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Modeling and control of inverter-based microgrids ⋆

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Abstract: Assuming the most common control structure for zero and primary control of inverter-based microgrids, i.e. three cascades with the highest one being droop control, the potential benefit of optimizing the control parameters is investigated. A detailed nonlinear plant model is derived that compactly describes the dynamics in local dq-coordinates. Then, the design of the decentralized, cascaded controllers is converted into the problem of designing one centralized static controller with structural restrictions. To tune the controller parameters, a direct method for pole-assignment is used. The simulations show that the oscillations in the transient response can be reduced greatly by choosing appropriate control parameters, while the speed of the system is restricted due to the low-pass filtering of the power for primary control.

Keywords: Microgrid, inverters, primary control, zero-level control, decentralized control, eigenstructure assignment

1. INTRODUCTION

The stability analysis of electricity grids has been of great research interest for a long time. Yet relatively few textbooks or publications actually treat the selection of the control parameters, and even fewer question the typically used control structure. This might be due to the fact that for large power systems the detailed model often is not available. But when considering microgrids, this should not be an issue. Another reason might be the difficulties arising due to the decentralized nature of grid control. And yet, the occurring problems have long been tackled by the control society, e.g. Litz (1983), Konigorski (1988), Siljak (1991), Lunze (1992).

Combining the modeling approaches broadly used in power system stability analysis and the results from the control society on the design of decentralized controllers, we tune the controller parameters of a microgrid to improve its transient behavior.

2. BASIC ASSUMPTIONS AND NOTATION

Assume a symmetrically constructed power system that is symmetrically operated. Then, only symmetrical signals occur. Let three-phase AC signals be written in vector notation: $\mathbf{x}_{\mathrm{abc}} = (x_{\mathrm{a}}\ x_{\mathrm{b}}\ x_{\mathrm{c}})^T$. Let $\mathcal{V}_{\mathcal{N}}$ be the set of vertices and $\mathcal{E}_{\mathcal{N}}$ the set of edges of the network. Let $\mathcal{I} \subset \mathcal{V}_{\mathcal{N}}$ be the set of vertices to which inverters are connected and $\mathcal{V}_{\mathcal{L}} \subset \mathcal{V}_{\mathcal{N}}$ the set of vertices to which loads are connected.

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After power flow calculation, edges are added to represent the loads. Denote the set of these edges $\mathcal{E}_{\mathcal{L}}$. They are connected to the ground node, which is the only element of $\mathcal{V}_{\mathcal{L}0}$. The sets of vertices and edges of the resulting network used for the dynamical analysis are $\mathcal{V} = \mathcal{V}_{\mathcal{N}} \cup \mathcal{V}_{\mathcal{L}0}$ and $\mathcal{E} = \mathcal{E}_{\mathcal{N}} \cup \mathcal{E}_{\mathcal{L}}$. Let $|\bullet|$ be the cardinality of the set $\bullet$. We assume following numbering of the busses

$$
\begin{aligned}
\mathcal{I} &= \{1, \ldots, |\mathcal{I}|\} \\
\mathcal{V}_{\mathcal{L}} &= \{|\mathcal{I}| + 1, \ldots, |\mathcal{I}| + |\mathcal{V}_{\mathcal{L}}|\} \\
\mathcal{V}_{\mathcal{N}} \setminus \mathcal{I} \setminus \mathcal{V}_{\mathcal{L}} &= \{|\mathcal{I}| + |\mathcal{V}_{\mathcal{L}}| + 1, \ldots, |\mathcal{V}_{\mathcal{N}}|\} \\
\mathcal{V}_{\mathcal{L}0} = \mathcal{V} \setminus \mathcal{V}_{\mathcal{N}} &= \{|\mathcal{V}_{\mathcal{N}}| + 1\} = \{|\mathcal{V}|\}
\end{aligned}
$$

and of the edges

$$
\begin{aligned}
\mathcal{E}_{\mathcal{N}} &= \{|\mathcal{V}| + 1, \ldots, |\mathcal{V}| + |\mathcal{E}_{\mathcal{N}}|\} \\
\mathcal{E}_{\mathcal{L}} = \mathcal{E} \setminus \mathcal{E}_{\mathcal{N}} &= \{|\mathcal{V}| + |\mathcal{E}_{\mathcal{N}}| + 1, \ldots, |\mathcal{V}| + |\mathcal{E}|\}.
\end{aligned}
$$

With this consecutive numbering of nodes and edges, currents injected into vertices and voltages at vertices, which have a subscript $i \in \mathcal{V}$, and voltages over edges and currents flowing through edges, which have a subscript $i \in \mathcal{E}$, can easily be distinguished. Let the subscripts $\mathcal{I}, \mathcal{V}_{\mathcal{L}}, \mathcal{V}_{\mathcal{N}}, \mathcal{V}_{\mathcal{L}0}, \mathcal{V}, \mathcal{E}_{\mathcal{N}}, \mathcal{E}_{\mathcal{L}}, \mathcal{E}$ denote ordered column vectors from the corresponding set. For example, the voltages at the vertices are denoted $\mathbf{u}_{\mathrm{abc},\mathcal{V}}^T := \left[\mathbf{u}_{\mathrm{abc},1}^T \ \cdots \ \mathbf{u}_{\mathrm{abc},|\mathcal{V}|}^T\right]$. Define the function diag, which creates a diagonal matrix from its argument. Let the Kronecker product be denoted by $\otimes$, the Hadamard product by $\circ$ and the inverse Hadamard product by $\mathbf{A}^{\circ(-1)} := (1/a_{ij})$. Define the dq-transformation

$$
\mathbf{T}_{\mathrm{dq}}(\theta) := \sqrt{\frac{2}{3}} \begin{bmatrix} \cos(\theta) & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ -\sin(\theta) & -\sin\left(\theta - \frac{2\pi}{3}\right) & -\sin\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix} \quad (1)
$$

and the inverse dq-transformation

$$
\mathbf{T}_{\mathrm{abc}}(\theta) := \mathbf{T}_{\mathrm{dq}}(\theta)^T \quad (2)
$$

such that

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$$\mathbf{x}_{\mathrm{dq}} = [x_{\mathrm{d}}\ x_{\mathrm{q}}]^T = \mathbf{T}_{\mathrm{dq}}(\theta)\mathbf{x}_{\mathrm{abc}} \qquad (3)$$
$$\mathbf{T}_{\mathrm{abc}}(\theta)\mathbf{x}_{\mathrm{dq}} = \mathbf{x}_{\mathrm{abc}}.$$

3. COORDINATE SYSTEMS

The coordinate systems are chosen as described in typical textbooks on the modeling of electricity networks, e.g. Kundur (1994), Anderson and Fouad (2003), Sauer and Pai (1997). The network will be described in dq-coordinates revolving with the angular velocity $\breve{\omega}$. The transformation angle is

$$\theta = \mathrm{mod}_{2\pi}(\breve{\omega}t) \in \mathbb{T}, \qquad (4)$$

where $\mathbb{T} := \{x \in \mathbb{R} | 0 \leq x < 2\pi\}$ and the operator $\mathrm{mod}_{2\pi}$ is used so that $\theta \in \mathbb{T}$. Signals in network dq-coordinates will be underlined. Each inverter is described in its own $\mathrm{dq}_{i\in\mathcal{I}}$-coordinate system. The rotation frequencies $\omega_{i\in\mathcal{I}}$ of these local coordinate systems are set by the power controllers of the respective inverters. Then, the angle between the global network dq-coordinates and the local dq_i-coordinates can be tracked by integration

$$\delta_i(t) = \delta_{0,i} + \int_0^t (\omega_i(\tau) - \breve{\omega})\mathrm{d}\tau, \quad i \in \mathcal{I}, \qquad (5)$$

where $\delta_{0,i}$ is the value of δ_i at $t = 0$. Since the signals in the final equations will be in local coordinates, signals in local coordinates will not specifically be marked. The transformation from local coordinates of inverter i to global coordinates is a rotation by δ_i

$$\mathbf{T}_{i\in\mathcal{I}} := \begin{bmatrix} \cos(\delta_i) & -\sin(\delta_i) \\ \sin(\delta_i) & \cos(\delta_i) \end{bmatrix}. \qquad (6)$$

Define the block-diagonal transformation matrix

$$\mathbf{T}_{\mathcal{I}} := \mathrm{diag}\left(\mathbf{T}_1, \ldots, \mathbf{T}_{|\mathcal{I}|}\right). \qquad (7)$$

Then, the voltages and currents at the inverter nodes can be transformed from local to global coordinates by

$$\underline{\mathbf{i}}_{\mathcal{I}} = \mathbf{T}_{\mathcal{I}}\mathbf{i}_{\mathcal{I}}, \qquad \underline{\mathbf{u}}_{\mathcal{I}} = \mathbf{T}_{\mathcal{I}}\mathbf{u}_{\mathcal{I}}. \qquad (8)$$

4. NETWORK MODEL

Let the electrical network be modeled by concentrated parameters. Conduct a power-flow calculation for a given operating point and compute the typical impedances as load models. To derive a dynamical model of the network with loads, we use the single-phase representation and Matlab's *power_statespace* command. This way, a state space model with voltages as input $\mathbf{u}_{\mathrm{a},\mathcal{I}}$ and currents as output $\mathbf{i}_{\mathrm{a},\mathcal{I}}$ is obtained:

$$\dot{\mathbf{x}}_{\mathrm{a}} = \mathbf{A}\mathbf{x}_{\mathrm{a}} + \mathbf{B}\mathbf{u}_{\mathrm{a},\mathcal{I}} \qquad (9)$$
$$\mathbf{i}_{\mathrm{a},\mathcal{I}} = \mathbf{C}\mathbf{x}_{\mathrm{a}}.$$

Denote the order of the model n. First, extend the model to represent all three-phases

$$\dot{\mathbf{x}}_{\mathrm{abc}} = [\mathbf{A} \otimes \mathbf{I}_3]\,\mathbf{x}_{\mathrm{abc}} + [\mathbf{B} \otimes \mathbf{I}_3]\,\mathbf{u}_{\mathrm{abc},\mathcal{I}} \qquad (10)$$
$$\mathbf{i}_{\mathrm{abc},\mathcal{I}} = [\mathbf{C} \otimes \mathbf{I}_3]\,\mathbf{x}_{\mathrm{abc}}.$$

Then, transform the model to global network dq-coordinates, which we denote by an underline, c.f. section 3. To do so, apply the inverse dq-transformation (3)

$$\dot{\mathbf{x}}_{\mathrm{abc}} = [\mathbf{A} \otimes \mathbf{I}_3]\,[\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{abc}}]\,\underline{\mathbf{x}} \qquad (11)$$
$$+ [\mathbf{B} \otimes \mathbf{I}_3]\,\left[\mathbf{I}_{|\mathcal{I}|} \otimes \mathbf{T}_{\mathrm{abc}}\right]\,\underline{\mathbf{u}}_{\mathcal{I}}$$
$$\mathbf{i}_{\mathrm{abc},\mathcal{I}} = [\mathbf{C} \otimes \mathbf{I}_3]\,[\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{abc}}]\,\underline{\mathbf{x}}$$

and multiply the first equation of (11) by $\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{dq}}$ and the second equation by $\mathbf{I}_{|\mathcal{I}|} \otimes \mathbf{T}_{\mathrm{dq}}$ from the left. Considering the mixed-product property of the Kronecker-product, this leads to

$$[\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{dq}}]\,\dot{\mathbf{x}}_{\mathrm{abc}} = [\mathbf{A} \otimes \mathbf{I}_2]\,\underline{\mathbf{x}} + [\mathbf{B} \otimes \mathbf{I}_2]\,\underline{\mathbf{u}}_{\mathcal{I}} \qquad (12)$$
$$\underline{\mathbf{i}}_{\mathcal{I}} = [\mathbf{C} \otimes \mathbf{I}_2]\,\underline{\mathbf{x}}.$$

To compute $[\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{dq}}]\,\dot{\mathbf{x}}_{\mathrm{abc}}$ consider

$$\dot{\underline{\mathbf{x}}} = \frac{\mathrm{d}\,[\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{dq}}]\,\mathbf{x}_{\mathrm{abc}}}{\mathrm{d}t} \qquad (13)$$
$$= \left[\mathbf{I}_{\mathrm{n}} \otimes \begin{bmatrix} 0 & \breve{\omega} \\ -\breve{\omega} & 0 \end{bmatrix}\right]\underline{\mathbf{x}} + [\mathbf{I}_{\mathrm{n}} \otimes \mathbf{T}_{\mathrm{dq}}]\,\dot{\mathbf{x}}_{\mathrm{abc}}.$$

Inserting (13) into (12) leads to

$$\dot{\underline{\mathbf{x}}} = \left[\mathbf{A} \otimes \mathbf{I}_2 + \mathbf{I}_{\mathrm{n}} \otimes \begin{bmatrix} 0 & \breve{\omega} \\ -\breve{\omega} & 0 \end{bmatrix}\right]\underline{\mathbf{x}} + [\mathbf{B} \otimes \mathbf{I}_2]\,\underline{\mathbf{u}}_{\mathcal{I}} \qquad (14)$$
$$\underline{\mathbf{i}}_{\mathcal{I}} = [\mathbf{C} \otimes \mathbf{I}_2]\,\underline{\mathbf{x}},$$

which is the network model in dq-coordinates. Since the inverters will all be described in local dq-coordinates, transform input and output of (14) to local coordinates, too. Application of (8) yields

$$\dot{\underline{\mathbf{x}}} = \tilde{\mathbf{A}}\underline{\mathbf{x}} + \tilde{\mathbf{B}}\mathbf{T}_{\mathcal{I}}(\boldsymbol{\delta}_{\mathcal{I}})\mathbf{u}_{\mathcal{I}} \qquad (15)$$
$$\mathbf{i}_{\mathcal{I}} = \mathbf{T}_{\mathcal{I}}^{-1}(\boldsymbol{\delta}_{\mathcal{I}})\tilde{\mathbf{C}}\underline{\mathbf{x}},$$

where we abbreviated

$$\tilde{\mathbf{A}} := \left[\mathbf{A} \otimes \mathbf{I}_2 + \mathbf{I}_{\mathrm{n}} \otimes \begin{bmatrix} 0 & \breve{\omega} \\ -\breve{\omega} & 0 \end{bmatrix}\right],$$
$$\tilde{\mathbf{B}} := [\mathbf{B} \otimes \mathbf{I}_2], \qquad \tilde{\mathbf{C}} := [\mathbf{C} \otimes \mathbf{I}_2].$$

In this formulation, $\boldsymbol{\delta}_{\mathcal{I}}$ is an input of the network model. Since $\boldsymbol{\delta}_{\mathcal{I}}$ will not be needed anywhere else, we augment the model by (5) which results in a model of the network with inputs $\boldsymbol{\omega}_{\mathcal{I}}$, $\mathbf{u}_{\mathcal{I}}$ and output $\mathbf{i}_{\mathcal{I}}$:

$$\dot{\mathbf{x}}_{\mathrm{net}} = \frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} \underline{\mathbf{x}} \\ \boldsymbol{\delta}_{\mathcal{I}} \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{A}}\underline{\mathbf{x}} + \tilde{\mathbf{B}}\mathbf{T}_{\mathcal{I}}(\boldsymbol{\delta}_{\mathcal{I}})\mathbf{u}_{\mathcal{I}} \\ \boldsymbol{\omega}_{\mathcal{I}} - \mathbf{1}_{|\mathcal{I}|}\breve{\omega} \end{bmatrix} \qquad (16)$$
$$\mathbf{i}_{\mathcal{I}} = \mathbf{T}_{\mathcal{I}}^{-1}(\boldsymbol{\delta}_{\mathcal{I}})\tilde{\mathbf{C}}\underline{\mathbf{x}}.$$

5. INVERTER AND PLANT MODEL

Fig. 1 shows the block diagram of the coupled inverters. The coupling inductance has been modelled as a line of the network. Therefore, the inverter node $v_{i\in\mathcal{I}}$ is actually inside the inverter and no loads are connected to these vertices, which justifies our indexing. Subscript f is used to denote the remaining filter parameters $L_{\mathrm{f},i}, C_{\mathrm{f},i}$, the current flowing through the filter inductance $\mathbf{i}_{\mathrm{f}}$ and the voltage over the filter $\mathbf{u}_{\mathrm{f}}$. Reference values given by the secondary controller are denoted with superscript o. Since the secondary controller is not investigated, these inputs are assumed to be constants corresponding to the operating point of the network. Setpoints from inner control loops are denoted by superscript *. The absolute value of the voltage is denoted by $\hat{u}$. Subscript m is used to differentiate low-pass filtered measured values from the actual values.

Before focusing on the controllers, the inverters must be modeled. As customary in these kinds of models, we neglect the switching process of the inverters: $\mathbf{u}_{\mathrm{f}} = \mathbf{u}_{\mathrm{f}}^*$. The relationship between voltage decline and current flowing through the filter is described by

$$\mathbf{u}_{\mathrm{abc,f},i} - \mathbf{u}_{\mathrm{abc},i} = R_{\mathrm{f},i}\mathbf{i}_{\mathrm{abc,f},i} + L_{\mathrm{f},i}\frac{\mathrm{d}\mathbf{i}_{\mathrm{abc,f},i}}{\mathrm{d}t} \qquad (17)$$
$$\frac{\mathrm{d}\mathbf{u}_{\mathrm{abc},i}}{\mathrm{d}t} = \frac{1}{C_{\mathrm{f},i}}\left(\mathbf{i}_{\mathrm{abc,f},i} - \mathbf{i}_{\mathrm{abc},i}\right), \qquad i \in \mathcal{I}. \qquad (18)$$