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Flatness-based tracking control for a pneumatic system with distributed parameters

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Abstract: This paper considers the flatness-based tracking control design for a pneumatic system, where a tank and a proportional valve are connected via a long transmission line of approximately 20 m. Motivated by this test bench set-up and the aim to implement fast pressure changes in the tank, three distributed-parameter models of different complexity and physical accuracy are presented, involving linear and quasi-linear hyperbolic partial differential equations. A flatness-based state feedback control is derived based on a linear distributed-parameter model of the pneumatic system. In combination with flatness-based feedforward controllers, designed for two of the three models, tracking controllers are obtained. Based on a simulation model of the test bench, verified to match the measurement data almost perfectly, the controllers are shown to execute fast pressure changes in the tank very accurately.

Keywords: pneumatic systems, distributed-parameter systems, partial differential equations, model-based control, feedforward control, feedback control, flatness

1. INTRODUCTION

Pneumatic systems are widely spread in industrial applications. One important class arises, for example, wherever compressed air is supplied from a source to a target process via a transmission line. Unfortunately, pneumatic systems are commonly known to have a relatively low energy efficiency (see Radgen (2001)), a problem that can be tackled by means of sophisticated controllers. In Rager et al. (2016), a pole placement approach is used to stabilize a lumped-parameter model of a transmission line, given in terms of linear ordinary differential equations (ODEs). A similar mathematical model serves as basis for the model predictive controller designed in Alaya and Fiedler (2016). Though further control approaches for pneumatic systems with long transmission lines can be found, collectively, they neglect or approximate the distributed character of the transmission line by discretizing its spatial coordinate. However, as soon as lines of significant length are involved, the finite speed of propagation therein should be taken into account. Rather than using linear ODE models of high order, the infinite-dimensional character is more accurately described by partial differential equations (PDEs) of hyperbolic type.

The set-up of the test bench to be controlled in this paper involves a transmission line of almost 20 m feeding a tank. Pressure changes in the tank of several bar are to be stabilized within less than 0.5 s. The same set-up is considered in Kern and Gehring (2017), where a stabilizing state feedback is designed using the backstepping approach in Deutscher et al. (2017) for bi-directionally coupled PDE-ODE systems. Likewise, the tracking controllers designed in this paper are based on the five distributed-parameter models derived for the pneumatic test bench in Kern (2017), three of which are essential here: a so-called plant model, a quasilinear model and a linear model. These models of decreasing complexity are well suited to represent the infinite-dimensional character of the transmission line and the nonlinearities occurring. This paper employs models of appropriate complexity and different control approaches. A state feedback is designed for the linear model by application of the flatness-based approach presented in Woittennek (2013). As a mere stabilization fails to achieve the control aim specified before, model-based feedforward controllers are derived for the linear and the quasilinear model following the flatness-based design in Knüppel and Woittennek (2015). In combination with the state feedback, the tracking controller using the quasilinear feedforward is shown to meet the control objective without exceeding the physical input limitations at the test bench.

In the next section, mathematical models of the pneumatic test bench are derived. Based on that, a flatness-based state feedback is designed for the linear model in Section 3. In combination with two flatness-based feedforward controllers given in Section 4, the performance of the resulting tracking controllers is discussed in Section 5.

2. MODELS OF THE TEST BENCH

The test bench shown in Fig. 1 basically consists of a proportional 5/3-directional pneumatic valve (used only in a 3/3-configuration, as illustrated in Fig. 2) that is

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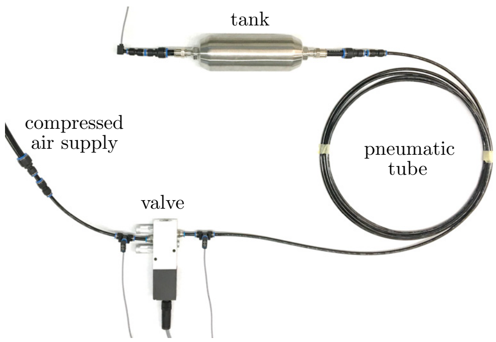

Fig. 1. A photo of the main components of the test bench [1] .

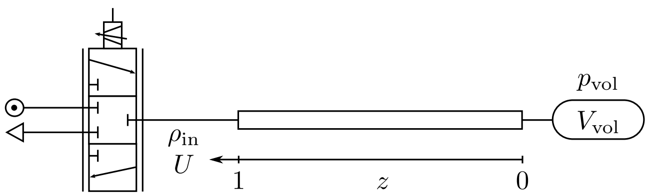

Fig. 2. Sketch of the pneumatic test bench.

connected to a tank via a tube. Compressed air of 9 bar is supplied to the valve. All essential parameters characterizing the test bench are summarized in Table 1 (see Kern (2017) and references therein for more details).

Due to the small inner diameter D of the tube as compared to its length L, the flow in the transmission line is considered as one-dimensional. Introducing a scaled spatial variable $z \in [0, 1]$, as sketched in Fig. 2, and denoting the time by t, the air flowing through the tube is described by the density $\rho(z, t)$, the pressure $p(z, t)$, the temperature $T(z, t)$, the velocity $v(z, t)$ in the axial direction of the tube and the total energy $(\rho e)(z, t)$. The tank of constant volume V_{vol} is modeled as a lumped-parameter system with its quantities denoted by $(\cdot)_{\mathrm{vol}}(t)$.

A detailed physical derivation of the equations modeling the pneumatic test bench, including all assumptions involved, can be found in Kern (2017). For comprehensibility, the essential relations are restated in the following.

2.1 Quasilinear third-order model (plant model)

In Kern (2017), the Euler equations for one-dimensional flow are augmented by taking effects of friction and heat transfer into account. The mathematical model thusly obtained is a set of three quasilinear PDEs:

$$\partial_t \rho - \frac{1}{L}\partial_z(\rho v) = 0 \tag{1a}$$

$$\partial_t(\rho v) - \frac{1}{L}\partial_z(\rho v^2 + p) = -f_{\mathrm{comp}}\frac{\rho v|v|}{2D} \tag{1b}$$

$$\partial_t(\rho e) - \frac{1}{L}\partial_z(v(\rho e + p)) = \alpha\frac{\pi D}{A}(T_0 - T) - f_{\mathrm{comp}}\frac{\rho v^2|v|}{2D}, \tag{1c}$$

where $A = \pi D^2/4$ is the cross-section area of the tube, and ∂_t and ∂_z denote partial derivatives w.r.t. to t and z. In (1c), the first term on the right-hand side models thermal losses due to the tube being not ideally insulated.

[1] Note that the dimensions of some components in the photo do not match the test bench parameters in Table 1, relevant in this paper.

Table 1. Test bench parameters.

D	inner tube diameter	$8 \cdot 10^{-3}$ m
L	length of the tube	19.83 m
ρ_0	ambient air density	1.21 kg/m^3
p_0	ambient air pressure	1.01 bar
T_0	ambient air temperature	293.15 K
R_{s}	specific gas constant of air	287.05 J/kgK
ε	height of roughness elements	$1.5 \cdot 10^{-6}$ m
γ	ratio of specific heats	1.4
V_{vol}	tank volume	$6.46 \cdot 10^{-4}$ m^3
R_{vol}	thermal resistance	$4 \cdot 10^{-3}$ K/W

As the heat capacity of the tube is much greater than the one of the air in the tube, the wall temperature can be considered to be constant and equal to the (constant) ambient air temperature T_0. By the terms proportional to f_{comp}, friction losses due to the viscosity of the air in the tube are accounted for. Both the heat transfer coefficient α and the friction factor f_{comp} depend on ρ, v and e and can be calculated by suitable correlations (Munson (2013); Idel'chik and Steinberg (1996)). As the air in the tube is assumed to be a polytropic, ideal gas, the pressure p, the density ρ and the temperature T are related in terms of the ideal gas law

$$p = \rho R_{\mathrm{s}} T, \tag{2}$$

with the specific gas constant R_{s}. Moreover, e, v, p and ρ satisfy

$$e = \frac{1}{2}v^2 + \frac{p}{\rho}\frac{1}{\gamma - 1}, \tag{3}$$

where γ is the ratio of specific heats.

A proportional valve fed by a compressed air supply is attached to the tube at $z = 1$. As the aperture of the valve can be controlled, it is assumed that the (scaled) mass flow $\dot{m}(1, t) = A(\rho v)(1, t)$ serves as the control input, giving rise to the boundary conditions (BCs)

$$\rho(1, t) = \rho_{\mathrm{in}}(t), \quad (\rho v)(1, t) = U(t) \tag{4}$$

at the inlet. The input U is specified by the valve's aperture. The density ρ_{in} depends on the characteristics of the valve, the compressed air supply and the mass flow. As the connection between the air supply and the valve is a long transmission line in itself, no mathematical model of ρ_{in} is available at the moment.

Using the laws of conservation for the tank, terminating the tube at $z = 0$, the third and final BC of (1) reads

$$\frac{\mathrm{d}}{\mathrm{d}t} m_{\mathrm{vol}}(t) = A(\rho v)(0, t) \tag{5a}$$

$$c_{v,\mathrm{vol}}\frac{\mathrm{d}}{\mathrm{d}t}\left(m_{\mathrm{vol}} T_{\mathrm{vol}}\right)(t) = A(\rho v e)(0, t) + A(p v)(0, t) + \frac{1}{R_{\mathrm{vol}}}(T_0 - T_{\mathrm{vol}}(t)). \tag{5b}$$

Therein, $c_{v,\mathrm{vol}}$ is the specific heat capacity of air at a constant volume and R_{vol} is the constant thermal resistance of the tank jacket.

2.2 Simplified models

If the thermal equilibrium of the air with its surroundings is reached almost instantaneously, the flow in the tube can be considered as isothermal, i.e. $T(z, t) = T_0$ (cf. Osiadacz and Chaczykowski (2001)). Subsequently, the