Available online at www.sciencedirect.com

ScienceDirect

IFAC-PapersOnLine 48-25 (2015) 172–177

On a Problem of Guarantee Optimization in Time-Delay Systems ⋆

Mikhail Gomoyunov * **Anton Plaksin** *

* *Institute of Mathematics and Mechanics of the Ural Branch of the Russian Academy of Sciences, S.Kovalevskaya str. 16, Ekaterinburg, 620990, Russia;*

Institute of Mathematics and Computer Sciences, Ural Federal University, Mira str. 19, Ekaterinburg, 620002, Russia,

(*e-mail: m.i.gomoyunov@gmail.com, a.r.plaksin@gmail.com*)

Abstract: A control problem under conditions of disturbances is considered for a linear time-delay dynamical system. The goal of the control is to minimize a non-terminal quality index that evaluates a motion history and realizations of control and disturbance actions. The control problem is posed within the game-theoretical approach. For calculating the optimal guaranteed result of the control and constructing a control scheme that ensures this result, two methods are proposed. The first one is based on an appropriate approximation of the quality index. The second one is based on a finite-dimensional approximation of the dynamical system. Both methods allow us to reduce the control problem to high-dimensional auxiliary differential games without delays and with terminal quality indices. An illustrative example is considered, simulations results are given.

Keywords: control theory; differential games; time-delay systems.

1. INTRODUCTION

The paper is devoted to a control problem under conditions of disturbances for a dynamical system described by a linear delay differential equation. The goal of the control is to minimize a non-terminal quality index that evaluates a motion history and realizations of control and disturbance actions. The control problem is posed within the game-theoretical approach of Krasovskii and Subbotin (1988); Krasovskii and Krasovskii (1995) (see also Osipov (1971) for time-delay systems).

Two methods are proposed for calculating the optimal guaranteed result of the control and constructing a control scheme that ensures this result. Both methods reduce the control problem to appropriate differential games without delays and with terminal quality indices. The first method is based on an approximation of the quality index. It uses a functional interpretation of the control process (see, e.g., Krasovskii (1959)) and certain predictions of system motions (see Lukoyanov and Reshetova (1998)). The second method is based on a finite-dimensional approximation of the dynamical system by a system of ordinary differential equations (see, e.g., Krasovskii (1964); Banks and Kappel (1979)). This approximating system is used as a leader (see, e.g., Krasovskii and Subbotin (1988)) for the initial system (see Lukoyanov and Plaksin (2013, 2015)). A solution to the obtained high-dimensional auxiliary differential games is constructed by the upper convex hulls method (see, e.g., Krasovskii (1987); Krasovskii and Krasovskii (1995); Lukoyanov (1994, 1998)).

The efficiency of the proposed solution methods is illustrated by an example. Results of numerical simulations are given.

2. STATEMENT OF THE PROBLEM

In the paper the following control problem is considered. A dynamical system is described by the delay differential equation

$$\dot{x}(t) = A(t)x(t) + A_h(t)x(t-h) + B(t)u(t) + C(t)v(t),$$
$$t_0 \le t < \vartheta, \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^r, \quad v \in \mathbb{R}^s, \qquad (1)$$

with the initial condition

$$x_{t_0}(\cdot) = \sigma(\cdot) \in C. \qquad (2)$$

Here t is the time variable, x is the state vector, $\dot{x}(t) = \mathrm{d}x(t)/\mathrm{d}t$, u is the control vector, and v is the vector of unknown disturbances; t_0 and ϑ are respectively the initial and the terminal instants of time; matrix functions $A(t)$, $A_h(t)$, $B(t)$ and $C(t)$ are continuous; $h = \text{const} > 0$ is the delay value; $x_t(\cdot)$ is the motion history on $[t-h, t]$ defined by $x_t(\xi) = x(t+\xi)$, $\xi \in [-h, 0]$; $C = C[-h, 0]$ is the set of all continuous functions from $[-h, 0]$ to $\mathbb{R}^n$.

It is assumed that admissible values of the control vector u and the disturbance vector v are restricted by the inclusions

⋆ This work is supported by the Russian Science Foundation (project no. 15–11–10018).

10.1016/j.ifacol.2015.11.079

$$u \in P = \{u \in \mathbb{R}^r : \|u\| \leq R\},$$
$$v \in Q = \{v \in \mathbb{R}^q : \|v\| \leq R\},$$

where the symbol $\|\cdot\|$ denotes the Euclidian norm and the constant $R > 0$ is sufficiently large in order to we can use results of (Krasovskii and Krasovskii, 1995, p. 179) (see also Lukoyanov (1994)).

Admissible control and disturbance realizations are Borel measurable functions

$$u[t_0[\cdot]\vartheta) = \{u(t) \in P,\, t_0 \leq t < \vartheta\},$$
$$v[t_0[\cdot]\vartheta) = \{v(t) \in Q,\, t_0 \leq t < \vartheta\}.$$

Such realizations uniquely generate a motion of system (1)

$$x[t_0 - h[\cdot]\vartheta] = \{x(t) \in \mathbb{R}^n,\, t_0 - h \leq t \leq \vartheta\}$$

that is an absolutely continuous function, which satisfies initial condition (2) and, together with $u(t)$ and $v(t)$, satisfies equation (1) for almost all $t \in [t_0, \vartheta]$. The triple $\{x[t_0-h[\cdot]\vartheta], u[t_0[\cdot]\vartheta), v[t_0[\cdot]\vartheta)\}$ is called a control process realization. The quality of this realization is evaluated by the index

$$\gamma = \Big(\int_{\vartheta-h}^{\vartheta} \|x(t)\|^2 \mathrm{d}t\Big)^{1/2} + \int_{t_0}^{\vartheta} \Big[\langle u(t), \Phi(t)u(t)\rangle - \langle v(t), \Psi(t)v(t)\rangle\Big]\mathrm{d}t. \quad (3)$$

Here the symbol $\langle\cdot,\cdot\rangle$ denotes the scalar product of vectors; $\Phi(t)$ and $\Psi(t)$ are symmetric continuous matrix functions such that the quadratic forms $\langle u, \Phi(t)u\rangle$ and $\langle v, \Psi(t)v\rangle$ are positive definite for $t \in [t_0, \vartheta]$.

The goal of the control is to minimize quality index (3). Let us note that, since disturbance actions are unknown, the worst-case may occur when disturbances maximize (3).

According to Osipov (1971); Krasovskii and Krasovskii (1995) the control problem (1)–(3) is posed as follows.

A control strategy $U(\cdot)$ is an arbitrary function

$$U(\cdot) = \{U(t, x_t(\cdot), \varepsilon) \in P,\, (t, x_t(\cdot)) \in [t_0, \vartheta] \times C,\, \varepsilon > 0\},$$

where $\varepsilon > 0$ is the accuracy parameter. The strategy $U(\cdot)$ acts onto system (1) in the discrete time scheme on the basis of a partition of the control interval $[t_0, \vartheta]$:

$$\Delta_\delta = \{\tau_j : \tau_1 = t_0,\, 0 < \tau_{j+1} - \tau_j \leq \delta,\, j = \overline{1,k},\, \tau_{k+1} = \vartheta\}. \quad (4)$$

A triple $\{U(\cdot), \varepsilon, \Delta_\delta\}$ defines a control law that forms a piecewise constant control realization according to the following step-by-step rule:

$$u(t) = U(\tau_j, x_{\tau_j}(\cdot), \varepsilon), \quad t \in [\tau_j, \tau_{j+1}), \quad j = \overline{1,k}.$$

Let us denote by $\Omega = \Omega(U(\cdot), \varepsilon, \Delta_\delta)$ the set of all control process realizations $\{x[t_0 - h[\cdot]\vartheta], u[t_0[\cdot]\vartheta), v[t_0[\cdot]\vartheta)\}$ such that $v[t_0[\cdot]\vartheta)$ is an admissible disturbance realization; $u[t_0[\cdot]\vartheta)$ is the control realization formed according to the law $\{U(\cdot), \varepsilon, \Delta_\delta\}$; $x[t_0 - h[\cdot]\vartheta]$ is the system motion generated by these realizations $u[t_0[\cdot]\vartheta)$ and $v[t_0[\cdot]\vartheta)$.

Let us define

$$\Gamma = \sup\Big\{\gamma : \{x[t_0 - h[\cdot]\vartheta], u[t_0[\cdot]\vartheta), v[t_0[\cdot]\vartheta)\} \in \Omega\Big\}.$$

Then, the optimal guaranteed result of the control is the following value:

$$\Gamma^0 = \inf_{U(\cdot)} \limsup_{\varepsilon\downarrow 0} \lim_{\delta\downarrow 0} \sup_{\Delta_\delta} \Gamma. \quad (5)$$

According to this definition, the value Γ^0 is the infimum of quality index values that can be ensured when the control scheme described above is used.

It is known (see, e.g., Osipov (1971)) that the infimum in (5) is attained. The corresponding strategy $U^0(\cdot)$ is called the optimal control strategy. Let us note that according to (5) the following property of the strategy $U^0(\cdot)$ is valid.

For any number $\zeta > 0$, there exist a number $\varepsilon^0 > 0$ and a function $\delta^0(\varepsilon) > 0$, $\varepsilon \in (0, \varepsilon^0]$, such that, for any value $\varepsilon \in (0, \varepsilon^0]$ and any partition Δ_δ (4) with $\delta \leq \delta^0(\varepsilon)$, the control law $\{U^0(\cdot), \varepsilon, \Delta_\delta\}$ ensures the inequality

$$\gamma \leq \Gamma^0 + \zeta \quad (6)$$

for any admissible disturbance realization $v[t_0[\cdot]\vartheta)$.

The problem under consideration is to find the value of the optimal guaranteed result and to construct a control scheme that ensures inequality (6).

In Sections 4 and 5 two solution methods for this problem are given. In both methods the control problem (1)–(3) is reduced to an auxiliary differential game of a special type. So, before we describe these methods let us consider in the next Section a differential game of this type.

3. AUXILIARY DIFFERENTIAL GAME

In this Section one of the classical problems of the positional differential games theory is considered. An effective solution to this problem is described. Detailed exposition can be found in Lukoyanov (1994); Krasovskii and Krasovskii (1995).

A zero-sum two-person differential game is described by the dynamical system

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)u(t) + \mathbf{C}(t)v(t),$$
$$t_0 \leq t < \vartheta, \quad \mathbf{x} \in \mathbb{R}^{\mathbf{n}}, \quad u \in P, \quad v \in Q, \quad (7)$$

with the initial condition

$$\mathbf{x}(t_0) = \mathbf{x}_0, \quad (8)$$

and the quality index

$$\boldsymbol{\gamma} = \|\mathbf{D}\mathbf{x}(\vartheta)\| + \int_{t_0}^{\vartheta} \Big[\langle u(t), \Phi(t)u(t)\rangle - \langle v(t), \Psi(t)v(t)\rangle\Big]\mathrm{d}t. \quad (9)$$

Here $\mathbf{x}$ is the state vector, u is the control vector of the first player, and v is the control vector of the second player; matrix functions $\mathbf{A}(t)$, $\mathbf{B}(t)$ and $\mathbf{C}(t)$ are piecewise-continuous; $\mathbf{D}$ is a constant $(\mathbf{d} \times \mathbf{n})$-matrix $(1 \leq \mathbf{d} \leq \mathbf{n})$. For the meanings of the other symbols see Section 2.

The first player aims to minimize index (9), while the second player aims to maximize it.

A pair of admissible realizations $u[t_0[\cdot]\vartheta)$ and $v[t_0[\cdot]\vartheta)$ uniquely generate a motion of system (7)