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Small Variations of Basic Solution Method for Non-numerical Optimization

A.I. Diveev

Institution of Russian Academy of Sciences Dorodnicyn Computing Centre of RAS Moscow, Russia (e-mail: aidiveev@mail.ru).

Abstract: The universal method for the solution of problems of non-numerical optimization is considered. Concepts of basic element, small and elementary variations were defined. Definitions of norm and metric distance on the code's space of non-numerical elements were introduced. A genetic algorithm on the basis of small variations for basic solution was presented. Examples of solutions of travelling salesman problem and synthesis of control were presented.

Keywords: numerical methods for optimization, optimization methods, evolutionary algorithms.

1. INTRODUCTION

Problems of mathematical expression search, structure of a network, program codes, etc. belong to non-numerical problems of optimization. In all such tasks, admissible decisions are described by codes, with which it is impossible to carry out numerical arithmetic operations. The most known problem of non-numerical optimization is the travelling salesman problem. Complexity of algorithms creation for search in non-numerical optimization consists in lack of metrics between admissible decisions. This circumstance complicates creation of solutions' neighbourhoods and the organization of search in them.

In this paper, a universal approach to algorithms' development of non-numerical optimization is offered. This approach named the principle of basic solution small variations. The essence of the principle consists that the researcher defines a set of small variations for codes of the admissible solution. In the course of search, the researcher sets one or several basic solutions and will organize algorithm of search on a limited set of small variations of this basic solution. During the search we replace the basic solution with the best solution found to this moment. This approach allows to reduce the space of search and to use intuition of the researcher for adjustment of the basic solution in complex challenges.This principle is used successfully by search of mathematical expression in the solution of control synthesis problem (Diveev, 2008, 2012) and identification (Gubaidullin, 2014) for different types of coding by method of the network operator, genetic programming and analytical programming. A solution of the travelling salesman problem by this approach is considered as a test.

2. PRINCIPLE OF SMALL VARIATIONS OF BASIC SOLUTION

Consider the universal generalized method for the organization of the optimum solution search in space of non-numerical elements. The method demands preliminary research of a code for a non-numerical element and the analysis of an optimization problem to find a possible solution.

$$\Xi = \{\mathbf{y}^1, \mathbf{y}^2, \dots\} \tag{1}$$

where $\mathbf{y}^i$ is a code of non-numerical element.

Code space is not restricted with unlimited length code. On the space codes (1) given a finite number of functions for evaluating the correctness of recording non-numerical code element

$$\theta_j(\mathbf{y}^i) \le 0, \; j = \overline{1,l}. \tag{2}$$

Non-numerical code of element in the general case is a finite length sequence of character codes

$$\mathbf{y}^i = (y_1^i, y_2^i, \dots, y_{n_i}^i), \tag{3}$$

where n_i is a code length of element i.

The length of the non-numerical element code is an important characteristic. It is advisable to limit the length of non-numerical element code. In future, let's consider the code in the space (1) to be only the elements of limited length

$$\Xi_n = \{\mathbf{y}^1, \mathbf{y}^2, \dots\}. \tag{4}$$

If the code has a constraint length than it is necessary to define a special character to describe the missing symbol. Without loss of generality, define the missing character code as zero. Each code character is selected from a finite set of basic characters.

$$\mathrm{A} = \{0, a_1, a_2, \dots, a_L\}. \tag{5}$$

Here the zero symbol 0 for designation of the absent element is entered into a basic set of symbols. Two elements of code space with limited length differ among themselves in discrepancy at least of one code symbol: $\forall \mathbf{y}^m \in \Xi$, $\mathbf{y}^m = (y_1^m, y_2^m, \dots, y_{n_m}^m)$ and $\forall \mathbf{y}^k \in \Xi$, $\mathbf{y}^k = (y_1^k, y_2^k, \dots, y_{n_k}^k)$ $\mathbf{y}^m \ne \mathbf{y}^k$ if $\exists y_i^m \ne y_i^k$, $1 \le i \le n$.

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The basic set of symbols (5) depending on complexity of the coded non-numerical elements consists of a set of subsets for symbols

$$\mathrm{A} = \{0\} \cup \mathrm{A}_1 \cup \ldots \cup \mathrm{A}_M , \tag{6}$$

where $\mathrm{A}_i = \{a_1^i, \ldots, a_{L_i}^i\}$, $i = \overline{1, M}$.

Definition 1. Elementary variation of an element code we call replacement of one symbol in the code by a symbol from the set of basic symbols (5).

Determine rules of creation of codes for non-numerical elements by code accessory to a subset of basic symbols. Checking up correctness of the coding means of functions (2) are used. Define in a set of codes elements (4) only (2) codes meeting requirements. A set of admissible element codes with the set length is received

$$\tilde{\Xi}_n = \{\tilde{\mathbf{y}}^1, \tilde{\mathbf{y}}^2, \ldots\} . \tag{7}$$

Not always replacement of one symbol in an admissible code leads to a new admissible code. Sometimes replacement of one symbol in an admissible code leads to violation of rules (2) for codes. The elementary variation is not enough for receiving from one admissible code another one.

Definition 2. The minimum set of elementary variations is called a small variation which is required to fulfil so that to receive from one admissible code another one.

In certain cases, the small variation consists of one elementary variation. In other cases, the small variation includes some elementary variations to receive a new admissible code.

For the given set of admissible codes (7) a final set of small variations are defined

$$\Omega(\tilde{\Xi}_n) = \{\delta_1(\mathbf{y}), \ldots, \delta_M(\mathbf{y})\}. \tag{8}$$

Definition 3. The set of small variations is complete if it is always possible to find limited number of variations for any two admissible codes to receive from one admissible code another admissible code.

$$\forall \tilde{\mathbf{y}}^i, \tilde{\mathbf{y}}^j \in \tilde{\Xi}_n, \ \exists \delta_{k_1}(\mathbf{y}) \in \Omega(\tilde{\Xi}_n), \ldots, \delta_{k_d}(\mathbf{y}) \in \Omega(\tilde{\Xi}_n) \to \tilde{\mathbf{y}}^j = \delta_{k_1}\left(\ldots \delta_{k_d}\left(\tilde{\mathbf{y}}^i\right)\right). \tag{9}$$

Definition 3. Distance between two admissible codes equals minimum quantity of small variations to receive from one code some other code.

$$\left\|\tilde{\mathbf{y}}^i - \tilde{\mathbf{y}}^j\right\|_\Omega = d , \ d = \min_r \left\{\tilde{\mathbf{y}}^j = \delta_{k_1}\left(\ldots \delta_{k_r}\left(\tilde{\mathbf{y}}^i\right)\right)\right\} , \tag{10}$$

where $\forall \tilde{\mathbf{y}}^i, \tilde{\mathbf{y}}^j \in \tilde{\Xi}_n$, $\delta_{k_p}(\mathbf{y}) \in \Omega(\tilde{\Xi}_n)$, $p = \overline{1, r}$.

Definition 4. The neighbourhood $\Delta(\tilde{\mathbf{y}})$ of a code of an element $\tilde{\mathbf{y}}$ we call a set of the codes located at distance Δ from code of $\tilde{\mathbf{y}}$.

$$\Delta(\tilde{\mathbf{y}}) \subseteq \tilde{\Xi}_n, \ \forall \tilde{\mathbf{y}}^i \in \Delta(\tilde{\mathbf{y}}), \ \left\|\tilde{\mathbf{y}}^i - \tilde{\mathbf{y}}\right\|_\Omega = \Delta . \tag{11}$$

The small variation from (9) represents function of display of a set of elements admissible codes.

$$\delta_i(\tilde{\mathbf{y}}) : \tilde{\Xi}_n \to \tilde{\Xi}_n , \ 1 \le i \le M . \tag{12}$$

We enter variation vector to describe a small variation.

$$\mathbf{w} = [w_1 \ \ldots \ w_r]^T , \tag{13}$$

where dimension r of a variations' vector is defined by quantity of necessary information on the performance of small variation $\delta_i(\tilde{\mathbf{y}})$, w_1 - number of a small variation, $w_1 = i$, w_2 - number of the varied symbol in a code, etc.

The variation vector also describes the display function of admissible codes set. We will define action of variations vector on any element of admissible codes set with limited length in the form of the operator action on a set element.

$$\tilde{\mathbf{y}}^j = \mathbf{w} \circ \tilde{\mathbf{y}}^i , \tag{14}$$

where $\tilde{\mathbf{y}}^j = \delta_k\left(\tilde{\mathbf{y}}^i\right)$, if $w_1 = k$.

We define variations of code $\tilde{\mathbf{y}}$ from the neighbourhood $\Delta(\tilde{\mathbf{y}})$ by a limited set of variations' vectors

$$\forall \tilde{\mathbf{y}}^j \in \Delta(\tilde{\mathbf{y}}), \ \tilde{\mathbf{y}}^j = \mathbf{w}^k \circ \ldots \circ \mathbf{w}^1 \circ \tilde{\mathbf{y}} , \ k \le \Delta . \tag{15}$$

The purpose of the entered designs is the organization of purposeful search for a set of codes of non-numerical elements. We will consider the generalized genetic algorithm to search for the non-numerical solution with use of small variations.

Let it is necessary to find the optimum admissible solution according to the set criteria on a set (8) codes of non-numerical elements

$$f_{0,i}(\tilde{\mathbf{y}}) \to \min , \ i = \overline{1, n_f} , \tag{16}$$

where

$$f_{0,i}(\tilde{\mathbf{y}}) : \tilde{\Xi}_n \to \mathrm{R}^1 , \ 1 \le i \le n_f . \tag{17}$$

We set the first basic solution

$$\tilde{\mathbf{y}}^0 \in \tilde{\Xi}_n . \tag{18}$$

We chance to choose the basic decision or find it from the analysis of the task and properties of the required optimum solution.

We generate a set of ordered sets for variations' vectors

$$\mathrm{W}^i = \left(\mathbf{w}^{i,1}, \ldots, \mathbf{w}^{i,d}\right), \ i = \overline{1, H} , \tag{19}$$

where $\mathbf{w}^{i,j} = \left[w_1^{i,j} \ \ldots \ w_r^{i,j}\right]^T$, $1 \le j \le d$, d - quantity of vectors in one ordered set, H is quantity of ordered sets or according to terminology of genetic algorithm a size of initial population.

Together with H ordered sets of vectors we also give the identical ordered set for the basic solution

$$\mathrm{W}^0 = \left(\mathbf{w}^{0,1}, \ldots, \mathbf{w}^{0,d}\right), \tag{20}$$