)+\Delta^2 w(x,t)=0, \quad x\in\Omega,\ t>0,\\ w(x,t)|_\Gamma=0, \quad t\ge 0,\\ \Delta w(x,t)|_\Gamma=u(x,t), \quad t\ge 0,\\ w(x,0)=w_0(x), w_t(x,0)=w_1(x), \quad x\in\Omega,\\ Y_{out}(x,t)=\{Y_{1,out}(x,t),Y_{2,out}(x,t)\}\\ \quad =\left\{\dfrac{\partial w(x,t)}{\partial\nu},\dfrac{\partial w_t(x,t)}{\partial\nu}+d(x,t)\right\}, \quad x\in\Gamma,\ t\ge 0, \end{cases} \tag{1.1}$$

where $\Omega\subset\mathbb{R}^n (n\ge 2)$ is a bounded domain with smooth C^4-boundary $\Gamma\partial\Omega$, $\Delta=\sum_{i=1}^n\frac{\partial^2}{\partial x_i^2}$ is the standard Laplacian, $\Delta^2=\sum_{i=1}^n\frac{\partial^2}{\partial x_i^2}\sum_{j=1}^n\frac{\partial^2}{\partial x_j^2}$, ν is the unit normal vector of Γ pointing to the exterior of Ω, $\gamma>0$ is usually a small number by which the Euler–Bernoulli plate is the limit case of the Kirchhoff plate as $\gamma\to 0$, $u(x,t)$ is the control input, $(w_0(x),w_1(x))$ is the initial value, and $d(t)$ is the unknown external disturbance which is supposed to satisfy that

$$d\in L^\infty(0,\infty;C(\Gamma))\cap C(0,\infty;C(\Gamma)), d_t\in L^\infty(0,\infty;C(\Gamma)). \tag{1.2}$$

The Kirchhoff plate is originally a two-dimensional mathematical model used to determine the stresses and deformations in thin plates subjected to forces and moments. This theory is an extension of the Euler–Bernoulli plate beam theory. The theory assumes that a mid-surface plane can be used to represent a three-dimensional plate in two-dimensional form. In one-dimensional case, it reduces to the Rayleigh beam by adding the rotary inertia effects to the Euler–Bernoulli beam [13].

We consider system (1.1) in the energy Hilbert state space $\mathcal{H}=(H^2(\Omega)\cap H_0^1(\Omega))\times H_0^1(\Omega)$ and the control space $U=L^2(\Gamma)$. Throughout the paper, w_t or $\dot{w}$ denotes the derivative of w respect to t which is clear from the context. Let us first explain why we choose such a type of output $Y_{out}(x,t)$. There are basically two reasons for this. First, when there is no disturbance, the velocity feedback

$$u(x,t)=-kY_{2,out}(x,t),\ \ k>0, \tag{1.3}$$

stabilizes exponentially system (1.1) provided that the following geometrical condition on Ω is satisfied [16]:

$$\begin{cases} \text{(i)} & \text{There exists a vector field } h(x) \text{ defined on } \Omega \text{ such that}\\ & h(\sigma)=p(\sigma)\nu(\sigma) \text{ for a smooth } p(\sigma),\ \sigma\in\Gamma;\\ \text{(ii)} & \text{For some constant } \rho>0 \text{ and all vectors } y\in(L^2(\Omega))^n:\\ & \int_\Omega H(x)y(x)\cdot y(x)\,dx\ge\rho\int_\Omega|y(x)|^2\,dx \text{ where } H(x)=\{\partial h_i/\partial x_j\}_{i,j=1}^n. \end{cases} \tag{1.4}$$

For $n=2$, condition (1.4) can be removed [14]. So throughout the paper, (1.4) is always assumed except $n=2$. Second, the mapping $w\to Y_{1,out}$ from $H^2(\Omega)$ to $L^2(\Gamma)$ is a compact mapping. The output feedback by $Y_{1,out}(x,t)$ only cannot stabilize exponentially system (1.1) at least through linear feedback when there is no disturbance (see, e.g., [3]). In fact, the observation $Y_{1,out}(x,t)$ is not an exact observable output for system (1.1) and hence is hard to be used to design an observer. To see this point, we introduce a positive self-adjoint operator A in $L^2(\Omega)$, defined by

$$A\phi=\Delta^2\phi,\ D(A)=\{\phi\in H^4(\Omega)|\,\phi|_\Gamma=\Delta\phi|_\Gamma=0\}. \tag{1.5}$$

By interpolation result in Theorem 11.6, Chapter 1 of [18], we have the following space identification:

$$\begin{cases} D(A^\theta)=H^{4\theta}(\Omega)\cap H_0^1(\Omega),\ \frac{1}{8}<\theta<\frac{5}{8},\\ D(A^\theta)=\left\{\phi\in H^4(\Omega)|\,\phi|_\Gamma=\Delta\phi|_\Gamma=0\right\},\ \frac{5}{8}<\theta\le 1, \end{cases} \tag{1.6}$$

which gives

$$\begin{cases} A^{1/2}\phi=-\Delta\phi,\ D(A^{1/2})=H^2(\Omega)\cap H_0^1(\Omega),\\ D(A^{1/4})=H_0^1(\Omega). \end{cases} \tag{1.7}$$

We endow $D(A^{1/4})=H_0^1(\Omega)$ with the following equivalent inner product induced norm:

$$\|\psi\|^2_{D(A^{1/4})}=\|\psi\|^2_{L^2(\Omega)}+\gamma\|\nabla\psi\|^2_{L^2(\Omega)}=\|(1+\gamma A^{1/2})^{1/2}\psi\|^2_{L^2(\Omega)}. \tag{1.8}$$

By this relation, the state space becomes $\mathcal{H}=D(A^{1/2})\times D(A^{1/4})$ and the inner product induced norm in $\mathcal{H}$ is given by

$$\begin{aligned}\|(f,g)^\top\|^2&=\|\Delta f\|^2_{L^2(\Omega)}+\|(1+\gamma A^{1/2})^{1/2}g\|^2_{L^2(\Omega)}\\ &=\|\Delta f\|^2_{L^2(\Omega)}+\|g\|^2_{L^2(\Omega)}+\gamma\|\nabla g\|^2_{L^2(\Omega)}, \quad \forall (f,g)^\top\in\mathcal{H}.\end{aligned}$$

Let $\mu_m>0$, $m=1,2,\ldots$, be the eigenvalues of operator A; $\mu_m\to\infty$ as $m\to\infty$; and let ϕ_m be the eigenfunction corresponding to λ_m with $\|\phi_m\|_{H_0^1(\Omega)}=1$. A simple calculation shows that $(\lambda_m=\mu_m/(1+\gamma\sqrt{\mu_m}),\phi_m)$ is an eigen-pair of $\mathscr{A}$ defined by

$$\mathscr{A}=(1+\gamma A^{1/2})^{-1}A, \quad D(\mathscr{A})=D(A). \tag{1.9}$$

It is seen that for any $m\ge 1$,

$$(w^m(x,t),w_t^m(x,t))=e^{i\sqrt{\lambda_m}t}\left(\frac{\phi_m(x)}{\sqrt{\lambda_m}},\ i\phi_m(x)\right) \tag{1.10}$$

is a solution of the following equation:

$$\begin{cases} w_{tt}^m(x,t)-\gamma\Delta w_{tt}^m(x,t)+\Delta^2 w^m(x,t)=0,\\ w^m(x,t)|_\Gamma=0,\\ \Delta w^m(x,t)|_\Gamma=0. \end{cases} \tag{1.11}$$

Since from (1.11),

$$\lambda_m[\|\phi_m\|^2_{L^2(\Omega)}+\gamma\|\nabla\phi_m\|^2_{L^2(\Omega)}]=\|\Delta\phi_m\|^2_{L^2(\Omega)}, \tag{1.12}$$

we have

$$\|(w^m(\cdot,0),w_t^m(\cdot,0))\|^2_{\mathcal{H}}=\left\|\left(\frac{\phi_m}{\sqrt{\lambda_m}},\phi_m\right)\right\|^2_{\mathcal{H}}=2, \quad m=1,2,\ldots. \tag{1.13}$$

However, by the Sobolev trace theorem and (1.6), for any $T>0$, there exists a constant $C>0$ independent of m such that

$$\begin{aligned}\int_0^T\left\|\frac{\partial w^m(\cdot,t)}{\partial\nu}\right\|^2_{L^2(\Gamma)}dt&=\frac{T}{\lambda_m}\left\|\frac{\partial\phi_m}{\partial\nu}\right\|^2_{L^2(\Gamma)}\le\frac{CT}{\lambda_m}\left\|\phi_m\right\|^2_{H^{3/2}(\Omega)}\le\frac{CT}{\lambda_m}\left\|A^{3/8}\phi_m\right\|^2_{L^2(\Omega)}\\ &=\frac{CT\mu_m^{1/4}}{\lambda_m}\left\|A^{1/4}\phi_m\right\|^2_{L^2(\Omega)}=\frac{CT(1+\gamma\sqrt{\mu_m})}{\mu_m^{3/4}}\left\|\phi_m\right\|^2_{H_0^1(\Omega)}\to 0 \text{ as } m\to\infty.\end{aligned} \tag{1.14}$$

By (1.13) and (1.14), the "observability inequality"

$$\int_0^T\left\|Y_{1,out}(\cdot,t)\right\|^2_{L^2(\Gamma)}dt=\int_0^T\left\|\frac{\partial w(\cdot,t)}{\partial\nu}\right\|^2_{L^2(\Gamma)}dt\ge C_T\|(w(\cdot,0),w_t(\cdot,0))\|^2_{\mathcal{H}} \tag{1.15}$$

does not hold for any $T>0$ and $C_T>0$.

Nevertheless, we still need the signal $Y_{1,out}(x,t)=\frac{\partial w(x,t)}{\partial\nu}\Big|_\Gamma$ for the purpose of output feedback stabilization. This is because if we come across the disturbance such that $d(x,t)=-\frac{\partial w_t(x,t)}{\partial\nu}\Big|_\Gamma$, then $Y_{2,out}(x,t)\equiv 0$. For this case, we are not able to stabilize the system without $Y_{1,out}(x,t)$.

The contributions of this paper lie in the following: (a) we design an adaptive (x,t)-dependent external disturbance estimator while in existing literature, only time dependent estimator is available; (b) we provide actually a numerical scheme by which for any fixed time, only a finite number of ODEs to be solved to get the