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# Calculation method for conjugate depths in quadratic parabolic channels

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ABSTRACT

Quadratic parabolic channels are widely used in irrigation and drainage engineering projects. Up until today there are no an effective method for computing the conjugate depths of quadratic parabolic channels. In this paper, calculation method for calculating the conjugate depths of quadratic parabolic channels was studied. Dimensionless depths were selected to simplify momentum equation for determining conjugate depths. Iterative equation for downstream depth was developed by suitable mathematical transformation on the momentum equation. If one knows dimensionless upstream depth, one can obtain dimensionless downstream depth by using iterative equation. Based on a large number of data of dimensionless conjugate depths, explicit equations with simple form for conjugate depths of quadratic parabolic channels were obtained by running a MATLAB program. Relative error analysis indicates that the proposed explicit equations which are employed to calculate conjugate depths (downstream depth and upstream depth) have high accuracy and wide application range. In the practical range of downstream depth to upstream depth ratio [1, 77.33], the maximum relative error is less than 0.30%.


## 1. Introduction

Many man-made parabolic cross sections are widely used in irrigation and drainage engineering applications and middle-small hydropower stations because of their structural and hydraulic characteristics, such as easy construction, anti-frost heaving, small loss of water head along the way, and large sediment-carrying capacity. Conjugate depths can be employed to analyze and judge the position of a hydraulic jump and the connection form of flow downstream discharge structures. Also, conjugate depths are used in the design of energy dissipation downstream flow discharge dams and discharge sluices. Since the governing equation for conjugate depths is high-order equation, there are no simple and accurate calculation methods at present. There exist some calculation methods for conjugate depths such as trial and error method, iterative method, and explicit equations with various degrees of errors. Iterative equations and explicit equations usually use two formulations to compute conjugate depths (depths upstream and downstream of a hydraulic jump). Ma et al. developed an iterative method without dimensionless variables which need lots of iterations [1]. Leng et al gained a relatively simple form of calculation formula by optimal iterative initial value and iterative calculation, but the scopes of application are small [2]. Based on GA algorithm and 1stop program, Fu et al. obtained initial value and highly exact values can be obtained after only iteration, however, the explicit equations for upstream and downstream depths are both complex [3]. Vatankhah and Valiani also developed complex explicit equations for conjugate depths [4–6].

In order to obtained explicit equations with simple forms for conjugate depths of parabolic channels, dimensionless depth was introduced to develop calculation equation for Froude number which can be used to judge the type of a hydraulic jump. Iterative formula was obtained by making transformation on the governing equation for conjugate depths. The proposed iterative formula was used to obtain the dimensionless depth downstream of a hydraulic jump at the given dimensionless depth upstream of a hydraulic jump. Based on a MATLAB program, explicit equations for the conjugate depths of quadratic parabolic channels were developed by the data of conjugate depths calculated by the proposed iterative formula.

## 2. The related parameters of parabolic channels

Assuming the equation for a quadratic parabola as follow:

$$y = ax^2 (a > 0) \tag{1}$$

where $a$ is the shape parameter of a parabola.

From Eq. (1), it is easy to derive the related parameters of a parabolic cross section as follows.

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**Nomenclature**

$a$ Shape parameter of a parabola
$A_1$ and $A_2$ Cross-sectional areas upstream and downstream of a hydraulic jump, respectively
$A_k$ Cross-sectional area corresponding to the critical depth
$B_1$ and $B_2$ Top widths at the water surface upstream and downstream of a hydraulic jump, respectively
$B_k$ Top width at the water surface corresponding to the critical depth
$F_{r1}$ and $F_{r2}$ Froude numbers upstream and downstream of a hydraulic jump, respectively
$g$ Gravitational acceleration
$h_1$ and $h_2$ Flow depths upstream and downstream of a hydraulic jump, respectively
$h_{c1}$ and $h_{c2}$ Water depths from the centroids of the cross-sectional areas to the water surface upstream and downstream of a hydraulic jump, respectively
$h_k$ Critical depth
$\overline{h}_1$ and $\overline{h}_2$ Average water depths upstream and downstream of a hydraulic jump, respectively
$Q$ Discharge
$x$ Dimensionless flow depth upstream of a hydraulic jump
$y$ Dimensionless flow depth downstream of a hydraulic jump
$x^*$ Numerical solution of $x$
$y^*$ Numerical solution of $y$
$\beta$ Dimensionless conjugate depth function

The cross-sectional areas are:

$$\begin{cases} A_1 = \dfrac{4{h_1}^{3/2}}{3\sqrt{a}} \\ A_2 = \dfrac{4{h_2}^{3/2}}{3\sqrt{a}} \end{cases} \tag{2}$$

in which $A_1$ and $A_2$ are cross-sectional areas upstream and downstream of a hydraulic

jump, respectively(m$^2$). $h_1$ and $h_2$ are the flow depths upstream and downstream of a hydraulic jump, respectively (m).

The water depths from the centroid of the cross-sectional area to the water surface are:

$$\begin{cases} h_{c1} = \dfrac{2}{5}h_1 \\ h_{c2} = \dfrac{2}{5}h_2 \end{cases} \tag{3}$$

where $h_{c1}$ and $h_{c2}$ are the water depths from the centroids of the cross-sectional areas to the water surface upstream and downstream of a hydraulic jump, respectively (m).

The top widths at the water surface are:

$$\begin{cases} B_1 = 2\sqrt{\dfrac{h_1}{a}} \\ B_2 = 2\sqrt{\dfrac{h_2}{a}} \end{cases} \tag{4}$$

where $B_1$ and $B_1$ are the top widths at the water surface upstream and downstream of a hydraulic jump, respectively (m).

## 3. Equations for the critical depth and Froude number of parabolic channels

### 3.1. Equation for the critical depth

The governing equation for critical depth computation is [7]:

$$\frac{Q^2}{g} = \frac{A_k^3}{B_k} \tag{5}$$

where $Q$ is the discharge (m$^3$/s), $g$ is the gravitational acceleration (m/s$^2$), $A_k$ is the cross-sectional area (m$^2$) corresponding to the critical depth, and $B_k$ is the top width (m) at the water surface corresponding to the critical depth.

Substituting Eqs. (2) and (4) into Eq. (5) results in:

$$h_k = \left[\frac{27aQ^2}{32g}\right]^{1/4} \tag{6}$$

### 3.2. Equation for Froude number of parabolic cross section

Froude number is an important parameter in hydraulics. Froude number upstream of a hydraulic jump is used to judge the form of the hydraulic jump and can be computed by as:

$$F_{r1} = \frac{Q}{A_1\sqrt{g\overline{h}_1}} \tag{7}$$

where $F_{r1}$ is Froude number upstream of a hydraulic jump, and $\overline{h}_1$ is the average water depth upstream of a hydraulic jump (m).

From Eqs. (2) and (4), we can obtain $\overline{h}_1$ as:

$$\overline{h}_1 = \frac{A_1}{B_1} = \frac{2}{3}h_1 \tag{8}$$

Substituting Eqs. (2) and (8) into Eq. (7) gives:

$$F_{r1} = \left[\frac{27aQ^2}{32g}\right]^{1/2}\cdot\frac{1}{h_1^2} \tag{9}$$

Substituting Eq. (6) into Eq. (9) yields:

$$F_{r1} = \left(\frac{h_k}{h_1}\right)^2 \tag{10}$$

Similarly, Froude number downstream of a hydraulic jump can be computed as:

$$F_{r2} = \left(\frac{h_k}{h_2}\right)^2 \tag{11}$$

Letting

$$\begin{cases} x = \dfrac{h_1}{h_k} \\ y = \dfrac{h_2}{h_k} \end{cases} \tag{12}$$

Froude numbers upstream and downstream of a hydraulic jump can be calculated as: