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Evolution of Stresses and Deformations in Hollow Cylinder with Variable Material Composition: Mathematical Modeling and Optimization ⋆

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Abstract: The present paper is aimed at modelling and optimization of stress-strain state for a multilayered structure, which is represented by a hollow hyperelastic cylinder. Two cases are considered. In the first case the structure is assembled by a finite number of hollow cylinders (discrete assembling process). The second case is a limit for the discrete processes, when a number of layers increases indefinitely with a fixed total thickness of the whole structure. It is convenient to use the non-Euclidean approach to investigate so obtained continuous structure. The optimization for stress intensity have been performed over the control function of inner pressure. The distributions of Ricci invariant that characterize measure of non-Euclidity are obtained for the uncontrolled and optimized processes. The proposed approach can be used to determine the optimal strategies for synthesis of 3D-details by DLP stereolithography.

Keywords: incompatible deformations, residual stresses, hollow cylinder, variable material composition, additive technologies, optimization

1. INTRODUCTION

At present there is an intensive development of multilayered thin coating technologies which are used in the manufacturing of microelectronics, photonics and microelectromechanical devices. Multilayered structures may be produced by such technologies as lithography, Xu Ma (2010), Layer-by-Layer (LbL) processing, Decher and Schlenoff (2012), self-assembling, Chen (2013), *etc.* These technologies are widespread, but there exist a lot of problems in their implementation. The presence of significant residual (internal) stresses and distortion in thin-walled structures are among such problems. In this regard the improving of script for technological process aimed at internal stresses decline is an actual structural optimization problem, Sigmund and Maute (2013). Moreover it is promising idea to control the physical properties of such structures by prescribed deformation of the substrate and the deposited layers during manufacturing.

The present paper is aimed at modelling of stress-strain state for a multilayered cylindrical LbL structure. The structure is modeled in two ways: as axisymmetric assemble of a finite number of hollow hyperelastic cylinders; as structurally inhomogeneous cylindrical solid considered as a limit case for the latter one. In order to take into account specific features of technological process that results in internal stresses we study the cylindrical structure and the structurally inhomogeneous cylinder as particular cases of a body with variable composition, Lychev (2017). For simplicity we describe the technological process as successive assembling accompanied by layer-by-layer shrinkage and assume that after shrinkage material becomes incompressible. These assumptions, on one hand, permit us to obtain stress and strain distributions analytically and greatly simplify numerical implementation for optimization problem. On the other hand, such assumptions make it possible to describe the evolution of internal stresses that arise in a part created by sequential curing of layers during the DLP stereolithography, Yu and Pan (2015). The analysis for finite assembling is based on the provisions of the conventional (Euclidean) continuum mechanics. Investigations of the stress-strain state of linearly elastic multilayer self-stressed cylinders are presented in numerous papers, Lee et al. (2001). Nonlinear problems are studied much less. The analysis for structurally inhomogeneous cylinder is based on an approach of geometrical (non-Euclidean) mechanics, Maugin (1993); Marsden and Hughes (1994); Lychev and Manzhirov (2013). This allows one to describe stress-free reference shape of a body in whole, even if this shape does not exists in conventional (Euclidean) sense, by embedding it into a space with non-Euclidean connection which is the novelty of the proposed approach.

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2. HOLLOW CYLINDER WITH DISCRETE INHOMOGENEITY

In this section we consider the following assembling process that generates cylindrical LbL structure. Suppose that the process is discrete, i.e., it is determined by a finite number of steps. Before the first step the body consists of cylindrical substrate only. At the beginning of the first step the first layer joins to the substrate and, being adhered to it, instantly shrinks. It causes internal stresses in the two-layered cylindrical structure. At the beginning of the second step the second layer adheres and, in turn, instantly shrinks too. It causes some internal stresses in the three-layered structure. The stress distribution in the first two layers changes. Such processes are repeated $N-1$ times.

Formalization of the above mentioned assembling process can be carried out as follows. The solid is represented by a family $\mathfrak{G} = \{\mathfrak{B}_s\}_{s=1}^{N}$ of bodies treated as three-dimensional smooth manifolds, Noll (1967). Bodies from $\mathfrak{G}$ are considered as smooth submanifolds, Lee (2012), of an ambient three dimensional smooth manifold $\mathcal{M}$. It means that $\mathcal{M}$ is sufficiently large to hold each element of the $\mathfrak{G}$. We refer to $\mathcal{M}$ as *material manifold*. Also introduce definition for layers $\mathfrak{I}_k$ which are smooth three dimensional manifolds $\mathfrak{I}_{s+1} = \mathrm{Int}(\mathfrak{B}_{s+1} \setminus \mathfrak{B}_s)$, $s = 1, \ldots, N-1$; $\mathfrak{I}_1 := \mathfrak{B}_1$, where Int is the *interior operation relatively to the topology of* $\mathcal{M}$. In what follows the symbol $\vee$ denotes the joining operation [1]:

$$\mathfrak{B}_{s+1} = \mathfrak{B}_s \vee \mathfrak{I}_{s+1} := \\ := \mathfrak{B}_s \cup \mathrm{Int}(\mathfrak{B}_{s+1} \setminus \mathfrak{B}_s) \cup \partial_{\mathfrak{B}_{s+1}}(\mathfrak{B}_{s+1} \setminus \mathfrak{B}_s),$$

where the operations of interior ($\mathrm{Int}_{\mathfrak{B}_{s+1}}$), and boundary ($\partial_{\mathfrak{B}_{s+1}}$) are considered in the *induced topology* of $\mathfrak{B}_{s+1}$.

The family $\mathfrak{G}$ satisfies the following conditions:

(i) Layers $\mathfrak{I}_k$ have stress free reference configurations in three-dimensional Euclidean point space [2] $\mathcal{E}$:
$$\varkappa_R^k : \mathfrak{I}_k \to \mathcal{E}, \quad k = 1, \ldots, N.$$
(ii) Images of the layers $\mathfrak{I}_k$ under configurations $\varkappa_R^k$, $\varkappa_R^k(\mathfrak{I}_k) \subset \mathcal{E}$, are represented by hollow cylinders with the inner radii R_i^k, the exterior radii R_e^k, and the common height h.

(iii) All bodies $\mathfrak{B}_s$, $s = 2, \ldots, N$, result by a sequential joining of the layers $\mathfrak{I}_k$ to $\mathfrak{B}_1$. The image of any body $\mathfrak{B}_s$ under the actual configuration $\varkappa_s$ is a hollow cylinder. The cylinders $\varkappa_s(\mathfrak{I}_k)$ are coaxial. In what follows we will denote actual inner radius of k-th layer by $r_{i,\,s-1}^k$, and actual exterior radius of k-th layer by $r_{e,\,s-1}^k$. The index $s-1$ after comma designates the number of assembly.

(iv) Hollow cylindrical layers join consequently and each layer undergoes shrinkage at once after joining. Instantaneous shrinkage can be formalized by the assumption: each layer before joining to assembly undergoes axisymmetric plane deformation from stress free state.

(v) The inner radius of the first layer in reference configuration is given by $R_i^1 = \rho$. The reference thicknesses $\Delta^k = R_e^k - R_i^k$, $k = 1, \ldots, N$, of the layers are prescribed. Other reference radii are unknown, but they can be determined from the recurrence relation: inner reference radius of layer $\mathfrak{I}_{s+1}$ is equal to $R_i^{s+1} = S^{s+1} r_{e,\,s-1}^s$, where $0 < S^{s+1} < 1$ is a shrinkage coefficient, and $r_{e,\,s-1}^s$ is exterior actual radius of the layer $\mathfrak{I}_s$.

An assembling process specified by these assumptions is illustrated on Fig. 1.

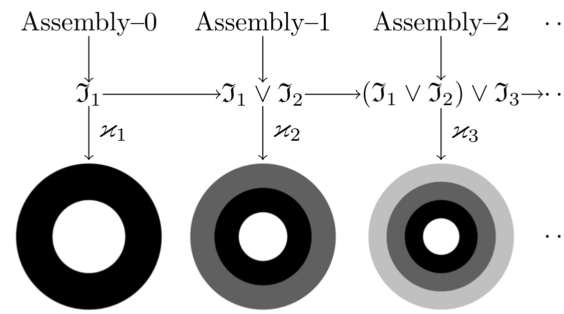

Figure 1. Assembling process

Let $k \in \{1, \ldots, s\}$ be an index number of a layer in the assembly with the index number of layers $s \in \{1, \ldots, N\}$. According to the assumptions stated above, the deformation of reference shape $\varkappa_R^k(\mathfrak{I}_k)$ into actual, $\varkappa_s(\mathfrak{I}_k)$, $d_k : \varkappa_R^k(\mathfrak{I}_k) \to \varkappa_s(\mathfrak{I}_k)$, is simply "bloating". In cylindrical coordinates it can be written as:

$$(r,\, \theta,\, z) = \left(\sqrt{(R^k)^2 + a_{s-1}^k},\, \Theta^k,\, Z^k\right),$$

where (R^k, Θ^k, Z^k) are the reference coordinates of points in the layer $\mathfrak{I}_k$ and (r, θ, z) are their coordinates in the assembly, a_{s-1}^k is the deformation parameter.

The deformation gradient $\boldsymbol{F}_k$ and the left Cauchy-Green tensor $\boldsymbol{B}_k = \boldsymbol{F}_k \boldsymbol{F}_k^{\mathrm{T}}$ are represented by decompositions [3]

$$\boldsymbol{F}_k = \frac{R^k}{\sqrt{(R^k)^2 + a_{s-1}^k}} \boldsymbol{e}_r \otimes \boldsymbol{e}^{R^k} + \boldsymbol{e}_\theta \otimes \boldsymbol{e}^{\Theta^k} + \boldsymbol{e}_z \otimes \boldsymbol{e}^{Z^k}, \quad (1)$$

$$\boldsymbol{B}_k = \frac{r^2 - a_{s-1}^k}{r^2} \boldsymbol{e}_r \otimes \boldsymbol{e}^r + \frac{r^2}{r^2 - a_{s-1}^k} \boldsymbol{e}_\theta \otimes \boldsymbol{e}^\theta + \boldsymbol{e}_z \otimes \boldsymbol{e}^z. \quad (2)$$

Assume that the layers are made of hyperelastic material. The corresponded constitutive relations have the form: $\boldsymbol{T}_k = -p\boldsymbol{I} + J_1 \boldsymbol{B}_k + J_{-1} \boldsymbol{B}_k^{-1}$, where $\boldsymbol{I}$ is the identity tensor, p is a hydrostatic component, and $J_1 = (1+\beta)\mu/2$, $J_{-1} = (\beta - 1)\mu/2$ are scalar response (constant) functions; here β and μ are material constants. Taken into account the diagonality of the tensor $\boldsymbol{B}_k$, defined by (2), one can derive the expression for the Cauchy stress tensor in the layer with index number k:

$$\boldsymbol{T}_k = T_k^{rr} \boldsymbol{e}_r \otimes \boldsymbol{e}_r + T_k^{\theta\theta} \boldsymbol{e}_\theta \otimes \boldsymbol{e}_\theta + T_k^{zz} \boldsymbol{e}_z \otimes \boldsymbol{e}_z,$$

[1] The operation $\vee$ has the following sense. When we obtain sequentially $\mathfrak{B}_{s+1}$ from $\mathfrak{B}_s$ and layer $\mathfrak{I}_{s+1}$ we have to include all interior points of $\mathfrak{B}_s$ and $\mathfrak{I}_{s+1}$ and, besides them, the points of common boundary of $\overline{\mathfrak{B}_s}$ and $\overline{\mathfrak{I}_{s+1}}$, which represent thin film between "gluing" solids.

[2] Physical space is represented by a three dimensional Euclidean point space with scalar product $(\cdot)$. The symbol $\mathcal{V}$ denote translation space. Vector and tensor fields are denoted by Latin boldface letters. By a configuration of the body $\mathfrak{B}$ (smooth three dimensional manifold) we mean a smooth embedding $\varkappa : \mathfrak{B} \to \mathcal{E}$. The symbol $\mathfrak{C}(\mathfrak{B};\, \mathcal{E})$ denotes the set of all configurations. The image $\varkappa(\mathfrak{B})$ is referred to as a shape, Noll (1967). Only simple materials, Truesdell and Noll (2004) are considered.

[3] The symbol $\boldsymbol{e}^c$ denotes the element of the dual frame, i.e. $\boldsymbol{e}^c \cdot \boldsymbol{e}_j = \delta_j^c$.