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H_∞ Control of a Parallel-Flow Heat Exchange Process ⋆

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Abstract: In this paper, we consider the H_∞ control problem of a coupled transport-diffusion system related to parallel-flow heat exchange process. It is shown that, by using our previous result for a single diffusion system, the H_∞ control problem can be solved by constructing a residual mode filter (RMF)-based controller which is of finite-dimension.

Keywords: Distributed parameter system, H_∞ control, residual mode filter, semigroup.

1. INTRODUCTION

Since the beginning of the 1980's, the design method of finite-dimensional stabilizing controllers for distributed parameter systems has been proposed by many researchers. In general, when one constructs a finite-dimensional model for a given distributed parameter system and applies a finite-dimensional controller designed for the model to the original infinite-dimensional system, spillover phenomenon may be occured by the influence of unmodeled modes. Sakawa (1983) first introduced two kinds of finite-dimensional observers for linear diffusion systems to reduce the influence of unmodeled modes for the closed-loop system with the finite-dimensional controller. After that, Balas (1988) called one of them as the residual mode filter (RMF, for short), and clarified that the RMF plays an essential role for the construction of finite-dimensional stabilizing controllers. On the other hand, Nambu (1985, 2005) gave the design method of infinite-dimensional stabilizing controllers applicable to a large class of linear parabolic systems, and further accomplished finite-dimensionalization of the controllers. Recently, the author showed that the method based on RMF by Sakawa (1983) was also applicable to a coupled transport-diffusion system related to chemical reaction process (Sano (2012)).

On the other hand, as for H_∞ controllers for distributed parameter systems, the research has been progressed since the beginning of the 1990's. The design method of infinite-dimensional H_∞ state feedback/output feedback controllers was first studied by van Keulen (1993). However, the algorithm was not feasible because one needed to solve two kinds of operator Riccati equations. After that, the design method of finite-dimensional H_∞ controllers for a single diffusion system was given by Sano & Sakawa (1999), in which the RMF was used in the output feedback controller design. The purpose of this paper is to show that our result is applicable to a coupled transport-diffusion system related to parallel-flow heat exchange process.

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2. SYSTEM DESCRIPTION AND FORMULATION

2.1 System Description

We shall consider the following coupled transport-diffusion system related to parallel-flow heat exchange process (Fig. 1):

$$\begin{cases} \dfrac{\partial z_1}{\partial t}(t,x) = D\dfrac{\partial^2 z_1}{\partial x^2}(t,x) - \alpha\dfrac{\partial z_1}{\partial x}(t,x) \\ \qquad\qquad +h_1(z_2(t,x) - z_1(t,x)), \\ \dfrac{\partial z_2}{\partial t}(t,x) = D\dfrac{\partial^2 z_2}{\partial x^2}(t,x) - \alpha\dfrac{\partial z_2}{\partial x}(t,x) \\ \qquad\qquad +h_2(z_1(t,x) - z_2(t,x)) \\ \qquad\qquad +b_1(x)w_1(t) + b_2(x)u(t), \\ \qquad\qquad (t,x) \in (0,\infty)\times(0,1), \\ z_1(t,0) = 0, \quad \dfrac{\partial z_1}{\partial x}(t,1) = 0, \\ z_2(t,0) = 0, \quad \dfrac{\partial z_2}{\partial x}(t,1) = 0, \quad t > 0, \\ z_1(0,x) = z_{10}(x), \ \ z_2(0,x) = z_{20}(x), \ \ x \in [0,1], \end{cases} \tag{1}$$

where $z_1(t,x)$, $z_2(t,x)$ denote the temperatures of fluids at time t and at the point $x \in [0,1]$, and $w_1(t) \in \mathbf{R}$ denotes the disturbance added through the influence function $b_1(x)$, $u(t) \in \mathbf{R}$ the control input added through the influence function $b_2(x)$. $D > 0$ is the heat diffusion coefficient, $\alpha > 0$ the fluid velocity, h_1, $h_2 > 0$ the heat exchange rates between two tubes. For system (1), let us set the controlled output $z_c(t) \in \mathbf{R}^2$ and the measured output $y(t) \in \mathbf{R}$ as follows:

$$\begin{cases} z_c(t) = \left[\displaystyle\int_0^1 c_1(x)z_1(t,x)dx, u(t)\right]^T, \\ y(t) = \displaystyle\int_0^1 c_2(x)z_1(t,x)dx + w_2(t), \quad t > 0, \end{cases} \tag{2}$$

where $c_1(x)$, $c_2(x)$ are the influence functions, and $w_2(t) \in \mathbf{R}$ denotes the disturbance included to the measurement.

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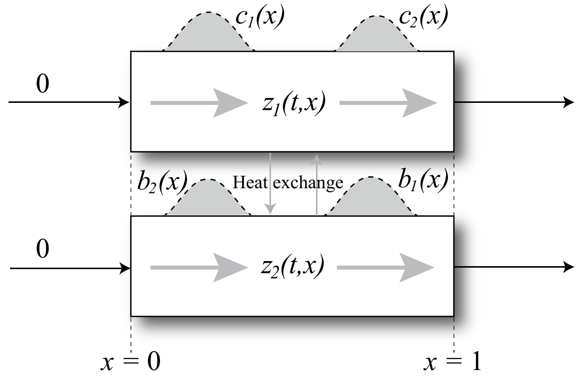

Fig. 1. Parallel-flow heat exchange process.

2.2 Formulation of the System

By defining the differential operator $\mathcal{L}$ as

$$\mathcal{L}\varphi(x) = -D\frac{d^2\varphi(x)}{dx^2} + \alpha\frac{d\varphi(x)}{dx} + h_1\varphi(x),\ x \in (0,1),$$

system (1) is written as

$$\begin{cases} \dfrac{\partial z_1}{\partial t}(t,x) = -\mathcal{L}z_1(t,x) + h_1 z_2(t,x), \\ \dfrac{\partial z_2}{\partial t}(t,x) = (-\mathcal{L} + h_1 - h_2)z_2(t,x) + h_2 z_1(t,x) \\ \qquad\qquad +b_1(x)w_1(t) + b_2(x)u(t), \\ \qquad\qquad (t,x) \in (0,\infty)\times(0,1), \\ z_1(t,0) = 0, \quad \dfrac{\partial z_1}{\partial x}(t,1) = 0, \\ z_2(t,0) = 0, \quad \dfrac{\partial z_2}{\partial x}(t,1) = 0, \quad t > 0, \\ z_1(0,x) = z_{10}(x), \ \ z_2(0,x) = z_{20}(x), \ \ x \in [0,1]. \end{cases} \tag{3}$$

Here, let us define the unbounded operator A as

$$A\varphi = \mathcal{L}\varphi, \quad \varphi \in D(A), \tag{4}$$
$$D(A) = \{\,\varphi \in H^2(0,1)\,;\,\varphi(0) = \varphi'(1) = 0\,\}.$$

Then, A is expressed as an operator of Sturm-Liouville type as follows:

$$(A\varphi)(x) = \frac{1}{w(x)}\left(-\frac{d}{dx}(p(x)\frac{d\varphi(x)}{dx}) + q(x)\varphi(x)\right),$$
$$w(x) = e^{-\beta x}, \quad p(x) = De^{-\beta x}, \quad q(x) = h_1 e^{-\beta x},$$

where $\beta := \alpha/D\,(> 0)$. Therefore, the operator A becomes self-adjoint in the weighted L^2-space $L^2_\beta(0,1)$ whose inner product is defined by

$$\langle\varphi,\psi\rangle_\beta = \int_0^1 \varphi(x)\psi(x)e^{-\beta x}dx, \quad \varphi,\psi \in L^2_\beta(0,1).$$

A has a set of eigenpairs $\{\lambda_i,\varphi_i\}_{i=1}^\infty$ in $L^2_\beta(0,1)$ such that $\{\varphi_i\}_{i=1}^\infty$ forms a complete orthonormal system in $L^2_\beta(0,1)$. Hence, any $f \in L^2_\beta(0,1)$ is expressed as

$$f = \sum_{i=1}^\infty \langle f,\varphi_i\rangle_\beta\varphi_i.$$

The eigenvalues and eigenfunctions of A are calculated as follows:

$$\lambda_i = \omega_i^2 D + \frac{\beta^2}{4}D + h_1, \quad \varphi_i(x) = \mu_i e^{\frac{\beta}{2}x}\sin\omega_i x, \tag{5}$$
$$\mu_i := \left(\frac{1}{2} + \frac{1}{\beta}\cos^2\omega_i\right)^{-\frac{1}{2}} \ (\le \sqrt{2}), \quad i \ge 1,$$

where $\omega_1 < \omega_2 < \cdots < \omega_i < \cdots$ are the solutions of $\tan\omega = -\frac{2}{\beta}\omega$ on $\omega > 0$. Hereafter, for the initial condition and the influence functions, we assume that $z_{10}, z_{20}, b_1, b_2, c_1, c_2 \in L^2_\beta(0,1)(= L^2(0,1))$.

Then, from (3) we have the following equation:

$$\begin{cases} \dfrac{dz_1(t,\cdot)}{dt} = -Az_1(t,\cdot) + h_1 z_2(t,\cdot), \\ z_1(0,\cdot) = z_{10}, \\ \dfrac{dz_2(t,\cdot)}{dt} = (-A + h_1 - h_2)z_2(t,\cdot) + h_2 z_1(t,\cdot) \\ \qquad\qquad +b_1 w_1(t) + b_2 u(t), \\ z_2(0,\cdot) = z_{20}. \end{cases} \tag{6}$$

As for the output equation (2), we can formulate as follows:

$$\begin{cases} z_c(t) = [\langle e^{\beta\cdot}c_1, z_1(t,\cdot)\rangle_\beta, u(t)]^T, \\ y(t) = \langle e^{\beta\cdot}c_2, z_1(t,\cdot)\rangle_\beta + w_2(t), \quad t > 0. \end{cases} \tag{7}$$

Here, by defining the bounded operators $B_i : \mathbf{R} \to L^2_\beta(0,1)$, $C_i : L^2_\beta(0,1) \to \mathbf{R}$ $(i = 1,2)$ as

$$B_i v = b_i v, \quad v \in \mathbf{R},$$
$$C_i\varphi = \langle e^{\beta\cdot}c_i, \varphi\rangle_\beta, \quad \varphi \in L^2_\beta(0,1),$$

system (6), (7) is written as follows:

$$\begin{cases} \dfrac{dz_1(t,\cdot)}{dt} = -Az_1(t,\cdot) + h_1 z_2(t,\cdot), \\ z_1(0,\cdot) = z_{10}, \\ \dfrac{dz_2(t,\cdot)}{dt} = (-A + h_1 - h_2)z_2(t,\cdot) + h_2 z_1(t,\cdot) \\ \qquad\qquad +B_1 w_1(t) + B_2 u(t), \\ z_2(0,\cdot) = z_{20}, \\ z_c(t) = \begin{bmatrix} C_1 z_1(t,\cdot) \\ u(t) \end{bmatrix}, \\ y(t) = C_2 z_1(t,\cdot) + w_2(t). \end{cases} \tag{8}$$

Moreover, by defining the unbounded operator $\mathcal{A} : [D(A)]^2 \subset [L^2_\beta(0,1)]^2 \to [L^2_\beta(0,1)]^2$, the bounded operators $\mathcal{B}_1 : \mathbf{R}^2 \to [L^2_\beta(0,1)]^2$, $\mathcal{B}_2 : \mathbf{R} \to [L^2_\beta(0,1)]^2$, $\mathcal{C}_1 : [L^2_\beta(0,1)]^2 \to \mathbf{R}^2$, $\mathcal{C}_2 : [L^2_\beta(0,1)]^2 \to \mathbf{R}$ and the matrices D_{12}, D_{21} as

$$\mathcal{A} := \begin{bmatrix} -A & h_1 \\ h_2 & -A + h_1 - h_2 \end{bmatrix},$$
$$\mathcal{B}_1 := \begin{bmatrix} 0 & 0 \\ B_1 & 0 \end{bmatrix}, \quad \mathcal{B}_2 := \begin{bmatrix} 0 \\ B_2 \end{bmatrix},$$
$$\mathcal{C}_1 := \begin{bmatrix} C_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathcal{C}_2 := [\,C_2 \ \ 0\,],$$
$$D_{12} := \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad D_{21} := [\,0 \ \ 1\,],$$