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Theoretical modelling of natural gas unsteady flow rate measurement using variable differential pressure method

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ABSTRACT

Variable differential pressure flowmeters are very often used in many industries. Therefore, within the framework of this work, this method is adapted for measuring the flow and quantity of natural gas using the theory of similarity approaches. The article presents a simple method of obtaining analytical correlations for the natural gas flow rate measurement, and produces an equation for natural gas expansion factor. It also investigates the limits of applicability of the standard equation for the steady flow expansion factor. The results of an analysis of the errors in gas flow rate measurement, without being taken into account by the Strouhal number, in different pipe sections and flow transducers, have been presented.

1. Introduction

Flow rate measurement is an important procedure for organizing various technological processes. Measuring the flow rate and the quantity of steady fluid and gas flows is a fairly well-solved problem, where an acceptable level of measurement error is achieved. But in various production processes when pumping oil or natural gas, where pump and compressor stations are available, there is always an unsteady fluid and gas flow. In this case, the methods that are designed for steady flows perform the flow rate measurement with large errors. Therefore, unsteady fluid and gas flow rate measurement is also an important issue. There is a large number of theoretical and experimental works that solve the problem of increasing the accuracy and reliability of pulsating flow rate and quantity measurement. Such works include studies [1–3], which describe the theoretical aspects of improving the measurement techniques and experimental works. There are papers that generalize previously obtained results like the type of article [4]. Pulsating flows are important for applied industries. For example, in the work [5], the authors also attempt to describe the motion of pulsating flows mathematically, then to check the theoretical calculations by experiment. The work [6] theoretically describes the behavior of viscous pulsating flows. One of the latest works, which outlines the attempts to improve methods of pulsating flow rate measurement, is the study [7]. In paper [8] the author gives damping criteria for pulsating gas flow measurement. The influence of pulsations strongly affects the correct operation of many flowmeters. The influence of pulsations on the flow measurement by the method of variable pressure drop is described in [9,10], and in [11] the effect of pulsations on the flow is also described and a model of such a flow is constructed. Many models and experimental works connected with the study of unsteady flows of liquids and gases are associated with the pulsation frequency that characterizes such flows.

It should be taken into account that the variable differential pressure method, which is often used for natural gas flow rate measurement [12], is widely used. In this regard, methods for measuring the flow rate and quantity of natural gas require improvement for use in pulsating flows.

The analysis of existing works shows the complexity of models and approaches in describing the flow of pulsating approaches, thereby complicating the process model of measuring the flow and quantity of fluids and gases. Some works have no direct connection with the existing equations of hydrodynamics that describe the fluid and gas flow, but introduce correction factors. It should also be noted that in almost all studies there is a direct relationship with the Strouhal number or, in case of viscous fluids, the Womersley number. Therefore, within the framework of this paper, the task is to obtain a simple model for the method of natural gas pulsating flow rate measurement. The problem is solved using methods of theory of similarity and dimensional theory, which is often used in hydrodynamics.

2. Theoretical bases necessary for modelling a pulsating flow

To obtain simple correlations, the purpose of which is to measure the pulsating natural gas flow rate, it is necessary to reconstruct the equations of fluid and gas flow. For this, the methods of theory of similarity are used. Thus, the author solved the problem of modelling the

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process of measurement of pulsating incompressible fluid flow in [13], the approach to the solution was previously described in paper of the author [14].

Therefore, on the one hand, the methods of theory of similarity are used to modify the equations of hydrodynamics in accordance with [15]. On the other hand, we shall leave the Strouhal number as the main parameter that describes the pulsations in the gas flow.

In accordance with [16], for one dimensional flow of pulsating ideal fluid, whose density is assumed to be constant within the framework of the problem, we shall write the Euler equation as follows:

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \tag{1}$$

where $V-$ fluid or gas velocity, $t-$ time, $\rho-$ fluid density, $p-$ fluid pressure in the pipe, $x-$ axis of movement.

Taking into account that the equation is one-dimensional in the given model, it is more convenient to use the total derivatives instead of partial derivatives Then, divide Eq. (1) into convective component of velocity derivative. Upon canceling out such values we have the following equation:

$$\frac{dx}{Vdt} + 1 = -\frac{1}{\rho}\frac{dp}{VdV}. \tag{2}$$

Substitute the differential dx in the last formula with resulting characteristic length L of channel, and time differential dt with the value contrary to the finite interval of time, i.e. frequency of flow pulsations $1/f$, afterward the formula (2) will be as follows:

$$\left(\frac{Lf}{V} + 1\right)VdV = -\frac{dp}{\rho}. \tag{3}$$

Expand the brackets and integrate formula (3), whereupon we will have the following formula:

$$\frac{V^2}{2}\left(2\frac{Lf}{V} + 1\right) + \frac{p}{\rho} = const. \tag{4}$$

The $Sh = \frac{Lf}{V}$ formula in Eq. (4) represents Strouhal number and will be the measure of ideal fluid pulsating flow unsteadiness.

Thus we shall finally rewrite Eq. (4) as solution of Eq. (1) for pulsating flow as follows:

$$\frac{V^2}{2}(2Sh + 1) + \frac{p}{\rho} = const. \tag{5}$$

Similarly, we can modify the equation of continuity of the flow, which will be as follows:

$$j(Sh + 1) = const, \tag{6}$$

where formula $j = \rho V$ is called gas or fluid flow density.

If $Sh < < 1$ or $Sh \to 0$, Eqs. (5) and (6) shall become a standard Bernoulli's equation for continuity of steady flow.

To model the processes of gas flow, we need to write Eq. (5) for the case of adiabatic flow. This will be achieved by integrating Eq. (3) with the following equation for adiabatic gas flow [17]:

$$p/\rho^{\gamma} = const,$$

where $\gamma-$ is a natural gas adiabatic exponent.

The final equation for the gas flow is obtained in the following form:

$$(2Sh + 1)\frac{V^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p}{\rho} = const. \tag{7}$$

The last equation contains the Strouhal number, which characterizes the pulsating flow and can be a quantitative measure of a similar process of adiabatic gas flow.

Thus, the theory of similarity methods allow to obtain simple approaches for describing correlations for a pulsating fluid or gas flow that are easier to apply for modelling measurement procedures. Therefore, using Eqs. (6) and (7), we simulate the method of pulsating gas flow rate measurement for the variable differential pressure method.

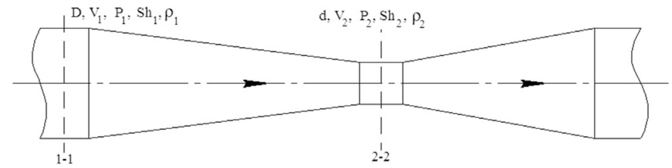

Fig. 1. Diagram of pulsating gas flow in a Venturi tube.

3. Unsteady natural gas flow rate measurement using a Venturi tube

In accordance with the traditional approach, which is described in many studies, as in [17], it is necessary to formulate equations for obtaining correlations in the pulsating flow of adiabatic gas according to Eqs. (6) and (7). We assume that in each measured cross-section there is a set of parameters of the pulsating gas, which includes the pipe diameter, gas velocity, pressure, Strouhal number and gas density. Hydraulic losses in this task are not taken into account because the purpose of our study is not the discharge coefficient, but the expansion factor for natural gas.

Fig. 1 shows a diagram of pulsating gas flow through a Venturi tube. In accordance with Fig. 1, we shall formulate the following equations for Sections 1–1 and 2–2 in the figure:

$$(2Sh_1 + 1)\frac{V_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_1}{\rho_1} = (2Sh_2 + 1)\frac{V_2^2}{2} + \frac{\gamma}{\gamma - 1}\frac{p_2}{\rho_2}, \tag{8}$$

$$(Sh_1 + 1)\rho_1 V_1\frac{\pi D^2}{4} = (Sh_2 + 1)\rho_2 V_2\frac{\pi d^2}{4}. \tag{9}$$

It is known that for density and pressure in an adiabatic gas there is the following expression [17]:

$$\frac{\rho_2}{\rho_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \tau^{\frac{1}{\gamma}}. \tag{10}$$

Let us express the flow velocity in the first Sections (1–1) in terms of the velocity in the second section using Eqs. (9) and (10):

$$V_1 = \tau^{\frac{1}{\gamma}}\beta^2 k_{Sh} V_2, \tag{11}$$

where $\beta = d/D-$ is a traditional diameter ratio, $k_{Sh} = \frac{Sh_2 + 1}{Sh_1 + 1}-$ is a relative coefficient associated with pulsations in the sections of the Venturi tube.

Taking into account the pulsating nature of Eqs. (6) and (7), the expression for the velocity of approach factor will also vary [13]:

$$E = \frac{\rho V_2^2}{2\Delta p} = \frac{\sqrt{(1 + 2Sh_2)^{-1}}}{\sqrt{1 - \beta^4 k_{Sh}^2 k_o}}, \tag{12}$$

where $\Delta p = p_1 - p_2-$ is differential pressure in Venturi tube, $k_o = \frac{2Sh_1 + 1}{2Sh_2 + 1}-$ is a coefficient depending also on the Strouhal numbers.

As a result, the simultaneous solution of Eqs. (8), (9), (11), (12) allows us to find the equation for the consumption of natural gas as follows:

$$Q = E\frac{\pi d^2}{4}\sqrt{2\rho_1\Delta p}\sqrt{\frac{\gamma}{\gamma - 1}\left(\frac{1 - \tau^{\frac{\gamma - 1}{\gamma}}}{1 - \tau}\right)\frac{(1 - \beta^4 k_{Sh}^2 k_o)\tau^{\frac{2}{\gamma}}}{1 - \beta^4 k_{Sh}^2 k_o \tau^{\frac{2}{\gamma}}}}. \tag{13}$$

In the last Eq. (13) we introduce the accepted standard notations from [17], and finally rewrite it as follows:

$$Q = E\frac{\pi d^2}{4}\sqrt{2\rho_1\Delta p}\,\varepsilon, \tag{14}$$

where $\varepsilon-$ is a gas expansion factor beyond a Venturi tube, which is