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Uncertainty evaluation for velocity–area methods

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ABSTRACT

Velocity–area methods are used for flow rate calculation in various industries. Applied within a fully turbulent flow regime, modest uncertainties can be expected. If the flow profile cannot be described as "log-like", the recommended measurement positions and integration techniques exhibit larger errors. To reduce these errors, an adapted measurement scheme is proposed. The velocity field inside a Venturi contour is simulated using computational fluid dynamics and validated using laser Doppler anemometry. An analytical formulation for the Reynolds number dependence of the profile is derived. By assuming an analytical velocity profile, an uncertainty evaluation for the flow rate calculation is performed according to the "Guide to the expression of uncertainty in measurement". The overall uncertainty of the flow rate inside the Venturi contour is determined to be 0.5% compared to ≈0.67% for a fully developed turbulent flow.

1. Introduction

Flow measuring devices can be grouped into three main categories: integrative methods like differential pressure measurements or magnetic inductive devices, semi-integrative methods like ultrasonic path meters, and sampling based techniques which rely on a discrete number of measurement positions such as velocity–area methods.

These methods have been successfully applied to a variety of flow conditions. Especially for larger pipes, open field acceptance tests or temporary measurements, they present a feasible alternative to conventional flow measuring devices. Another advantage is that these methods can easily be applied within an arbitrary measurement section or open channel flow. Though traditionally used with pitot tubes or inserted devices, they can also be applied to optical velocity measurement techniques like laser Doppler anemometry (LDA).

In order to convert the pointwise velocities to a flow rate, a precise numerical integration method is preferable. The overall idea is to get an acceptable uncertainty for the flow rate with the least possible number of sample positions. The most general formulation of the measurement problem is: $Q=\int_0^R\int_0^{2\pi} v{\cdot}r\ d\varphi\ dr$. For the case of a rotational symmetric turbulent pipe flow in a circular measurement section the problem is reduced to a path integration: $Q=2\pi\int_0^R r{\cdot}v\ dr$. For a known velocity profile the metrological effort can be even further reduced from a path measurement to a single point measurement. Aichelen [1] proposed placing the probe at the position where the volumetric flow velocity occurs. The point of the average velocity varies for different models of turbulent pipe flow depending on the Reynolds number, as shown in Fig. 1. A different approach is to measure the centreline velocity while computing the flow rate using a calibration factor as proposed by Strunck et al. [11]. However, by restricting the measurement of an unknown profile to just one point, there is no way to tell whether the implicit assumptions concerning the shape of the profile are viable. Therefore, in general multiple radial positions are necessary. Quite a few integration techniques and guidelines for optimized measurement positions have been published. Based on Winternitz and Fischl [15] the commonly used integration procedures are described in the standards ISO 3354 [3], ISO 3966 [4] and VDI 2640 [12]. These methods are based on the assumption of a fully developed turbulent pipe flow where the velocity can be described by "log-like" behavior. This can only be achieved by a long undisturbed entrance length or flow conditioning both are often not feasible. The *true* velocity field in the measuring plane is therefore in general unknown, thus making the uncertainty evaluation of the standard methods quite cumbersome.

In order to create well-defined conditions, a Venturi contour is investigated. Due to the different shapes of the velocity distribution in the Venturi nozzle, it will be shown that the standard velocity–area methods exhibit higher errors. To reduce the uncertainty for the flow rate calculation, optimized measurement

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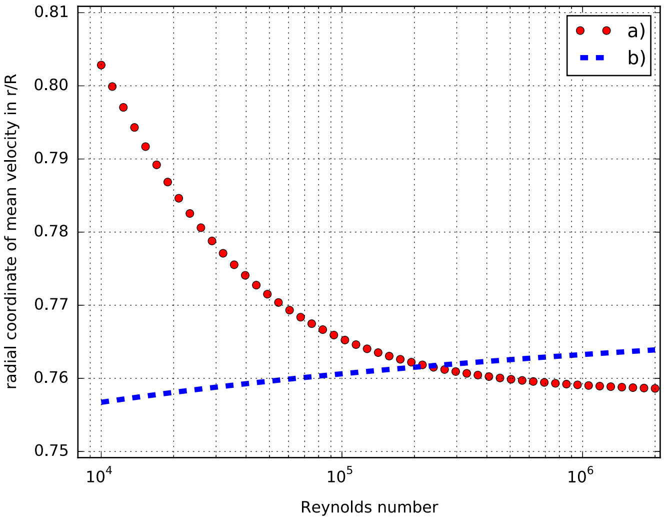

Fig. 1. Radial location of the average velocity: (a) Gersten/Herwig profile acc. to [8], (b) power-law profile acc. to Miller [9].

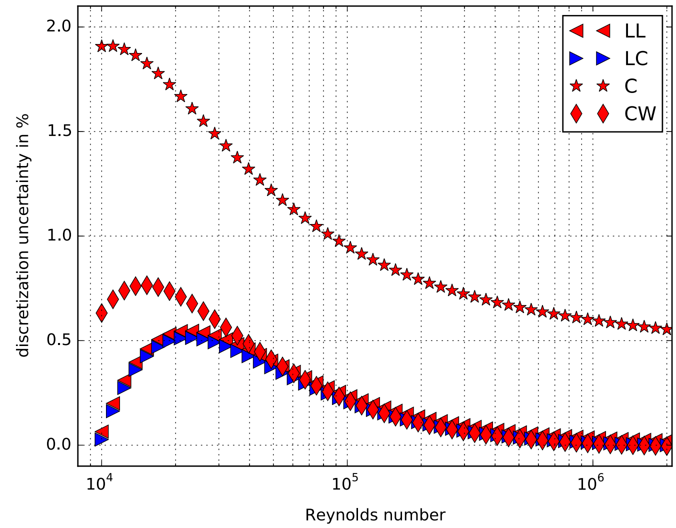

Fig. 2. Discretization error of velocity–area methods applied to the Gersten/Herwig profile ($k=1$); log linear (LL), log Chebyshev (LC), centroid (C) and centroid with wall correction (CW).

positions are derived. To compare the performance of the new method, an uncertainty evaluation based on analytical velocity profiles is applied. We consider the fully turbulent pipe flow first, since the descriptions of the uncertainties in the standards are rather short.

2. Uncertainty evaluation for fully developed turbulent pipe flow

Velocity–area methods calculate the flow rate as follows: the cross section is divided into equally sized parts. The measurement position for each piece is given either by the centroid or the position of the average velocity in this area. The flow rate is determined as the mean measured velocity multiplied with the area of the cross section. Some integration techniques apply an extrapolation procedure/a wall correction. Recommended locations for up to five radial sample positions are tabulated in the standards ISO 3354 [3], ISO 3966 [4] and VDI 2640 [12]. These procedures, developed to cope with only limited access to data processing and automation, are log linear (LL), log Chebyshev (LC), centroid (C), and centroid with wall correction (CW). Due to the pointwise sampling of the continuous velocity field, an intrinsic discretization error must be taken into account. To assess this error, an analytical reference profile with known flow rate is required. It is mandatory that this profile conveys the essential geometric and hydraulic phenomena of the emulated flow. A generic velocity formulation according to Gersten and Herwig [8] is used for the investigation of the discretization error. This profile, referred to as the GH profile, is a closed formulation for the streamwise velocity component of a flow in a round pipe. The pipe flow model is valid in the fully turbulent range for Reynolds numbers between 4×10^4 and 1×10^7.

All methods were analyzed for the recommended five radial sample positions. The derived flow rate was then compared to the exact integration of the GH profile. This procedure is performed for a Reynolds number range of 1×10^4–2×10^6. All methods show a Reynolds number dependence. It is worth noting that despite measuring on only five radial sample positions, even the highest discretization error is smaller than 2%. For the LL and LC, the errors are smaller than 0.6% compare Fig. 2.

To point out the importance of the discretization error, the overall uncertainty has to be derived. Neglecting any radial asymmetries, a minimal measurement uncertainty can be established based on the discretization error, the accuracy of the traverse system, the uncertainty of the velocity measurement and the uncertainty of the cross-sectional area.

The influence of the accuracy of the traverse system on the measurement positions and its effect on the uncertainty of the flow rate are determined as follows. The sensitivity coefficient for each individual measurement position is estimated by a numerical differential quotient as proposed in the "Guide to the expression of uncertainty in measurement (GUM)" [5]. Each emulated measurement is repeated with slightly shifted sample positions. For the sake of simplicity all measurement positions are confined to a dimensionless radial coordinate r/R between 0 and 1. If for any shifted sample position a radial coordinate outside of the conduit occurs, it is mirrored either on the wall or on the centreline. Furthermore, it is assumed that all positional errors are uncorrelated and of the same magnitude. In ISO standard 3966 [4] a maximum permissible positional error of 0.5% pipe diameter D is given. The following configurations will be discussed: $\Delta r/R = 0.50\%$ D, 0.25% D, 0.1%D and 0.05%D. For large Reynolds numbers the influence of the positioning precision declines. This is due to the rather flat velocity profile. The effect of the steeper curvature in the proximity of the wall cannot be sampled by the (recommended) five measurement positions. The uncertainty contribution of the other velocity–area methods is of the same order. The resulting traverse uncertainties for the log Chebyshev method are shown in Fig. 3.

As an example, the uncertainty of the log Chebyshev method at a Reynolds number of 1×10^5 is presented. Fig. 2 yields the discretization uncertainty with 0.21%. Fig. 3 yields the positioning contribution for an uncertainty of 0.1%D with 0.1%. The uncertainty of the velocity measurement, based on laser Doppler anemometry, is estimated to be 0.2%. The uncertainty of the cross section's diameter, nominally 75 mm, is 0.03 mm thus accounting for an uncertainty of the flow area of 0.1%.

The combined standard uncertainty for the log Chebyshev method can be stated to be 0.32% ($k=1$) or 0.65% ($k=2$) for this particular configuration, as shown in Table 1, column 1. For a different measurement setup, e.g. a different velocity uncertainty, these values can be easily adapted.

The example of the uncertainty assessment shows that the discretization error accounts for 40% of the overall uncertainty. It is obvious that with an increased number of sample positions, the intrinsic discretization error can be reduced. Depending on the application, the proper ratio between measuring time and accuracy has to be weighed.