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Successive Approximations of Model Reference Adaptive Control Design for Nonlinear Systems

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Abstract: In this study, model reference adaptive control algorithm with successive approximation method is suggested for nonlinear systems. The proposed method uses Linear Time Varying (LTV) approximations of the nonlinear model to design the controller. Provided that an adaptive control exists for the approximated LTV system, it is shown that the responses of the approximated LTV systems converge to the response of the nonlinear system. Then the model reference adaptive control for nonlinear system is designed by using successive LTV approximations. The proposed control design method is exemplified with a nonlinear dynamical system.

Keywords: Nonlinear systems, model reference adaptive control, successive approximations technique, linear time varying systems, adaptive control.

1. INTRODUCTION

One of the main objectives of adaptive control method is to deal with plant uncertainty and/or time varying plant parameters. The basic concept in the adaptive control is to design the controller which adapts itself to plant uncertainty or time variance in the plant dynamics (Ioannou and Sun (1996)). Within the adaptive control methodologies, Model Reference Adaptive Control (MRAC) is widely studied to adjust the output of unknown plant for a known model output. Thus the output of the plant is compared with the output of the reference model and the error is minimized by the MRAC design (Astrom and Wittenmark (1995)).

MRAC has been used for linear time invariant (LTI) systems first and extended to slowly time varying systems (Tsakalis and Ioannou (1987)). There are also studies for continuous-time systems (Ioannou and Sun (1996). Astrom and Wittenmark (1995). Tsakalis and Ioannou (1987). Narendra and Annaswamy (1989). Tao (2003)) and discrete-time systems (Goodwin et. al. (1980). Goodwin and Sin (1984). Akhtar and Bernstein (2005). Hoagg et. al. 2008)). There are also some studies which consider LTI reference models for nonlinear dynamical systems (Nam and Arapostathis (1988). Jagannathan et al (1994). Lee et al. (1995)).

SDRE based MRAC is proposed for nonlinear systems in which the nonlinear model and plant dynamics are frozen at each time increment and then adaptive control theory for LTI systems is applied to the model and plant systems (Babaei and Salamci (2015)). It has been shown that nonlinear reference models for nonlinear plants may result in faster adaptation than LTI reference models for nonlinear systems. Although SDRE methodology simplifies the controller design, it is well known that it guarantees only the local stability. Therefore, there are still limited interests for the implementation of SDRE control methods. Another approach to the controller design for nonlinear systems which is factorized by state dependent coefficient (SDC) matrices is the successive approximation approach. The approach is based on LTV approximations of the nonlinear system which allows one to design the controller for the approximated LTV systems. The method is well defined and applied to nonlinear systems with different control methods (Salamci et al. (2000), Tomás-Rodríguez and Banks (2010), Bilgin and Salamci (2014), Bilgin and Salamci (2011)).

The main contribution of this paper is to suggest a nonlinear reference model for the adaptive control of nonlinear plants. Therefore we propose a method for MRAC design for a class of nonlinear systems. We consider a nonlinear reference model having a stable dynamics and the nonlinear plant output is controlled with the proposed MRAC method. The successive approximations of nonlinear reference model and plant are generated in order to convert the nonlinear design problem to the design of LTV systems. Assuming that there exists an adaptive control for the LTV systems, it is shown that the responses of approximated systems converge to the response of nonlinear system. By designing MRAC for LTV approximations, the design is performed for the nonlinear system.

The organization of the paper is as follows; in section II, the mathematical preliminaries are given. Section III gives the main results of this paper, which is used to design MRAC for nonlinear system. In Section IV, the proposed method is applied to a nonlinear system. Section V draws the conclusions.

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2. PRELIMINARIES

The successive approximation approach is studied by many researchers and well documented in (Tomás-Rodríguez and Banks (2010)). The basic results of this technique are given here for the sake of completeness.

Theorem 1 (Salamci, Ozgoren and Banks (2000)): Consider the following type of nonlinear systems

$$\dot{x} = A(x)x \tag{1}$$

The response of the following linear successive approximations converges to the response of the nonlinear system provided that $A(x)$ is Lipschitz

$$\dot{x}^{[i]} = A\left(x^{[i-1]}(t)\right)x^{[i]}, \quad x^{[i]}(0) = x(0), for\ i = 1,2,\ldots \tag{2}$$

Theorem 2 (Tomás-Rodríguez and Banks (2010)): Consider the following nonlinear system and its successive approximations

$$\dot{x} = A(x)x + B(x)u \tag{3}$$

$$\dot{x}^{[i]} = A\left(x^{[i-1]}(t)\right)x^{[i]} + B\left(x^{[i-1]}(t)\right)u^{[i]},$$

$$x^{[i]}(0) = x(0), for\ i = 1,2,\ldots \tag{4}$$

Then the response of (4) converges to the response of (3), if the control is in the form of

$$u^{[i]} = -K\left(x^{[i-1]}(t)\right)x^{[i]} \tag{5}$$

The above theorems allow one to design controller for the LTV approximations with known methods such as optimal control, pole placement and even sliding mode control. The controller designed for the LTV approximations eventually converges to the controller for the nonlinear system provided that a nonlinear control exists for the nonlinear system.

The idea may be extended to adaptive control. The following example is barrowed from (Marino and Tomei (1995)) to illustrate the approach.

Example (Marino and Tomei (1995)). Consider the following nonlinear system.

$$\dot{x} = f(x) + q\left(x, \theta(t)\right) + q(x)u \tag{6}$$

where

$$\dot{x}_1 = x_2 + x_1^{\theta} \quad 1 \leq \theta \leq 2$$

$$\dot{x}_2 = u \tag{7}$$

where θ is unknown. Marino and Tomei (Marino and Tomei (1995)) suggested the following adaptive control for the system;

$$v_1(x_1) = -k_1x_1 - x_1\alpha_1(x_1) \tag{8}$$

$$\alpha_1(x_1) = \sqrt{1 + x_1^2} \geq \frac{x_1^{\theta}}{x_1} = \Psi_1(x_1, \theta)\ for\ all\ \theta \in [1,2] \tag{9}$$

$$\tilde{x}_2 = x_2 - v_1(x_1) \tag{10}$$

and the control is

$$u = -x_1 - k_2\tilde{x}_2 - \tfrac{1}{4}\tilde{x}_2\alpha_2^2(x_1, x_2) \tag{11}$$

$$\alpha_2(x_1, x_2) = 2 + 4x_1^2 + -2k_1\sqrt{1 + x_1^2} \tag{12}$$

The nonlinear system can also be expressed as in the form of (3) as follows;

$$\dot{x} = A(x, \theta(t))x + B(x)u \tag{13}$$

For the given system;

$$\dot{x} = \underbrace{\begin{bmatrix} x_1^{\theta-1} & 1 \\ 0 & 0 \end{bmatrix}}_{A(x,\theta)} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B(x)} u \qquad 1 \leq \theta \leq 2 \tag{14}$$

Now consider the following type of successive approximations

$$\dot{x}^{[i]} = A\left(x^{[i-1]}, \theta(t)\right)x^{[i]} + B(x^{[i-1]})u^{[i]} \tag{15}$$

$$u^{[i]} = -k(x^{[i-1]})x^{[i]} \tag{16}$$

Notice that the controller is in the form of nonlinear state feedback as;

$$u = -K(x)x \tag{17}$$

where the nonlinear state feedback is

$$K(x) = [1 + k_1k_2 + \alpha_1k_2 + \frac{1}{4}\alpha_2^2k_1 + \frac{1}{4}\alpha_1\alpha_2^2, k_2 + \frac{1}{4}\alpha_2^2]$$

By using Theorem 2, we re-design the controller as follows;

$$\dot{x}^{[i]} = \begin{bmatrix} (x^{[i-1]})_1^{\theta-1} & 1 \\ 0 & 0 \end{bmatrix} x^{[i]} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u^{[i]}$$

$$u^{[i]} = -K(x^{[i-1]})x^{[i]}$$

Fig.1 shows the simulation results of two methods. For the simulations, θ is taken as 2. Only three iterations (successive approximations) are performed in this particular example. It is seen that the same system responses are obtained with the successive approximations. Adaptive control input of (Marino and Tomei (1995)) Equation (11) is given in Fig. 2 and adaptive control input obtained via successive approximations (16) is given in Fig. 3.