Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-25 (2015) 091–096

Optimal Control of a Two-body Vibration-driven Locomotion System in a Resistive Environment ⋆

Nikolay Bolotnik, Felix Chernousko, Tatiana Figurina

Institute for Problems in Mechanics of the Russian Academy of Sciences, 101-1 Vernadsky Ave, Moscow 119526 Russia (e-mail: bolotnik@ipmnet.ru)

Abstract: A two-body system moving along a horizontal line in a resistive medium with a nonlinear resistance law is considered. One of the bodies (main body) interacts with the environment and with the other body (internal body), which interacts with the main body but does not interact with the environment. A periodic motion of the internal body relative to the main body sustains the progressive motion of the main body with periodically changing velocity. By solving an optimal control problem, the optimal motion of the internal body that maximizes the average velocity of the system is found.

Keywords: locomotion; vibration-driven systems; optimal control.

1. INTRODUCTION

A rigid body with internal masses that perform periodic motions can move progressively in a resistive medium with nonzero average velocity. This phenomenon can be used as a basis for the design of locomotion systems able to move without special propelling devices (wheels, legs, caterpillars, or screws) due to direct interaction of the body with the environment. Such systems have a number of advantages over systems based on the conventional principles of motion. They are simple in design, do not require gear trains to transmit motion from the motor to the propellers, and their body can be made hermetic and smooth, without any protruding components. The said features make this principle of motion suitable for capsule-type microrobots designed for motion in a constrained space (e.g., inside narrow tubes) and in vulnerable media, for example, inside a human body for delivering a drug or a diagnostic sensor to an affected organ.

An optimization problem for the motion of a body that is controlled by its interaction with a movable internal body and moves in a resistive environment was first stated by Chernousko (2005, 2006). The case where the main body moves along a straight line on a horizontal plane and is acted upon by Coulomb's dry friction was considered. Periodic control modes were constructed for the relative motion of the internal body, such that the main body moves with periodically changing velocity and passes the same distance in a prescribed direction for each period. The internal body is allowed to move within fixed limits. It was assumed that at the beginning and at the end of each period, the velocity of the main body vanishes, and the internal body rests in one of its extreme positions. Velocity-controlled and acceleration-controlled modes were considered for the motion of the internal body. The optimal parameters for which the average velocity of the main body is a maximum were found for both modes. Later (Chernousko (2008)), the problems described above were solved without the assumption that the velocity of the main body vanishes when the internal body is in its extreme position. The optimal parameters for the velocity-controlled mode of motion of the internal body were found not only for the dry friction environment but also for the environments with piecewise linear and quadratic laws of resistance of the environment. The optimal parameters of the acceleration-controlled mode of motion of the internal body were obtained numerically by Fang and Xu (2011) for the quadratic resistance law.

Figurina (2007) solved an optimal control problem for the motion of the mechanical system described above along a straight line in a horizontal plane, provided that Coulomb's friction acts between the main body and the plane. The acceleration of the internal body relative to the main body is used as the control variable. A constraint is imposed on the absolute value of this variable. A periodic control with zero average and the corresponding motion of the main body with periodically changing velocity and maximum displacement for the period were constructed. The control constructed allows one to restore the periodic law of motion of the internal body that generates the optimal motion of the system. A similar problem was solved by Bolotnik and Figurina (2008) for a system with two internal bodies, one of which moves periodically along a horizontal line, parallel to the line of motion of the main body, while the other moves along a vertical line. The internal body moving along the vertical enables one to control the normal pressure of the main body on the supporting plane and, as a consequence, the force of friction that acts on the main body when it is moving.

⋆ This study was partly supported by the Russian Foundation for Basic Research (projects 14-01-00061, 14-01-91153, and 15-51-12381).

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2015.11.065

Yegorov and Zakharova (2010) constructed and investigated energy-optimal control modes for the system with one internal body moving in the environments with power-law resistance. The energy consumption was characterized by the work produced by the resistance force for the period of motion of the system. When constructing the optimal control, the period of the relative motion of the internal body and the average velocity of the system were prescribed; no other constraints were imposed on the motion of the system.

In the present paper, the problem of the optimal control of the rectilinear motion of the main body excited by periodic relative motion of the internal body is considered for a wide class of laws of resistance of the environment. The relative acceleration of the internal body is used as the control variable. The absolute value of the control variable is subjected to a constraint. A periodic control function with zero average that corresponds to the motion of the main body with periodically changing velocity and maximum displacement for the period is sought. It is assumed that the period of the relative motion of the internal body is fixed equal to the period of change in the velocity of the main body. The qualitative features of the optimal motion, including the number of switchings of the control function for the period, the presence / absence of singular modes in which the control variable does not take on its maximal or minimal allowed values, and the conditions for entering a singular mode and leaving it, are identified. It is proved that a singular mode necessarily occurs in the optimal motion in the environments with the law of resistance described by an odd function of the velocity, which, in addition, is either concave or convex when the velocity is constant in sign. This is important in view of the fact that functions of this class are frequently used for modelling the resistance of viscous media to the motion of rigid bodies. Algorithms for calculation of the optimal control reduced to solving systems of finite equations are given for some particular classes of resistance laws. The optimal motion of the system in the environments with low-friction power-law resistance is studied.

2. STATEMENT OF THE PROBLEM

A two-body mechanical system moving in a resistive environment is considered (Fig. 1). The system consists of the main body that interacts with the environment and an auxiliary internal body that interacts with the main body but does not interact with the environment. Both bodies move translationally along the same horizontal line. The motion of the system is controlled by the motion of the internal body relative to the main body. Let x denote the displacement of the main body relative to the fixed environment, ξ the displacement of the internal body relative to the main body, M the mass of the main body, m the mass of the internal body, and $R(\dot{x})$ the resistance force exerted by the environment on the moving main body.

The motion of the system is governed by the differential equation

$$(M+m)\ddot{x} = -m\ddot{\xi} + R(\dot{x}). \quad (1)$$

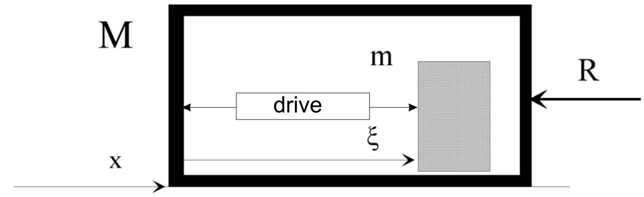

Fig. 1. Two-body system

For this system, we will seek a T-periodic motion of the internal body $\xi(t)$ constrained by $|\ddot{\xi}| \leq A$, where A is a given positive quantity, and the corresponding motion of the main body $x(t)$ such that its velocity $\dot{x}(t)$ is T-periodic and the distance travelled for the period is a maximum. The period T is fixed.

Introduce the dimensionless variables

$$v = \frac{M+m}{mAT}\dot{x},\ w = \frac{\dot{\xi}}{AT},\ u = \frac{\ddot{\xi}}{A},\ \tilde{t} = \frac{t}{T}, \quad (2)$$
$$r(v) = -\frac{1}{mA}R\left(\frac{mAT}{M+m}v\right)$$

to state the optimal control problem for the system under consideration as follows.

Problem. For the system

$$\dot{v} = -u - r(v), \quad \dot{w} = u \quad (3)$$

find a control $u(t)$ that satisfies the constraint

$$|u| \leq 1 \quad (4)$$

and maximizes the performance index

$$J = \int_0^1 v(\tau)d\tau, \quad (5)$$

subject to the conditions

$$v(0) = v(1), \quad w(0) = w(1). \quad (6)$$

The boundary conditions (6) are implied by the periodicity of the functions $v(t)$ and $\xi(t)$.

We assume that the function $r(v)$ is continuous, vanishes for $v = 0$, monotonically increases, does not have linear segments, and is twice differentiable everywhere, except possibly for the point $v = 0$, i.e.,

$$r(0) = 0, \quad r'(v) \geq 0, \quad r''(v) \neq 0 \text{ for any interval.} \quad (7)$$

From the relations (3) and (6) it follows that $\int_0^1 r(v(t))dt = 0$, which implies that the function $v(t)$ changes in sign or is identically zero.

The solution of the optimal control problem possesses a number of important properties that are stated by the following propositions (proofs are omitted).

Proposition 1. For any admissible motion, $|r(v(t))| < 1$.

Proposition 2. For any resistance law $r(v)$ satisfying the conditions of (7), there exists an admissible control for which the displacement of the main body for the period is not equal to zero.