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Computational Aspects of Electrocardiological Inverse Solutions

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Abstract: The purpose of this work is to investigate the effect of certain numerical factors on the electrocardiographic inverse solution. Since usually the zero-order Tikhonov regularization is applied in order to solve the inverse problem, choosing the right regularization parameter is very important. The used linear equation system solver also plays an important role in the accuracy of the solution. Therefore these two factors were studied in a well defined model environment. Reference epicardial and body surface potential maps were generated in 1003 epicardial and 344 body surface points, respectively, by the Wei-Harumi model, for the QRS interval. The inverse problem has been solved with various regularization parameters and linear equation system solvers. The highest correlations (> 0.9) occurred in the first half of the QRS interval and the correlation fell below 0.6 at the end of the QRS. The result was highly dependent on the chosen regularization parameter and equation system solver, and it was significantly influenced also by the complexity of the epicardial potential distribution. The optimal solving configuration proved to be the application of the Cholesky factorization with the regularization parameter of 10^{-14}.

Keywords: Inverse Problem of Electrocardiography, Modelling, Tikhonov Regularization, Linear Equation System Solvers

1. INTRODUCTION

High spatial and temporal resolution body surface potential maps contain much more diagnostic information about the electrical activity of the heart than standard 12-lead ECG recordings. This difference is particularly significant in the case of the early acute myocardial infarction diagnosis and malignant ventricular arrhythmia risk assessment, Owens et al. (2008) and Hubley-Kozey et al. (1995).

Obviously, the available diagnostic information can be increased further, if the epicardial potential distributions were considered instead of surface maps. To obtain the epicardial potential distribution there are two possible ways: performing epicardial measurements or estimating the epicardial distribution based on high resolution body surface potential measurements by solving the inverse problem of electrocardiography. Since the former one is an invasive procedure, it is not considered to be practical in human diagnostics. Therefore, developing and improving inverse solution methods is very important.

The inverse problem of electrocardiography has been a research area since the 1980s, Yamashita (1982). However, there are several recent studies about it, proving the actuality of the topic. For example, Tyšler et al. (2014) studied the impact of the torso model on the solution, and Milanič et al. (2014) assessed regularization methods for electrocardiographic imaging. Still no study can be found in the literature about the exact optimal value of the regularization parameter related to the widely used Tikhonov regularization, Tikhonov (1977) and Shahidi et al. (1994). Similarly, the impact of linear equation system solving algorithms on the solution has not yet been studied. Therefore our aim was to investigate the achievable epicardial potential distribution quality by solving the inverse problem with the Tikhonov regularization, using various regularization parameters and linear equation system solvers.

2. MATERIALS AND METHODS

2.1 Heart and torso model

In this work, we use the body surface and epicardial electrocardiograms of the Wei-Harumi model, Wei (1995) and Wei (1997). This model has been developed by Wei et al. to simulate the body surface, epicardial and endocardial electrocardiograms based on anatomical and electrophysiological settings at a whole-heart level. It is composed of about 50,000 normal model cells such as the sinus node, atria, atrioventricular (AV) node, His-bundle (HB), bundle branch, Purkinje fiber and ventricles. Other specified model cells, known as user defined types, can be defined by modifying their electrophysiological parameters for simulating cardiac arrhythmias or ischemias. The electrophysiological parameters of model cells are associated with the action potential, conduction velocity, automaticity, and pacing. The spatial resolution of our model is 1.5 × 1.5 × 1.5 mm. Furthermore, the cell-by-cell fiber direction in the ventricle is computed to incorporate rotating anisotropy. The Wei-Harumi model uses the propagation of Huygens' type to simulate the depolarization and repolarization of the myocardium. The equivalent dipole density source, i.e., the electric cardiac source, is derived with the bidomain approach from the transmembrane action potentials simulated in the

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heart model. Then the body surface and epicardial electrocardiograms are forwardly calculated based on Green's theorem, by the boundary element method within a piecewise heterogeneous volume conductor model. This volume conductor model is composed of body, epicardial, right endocardial and left endocardial surfaces with 344, 1003, 307 and 278 nodes; and 684, 2002, 610 and 552 triangles, respectively. Previous researches based on the Wei-Harumi model have successfully implemented the computer simulation of normal heart, bundle branch blocks, WPW syndrome, Wenckebach rhythm, supraventricular tachycardias, ventricular tachycardias, ventricular fibrillation, ventricular infarction, Brugada syndrome and cardiac electrophysiological study, Wei (1997).

Our reference epicardial ECG signals were generated in 1003 epicardial points by the Wei-Harumi model during the QRS interval, considering normal heart activation, Wei (1997). The body surface potential distributions were calculated in 344 points by solving the forward problem of electrocardiography according to (1):

$$\mathbf{\Phi}_{\mathrm{b}} = \mathbf{Z}\mathbf{\Phi}_{\mathrm{h}} \tag{1}$$

where

$\mathbf{\Phi}_{\mathrm{b}}$: vector of body surface potential distribution (344 x 1)

$\mathbf{\Phi}_{\mathrm{h}}$: vector of epicardial potential distribution (1003 x 1)

$\mathbf{Z}$: transfer matrix between the epicardial and thoracic surfaces (344 x 1003, related to the chest model), Barr et al. (1977).

2.2. Tikhonov regularization

Since the most frequently used form of the Tikhonov regularization is the zero-order Tikhonov regularization, we decided to use this form. Equation (2) represents the inverse solution according to the zero-order Tikhonov regularization:

$$\mathbf{\Phi}_{\mathrm{h}} = \left[\mathbf{Z}^{\mathrm{T}}\mathbf{Z} + \gamma\mathbf{I}\right]^{-1}\mathbf{Z}^{\mathrm{T}}\mathbf{\Phi}_{\mathrm{b}} \tag{2}$$

where

$\mathbf{I}$: identity matrix

γ : regularization parameter.

The regularization parameter is responsible for making the singular matrix to be regular, thus transforming the ill-posed problem into a well-posed problem. According to the literature the value of γ should be chosen small enough to avoid over-regularization, but large enough to make the computations numerically sufficiently accurate. The optimal value of γ is usually chosen empirically, Tikhonov (1977).

2.3. Solving the linear equation system

After rearranging (2) and substituting **A**, **x** and **b**, we get the linear equation system of (3):

$$\mathbf{A}\mathbf{x} = \mathbf{b} \tag{3}$$

where

$$\mathbf{A} = \mathbf{Z}^{\mathrm{T}}\mathbf{Z} + \gamma\mathbf{I} \quad (1003 \times 1003)$$
$$\mathbf{b} = \mathbf{Z}^{\mathrm{T}}\mathbf{\Phi}_{\mathrm{b}} \quad (1003 \times 1)$$
$$\mathbf{x} = \mathbf{\Phi}_{\mathrm{h}} \quad (1003 \times 1).$$

To solve (3) we chose six methods offered by the MATLAB library: Cholesky factorization, conjugate gradients, minimum residual, quasi-minimal residual, least-squares QR (LSQR) and symmetric LQ. From these algorithms the first one is a direct method, while the others are iterative methods.

To put the accuracy of the inverse solution to the test, we did the following for each time instant of the QRS interval: we forwarded the reference epicardial potential distribution (potential map) to the body surface according to (1). After that we performed the inverse computation by solving the linear equation system of (3) with various regularization parameters and solvers. Finally we compared the solution to the reference epicardial potential distribution by Pearson's linear correlation coefficient.

3. RESULTS

We experienced that using very small γ values ($\gamma < 10^{-16}$) makes (3) numerically very difficult to solve. To quantify the numerical difficulty, we calculated the condition number, Cheney et al. (2012), of matrix **A** defined by

$$\kappa = \left\| \mathbf{A}^{-1} \right\| \cdot \left\| \mathbf{A} \right\| \tag{4}$$

Table 1 shows the condition number as the function of different γ values.

Table 1. Condition numbers as function of *γ values*

Regularization parameter (γ)	Condition number (κ)
1.00E-17	2.14E+18
1.00E-16	4.15E+16
1.00E-15	2.99E+14
1.00E-14	2.81E+13
1.00E-13	2.78E+12
1.00E-12	2.78E+11
1.00E-11	2.78E+10
1.00E-10	2.78E+09
1.00E-09	2.78E+08
1.00E-08	2.78E+07
1.00E-07	2.78E+06
1.00E-06	2.78E+05
1.00E-05	2.78E+04
1.00E-04	2.78E+03
1.00E-03	2.79E+02
1.00E-02	2.88E+01
1.00E-01	3.78E+00
1.00E+00	1.28E+00