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Discharge coefficient equation for critical-flow toroidal-throat venturi nozzles covering the boundary-layer transition regime

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ABSTRACT

A single, simple correlating equation between the discharge coefficient of critical-flow Venturi nozzles (CFVNs) having an ISO 9300 toroidal throat and their Reynolds number is proposed in the Reynolds number range from 2.1×10^4 to 3.2×10^7. The equation covers the whole Reynolds number range from laminar to turbulent boundary-layer regimes and can thus be used instead of the two correlating equations defined in ISO 9300 : 2005. The deviation of the discharge coefficients of well-made CFVNs is expected to be less than $\pm0.2\%$ throughout the Reynolds number range. Tolerances for the diffuser length, inlet curvature and inlet diameter are also proposed. It is shown that the widely-accepted theories that estimate the core flow distribution result in significant error when the inlet curvature is small and that removing the third term in Hall's equation results in very good agreement with the experimental data regardless of the magnitude of the inlet curvature. The use of CFVNs with the inlet curvature of 1.0*D* is discussed in order to reduce the uncertainty owing to undefined boundary-layer transition Reynolds number. A possibility is shown that such a CFVN may not have an apparent boundary-layer transition in the investigated Reynolds number range from 1.5×10^4 to 2.0×10^6.

1. Introduction

The 1960s was one of the most active periods for critical-flow Venturi nozzles (CFVNs). In 1962, Arnberg [1] published a review that summarized the whole history of CFVNs from 175 B.C. He discussed and organized many matters such as the advantages of CFVNs, whether the stagnation or static pressure should be used to calculate the flow rate, definitions of working equations, importance of real gas effect, potentials of various nozzle shapes, and so on, thus paving the way for the modern CFVNs now in use.

In that period, the standard shapes of CFVNs to be defined by codes were still under discussion. In the early applications, ASME long-radius flow nozzles that were originally developed for the subsonic condition were tested in the critical condition [2,3]. In 1962, Smith and Matz [4] recommended a geometry that has, apart from the ASME long-radius flow nozzles, a toroidal throat with a constant inlet curvature radius that connects smoothly to a frustum diffuser. In 1964, Stratford [5] recommended making the curvature of the inlet contraction about twice the throat diameter in order to have a smaller discontinuity of the discharge coefficient along the Reynolds number across the boundary-layer transition. These recommendations are all taken into account in most of today's CFVNs [6,7]. However, they recommended these geometries in fact to increase the accuracy of their theoretical predictions of the discharge coefficient because they did not have good facilities for calibrating CFVNs at that time [4,5], but measurement data have been accumulated since then upon the foundation established by the theories. Although there is another kind of common CFVN geometry that has a cylindrical throat, this paper considers only CFVNs with a toroidal throat complying with ISO 9300 unless otherwise stated.

In 1978, Brain proposed a correlating equation between the discharge coefficient and the Reynolds number based on UK [8] and USA measurements [9], and his equation was finally introduced in the first version of ISO 9300 published in 1990. The correlating equation was later found to have a certain bias at low Reynolds numbers where the boundary layer at the throat is laminar. One of the main reasons for the bias was because the benchmark measurements in that era were performed mainly at high Reynolds numbers [9]. Szaniszlo [10] even recommended putting an edge on the front of the throat to let the transition occur earlier in order to reduce the dependence of the discharge coefficient on the Reynolds number by making the boundary layer turbulent from a smaller Reynolds number. On the other hand, theories [4,5] predicted different tendencies from the code's

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Nomenclature

C_d Discharge coefficient (-), $C_d = \frac{Q_{true}}{Q_{theo}}$

C_d^a Correlating equation referred as 'aCURVE' that is defined in ISO 9300: 2005 for the laminar boundary-layer regime (-), $C_d{}^a = 0.9985 - \frac{3.412}{\sqrt{R_e}}$ $(2.1 \times 10^4 < R_e < 1.4 \times 10^6)$

C_d^n Correlating equation referred as 'nCURVE' that is defined in ISO 9300: 2005 for whole Reynolds number range (-), $C_d{}^n = 0.9959 - \frac{2.720}{\sqrt{R_e}}$ $(2.1 \times 10^4 < R_e < 3.2 \times 10^7)$

C_d^s Correlating equation referred as 'sCURVE' that reflects the boundary-layer transition and connects aCURVE to nCURVE (-), $C_d{}^s = \left(0.99845 - \frac{3.412}{\sqrt{R_e}}\right) - \frac{0.00255 - \frac{0.692}{\sqrt{R_e}}}{1 + \exp\left(19.3 - \frac{R_e}{7 \times 10^4}\right)}$ $(2.1 \times 10^4 < R_e < 3.2 \times 10^7)$

C_d^b Correlating equation for CFVNs with $R = 1.0D$ inlet curvature assuming the laminar boundary layer referred as 'bCURVE' (-), $C_d{}^b = 0.9958 - \frac{2.912}{\sqrt{R_e}}$ $(1.5 \times 10^4 < R_e < 2.0 \times 10^6)$

C_d^t Correlating equation referred as 'tCURVE' for CFVNs with $R = 1.0D$ inlet curvature using third power function of $R_e^{-0.2}$ (-), $C_d{}^t = 1.0118 - 0.5476 R_e{}^{-0.2} + 5.5616 R_e{}^{-0.4} - 25.795 R_e{}^{-0.6}$ $(1.5 \times 10^4 < R_e < 2.0 \times 10^6)$

$C_d^{laminar}$ Correlating equation as a function of $R_e^{-0.5}$ (-)

$C_d^{turbulent}$ Correlating equation as a function of $R_e^{-0.2}$ (-)

C_d^{HG} Theoretical discharge coefficient based on Hall's core flow distribution and Geropp's laminar boundary layer (-)

C_d^{H2G} Modified theoretical discharge coefficient based on Hall and Geropp's theories by neglecting the third term of Hall's equation (–)

a A coefficient reflecting the flow rate deficit by the core flow distribution (-)

a_{Hall} Original coefficient a based on Hall's theory (-)

a_{Hall2} Modified coefficient a by ignoring the third term of Hall's equation (-)

b A coefficient reflecting the flow rate deficit by the boundary layer (-)

b_{Geropp} Coefficient b based on Geropp's theory (-)

R_e The Reynolds number defined in ISO 9300 using the values at the critical point for length, density and velocity and that of the viscosity at the stagnation point, $R_e = \frac{4Q_{true}}{\pi D \mu_0}$

R_e^{trad} The traditional Reynolds number defined using the values at the critical point for all parameters, $R_e = \frac{4Q_{true}}{\pi D \mu^*}$

R_e^{theo} The Reynolds number defined by using the stagnation viscosity and the theoretical flow rate, $R_e{}^{theo} = \frac{4Q_{theo}}{\pi D \mu_0} = \frac{R_e}{C_d}$

Q_{true} Actual flow rate through CFVN (kg s^{-1}), $Q_{true} = C_d Q_{theo}$

Q_{theo} Theoretical flow rate through CFVN assuming one-dimensional isentropic flow of perfect gas (kg s^{-1})

Q Size parameter for Hall's equation (-)

m Size parameter for Geropp's equation (-)

D Throat diameter (m)

D_{pipe} Pipe inner diameter of the nozzle holder (m)

k Coverage factor (-)

κ_0 Stagnation specific heat (-)

C^* Critical flow function for real gas

C_c Critical flow function for perfect gas

P_0 Stagnation pressure upstream of CFVN (Pa)

T_0 Stagnation temperature upstream of CFVN (K)

R_u Universal gas constant (8.31451 according to 1986 CODATA) (J K^{-1} mol^{-1})

R Inlet curvature (m)

D_{in} inlet diameter(m)

μ_0 Viscosity at the stagnation point (Pa s)

μ^* Viscosity at the critical point (Pa s)

ρ^* Density at the critical point (kg m^{-3})

x Axis on the nozzle center line (m)

equation in the laminar boundary regime, which Arnberg's measurements had already suggested were valid [9].

As the use of CFVNs spread, measurement data in the low Reynolds number regime were also accumulated [9–14]. In the 1990s, high-precision nozzles (HPN) were machined by super-accurate lathes to produce a mirror finish with average surface roughness of 0.04 μm without being polished, thus almost eliminating the effect of geometry errors on the discharge coefficient in a certain flow rate range, and so their calibration data showed negligible scattering in the measured discharge coefficients [15–18]. The experimental correlating equation based on calibration of the HPNs complying with ISO 9300 in the laminar boundary-layer regime was then verified by accurate analytical and also numerical calculations with an agreement of about 0.03% [19–23].

In 2005, the second (and current) version of ISO 9300 was published, which defines two correlating equations of the discharge coefficient for toroidal-throat CFVNs: one is applicable only in the laminar boundary–layer regime and the other is applicable throughout the Reynolds number range from laminar to turbulent boundary–layer regimes. The former equation assumes the use of 'accurately machined nozzles' that have small machining error such as HPNs to reduce the uncertainty to 0.2%, whereas the latter is applicable to 'normally machined nozzles' and allows a larger uncertainty of 0.3%. The accurately machined nozzles and the normally machined nozzles are distinguished by whether they were polished or not to achieve the required surface roughness. Hereinafter, the correlating equations defined in ISO 9300: 2005 are referred to as 'aCURVE' and 'nCURVE', where 'a' denotes an 'accurately' machined nozzle and 'n' denotes a 'normally' machined one, as shown in Fig. 1. The tendency of aCURVE is clearly different from that of nCURVE: it has a larger gradient against the Reynolds number and reaches a maximum deviation from nCURVE of more than +0.2% at the Reynolds number of 1.4×10^6.

nCURVE was originally developed by Arnberg as the 'Universal Curve' [24], and was a compromise equation that offered the best accuracy in both the laminar and the turbulent boundary-layer regimes without any information on the sort of boundary layer. Since there is no need to consider the sort of boundary layer or to detect the boundary-layer transition, nCURVE is very convenient for code users. As seen in Fig. 1, aCURVE lies within the uncertainty limits of nCURVE and vice versa, so it was a good compromise. However, nCURVE abandoned the better accuracies in each regime, especially in the laminar boundary-layer regime, when there is no ambiguity as to the sort of boundary layer. This caused controversy at the revision committee of ISO 9300 but finally the majority agreed because the behavior of the discharge coefficient in the boundary-layer transition regime was still unclear at that time.

Recent measurements [25–30] revealed stable boundary-layer transition in well-made CFVNs that yielded systematic variation of the discharge coefficient along the Reynolds number during the transition. Theoretical investigation of the experimental data in