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Optimal Control of an Optical System for Material Testing

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Abstract: We implement a prototype for an application of automatic optical material testing in the manufacturing of glass panels. Based on a nonlinear optimal control approach, we present a numerical method for optimal control at run time. The algorithm will be demonstrated and tested with the help of an illustrative example where it turns out that the optimal control is of bang-bang or bang-zero-bang type, depending on the state constraints.

Keywords: Optimal Control, Bang-Bang Control, Bang-Zero-Bang Control, Discretization, Control Oriented Models, Differential Equations, Large-Scale Systems

1. INTRODUCTION

Glass is a very universally applicable composite material, whether as coverage for displays of smart phones, monitors, TV sets and solar panels or in classical applications of the automotive and building industry. Especially due to the strong development of information technology, the world wide demand for glass panels of different sizes has rapidly increased over the last decades.

A naturally arising problem within the manufacturing process is the proper material testing to avoid the delivery of glass panels of poor quality. Typically, the produced glass panels are of rectangular shape and different sizes. Due to technical reasons, the panels cannot be arranged in a row but are laying randomly distributed on a conveyor belt, which is moving with constant speed. Perpendicular to the belt, there is a rail on which an optical sensor can be steered. This sensor is able to detect possible production errors, like scratches, opaque areas and broken edges, inside a small circular neighborhood of its actual position. The general setup of the production line is illustrated in Figure 1, where the borders of the conveyor belt are denoted by b_ℓ and b_r.

The standard strategy of steering the optical system is independent of the configuration of glass panels on the conveyor belt. It is steered from the lower to the upper bound as fast as possible and thereby scans the randomly passed panels for production errors (see Figure 2).

Since the sensor is very sensitive, there are restrictions on the forces acting upon it. Hence the acceleration and speed of the sensor are bounded from above. Therefore, the naive sensor steering strategy, without taking any information on the configuration of the glass panels into account, is very ineffective. Lots of manufacturing errors will not be detected and the profit of the company decreases.

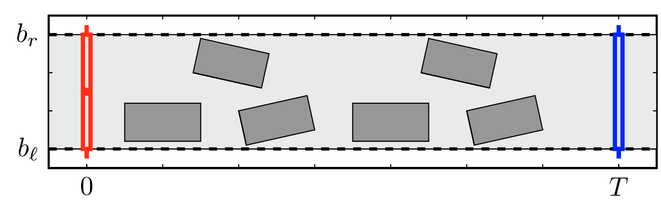

Fig. 1. Glass configuration on conveyor belt with sensor rail on the left (red) and scanner on the right (blue).

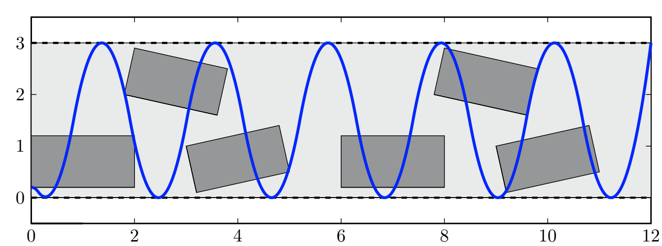

Fig. 2. Standard strategy for steering the optical system.

Within a new production line, one is interested in improving the performance of the material testing process. Therefore a scanner is installed which detects the exact configuration of glass panels on the conveyor belt. The central task is to use these information to improve the process of material testing. For a best possible quality assurance, one is interested in steering the optical sensor such that the way it is passing over glass is maximized.

From the viewpoint of optimal control theory (see, e.g. Macki and Strauss (1982); Betts (2010); Gerdts (2012); Clarke (2013); Berkovitz and Medhin (2013); Alt et al. (2013)), the arising problem is closely related to the well-known rocket car problem (cf. Bushaw (1953)): A car, equipped with a two-sided rocket engine, runs on a rail track. From a given initial position, the car has to be controlled to its destination in a given time. The variables

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in this problem are the position of the car, its velocity and the thrust of the rocket, which serves as control. In Section 2, we will model the movement of the sensor in a very similar way. But for the optimal control problem, we use a different cost functional in comparison to the rocket car problem. The resulting problem turns out to be difficult to solve. Therefore, we use the direct solution approach of discretization in Section 4 and present numerical results in Section 5.

2. THE MATHEMATICAL MODEL

In order to find a suitable model for our real world application problem, we have to describe the different requirements mathematically. Let us start with the definition of appropriate variables. We want to describe the process in the closed time interval $t \in [0, T]$, $T > 0$. We assume that the position of all glass panels that will be tested within this time interval, has already been detected by the scanner. By $u(t)$ we denote the acceleration of the optical sensor across the rail at time t. It is bounded by $u_{\max}$ in each direction, i.e.

$$-u_{\max} \le u(t) \le u_{\max} \quad \text{a.e. on } [0,T]\,.$$

Since the sensor cannot leave the rail across the conveyor belt, the position $x_1(t)$ of the sensor is bounded by

$$b_\ell \le x_1(t) \le b_r \quad \text{for } t \in [0,T]\,.$$

The velocity $x_2(t)$ of the sensor is bounded by $v_{\max}$, i.e.

$$-v_{\max} \le x_2(t) \le v_{\max} \quad \text{for } t \in [0,T]\,.$$

The relation between acceleration, speed, and position of the sensor can be described with the help of linear ordinary differential equations of first order with given starting position s_0 and starting velocity v_0:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) && \text{a.e. on } [0,T]\,,\\ x_1(0) &= s_0\,,\\ \dot{x}_2(t) &= u(t) && \text{a.e. on } [0,T]\,,\\ x_2(0) &= v_0\,. \end{aligned}$$

Furthermore we need some parameter functions. By v we denote the constant horizontal speed of the conveyor belt. Due to the combination of the movement of the conveyor belt and the movement of the sensor, the resulting speed $v_s(t)$ of the sensor relative to the glass panels can be computed via

$$v_s(t) := \sqrt{x_2(t)^2 + v^2}$$

for all $t \in [0, T]$. As already mentioned, we are interested in maximizing the distance of the sensor passing over glass panels. Basically, this distance can be described as the integral of the velocity $v_s(t)$ over all t where the sensor is located over a glass panel.

The tricky part is to describe whether it fulfills this condition or not. To this end, we define an "indicator" function by

$$\mathcal{I}(t) = \begin{cases} 1\,, & \text{sensor is above glass panel at time } t\,,\\ 0\,, & \text{otherwise.} \end{cases}$$

Based on this function we can theoretically define the function

$$g(x_1(t)) := \begin{cases} 1\,, & \text{sensor is above glass panel at time } t\,,\\ 0\,, & \text{otherwise,} \end{cases}$$

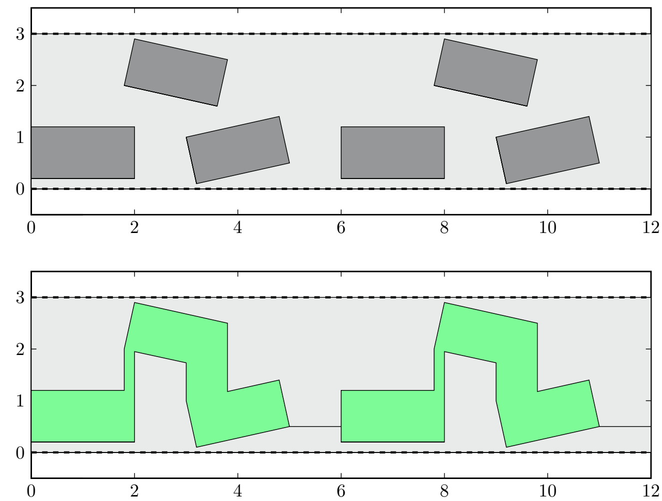

Fig. 3. Convexification of the sets $\Sigma(t)$: Initial glass configuration (top) and transformed area of glass (bottom).

and use the cost functional

$$f(x_1, x_2, u) := \int_0^T g(x_1(t))\, v_s(t)\,\mathrm{d}t\,. \tag{1}$$

This theoretical approach has two drawbacks in view of the practical solution of the problem. Let $\Sigma(t) \subseteq \mathbb{R}$ denote the set where the sensor is placed above a glass panel at time t. This set is the union of finitely many closed intervals and this union does not have to be convex. In addition, the function g is discontinuous, which will cause numerical problems. We will therefore apply a convexification of the set $\Sigma(t)$ and use a smooth approximation of the indicator function.

$\Sigma(t)$ describes the set of glass, which can possibly be tested by the sensor at time t. When at least two plates, which are not touching, pass the rail at the same time, the set of glass underneath the sensor rail will be the union of two intervals—the vertical dimensions of both glass panels. For simplification, we will apply a convexification of this set of glass, by simply assuming that there is glass all over the convex hull $\operatorname{conv}\Sigma(t)$ of this set. This can be realized by defining

$$s_\ell(t) := \text{leftmost position,}$$

and

$$s_r(t) := \text{rightmost position,}$$

where there is glass underneath the sensor rail at time t. The application of this convexification concept to our test setting is shown in Figure 3.

We have full access to the parameters s_ℓ and s_r via the scanner of the conveyor belt configuration. If there is no glass panel on the conveyor belt at time t, we define $s_\ell(t)$ and $s_r(t)$ by their last given values. This is done in Figure 3 for $t \in [5, 6]$, where $s_\ell(t) = s_r(t) = 0.5$.

Next we apply a cubic smoothing for the indicator function. With an arbitrary constant $\varepsilon > 0$ we define

$$g_\varepsilon(x,t) = \begin{cases} \varepsilon\left((x - s_\ell(t))\right)^3 + 1\,, & x < s_\ell(t)\,,\\ 1\,, & x \in [s_\ell(t), s_r(t)]\,,\\ -\varepsilon\,(x - s_r(t))^3 + 1\,, & x > s_r(t)\,. \end{cases}$$

The function $g_\varepsilon(x,t)$ is continuous and twice continuously differentiable with respect to x. With the help of the